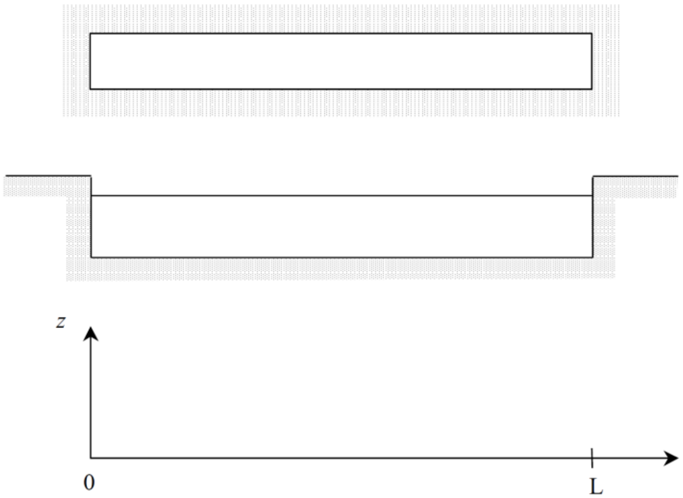
Deveto predavanje
(13. svibnja 2022.)

Generiranje seša
u uskom pravokutnom bazenu
Polazne jednadžbe

$$\frac{\partial U}{\partial t} = -g\frac{\partial \zeta}{\partial x} + \frac{1}{\rho D}\left[\tau_x - \tau_{xD}\right]$$

$$\frac{\partial V}{\partial t} = -g\frac{\partial \zeta}{\partial y} + \frac{1}{\rho D}\left[\tau_y - \tau_{yD}\right]$$

$$\frac{\partial (DU)}{\partial x} + \frac{\partial (DV)}{\partial y} + \frac{\partial \zeta}{\partial t} = 0$$



Zanemarujemo pridneno trenje,
pretpostavljamo $V = 0$ i $D = \text{const.}$

Situacija prije isključivanja vjetra, uz postignuto stacionarno stanje

$$-\infty < t < 0$$

$$0 = -g\frac{\partial \zeta}{\partial x} + \frac{\tau_x}{\rho D}$$

$$\frac{\partial U}{\partial x} = 0\,.$$

$$\zeta = \frac{\tau_x}{g\rho D}\left(x - \frac{L}{2}\right),$$

$$U = 0\,.$$

Situacija nakon isključivanja vjetra

$$\mathbf{0} \leq \boldsymbol{t} < \infty$$

$$\frac{\partial U}{\partial t} = -g \frac{\partial \zeta}{\partial x}$$

$$D \frac{\partial U}{\partial x} + \frac{\partial \zeta}{\partial t} = 0 \; .$$

$$\zeta(t=0) = \frac{\tau_x}{g \rho D}\left(x - \frac{L}{2}\right),$$

$$U(t=0) = 0 \, , \;\; \forall x \, ,$$

$$U(x = 0, L) = 0 \, , \;\; \forall t \; .$$

Rješavanje

$$\frac{\partial^2 \zeta}{\partial t^2} - gD\frac{\partial^2 \zeta}{\partial x^2} = 0$$

$$\zeta(t=0) = \frac{\tau_x}{g\rho D}\left(x - \frac{L}{2}\right),$$

$$\frac{\partial \zeta}{\partial t}(t=0) = 0\,, \ \forall x\,,$$

$$\frac{\partial \zeta}{\partial x}(x=0,L) = 0\,, \ \forall t\,.$$

Rješenje (1)

$$\zeta = \sum_{n=1}^{\infty} A_n \cos\frac{n\pi}{L} x \cos\sigma_n t$$

$$\sigma_n = \frac{n\pi}{L}\sqrt{gD}\,, \qquad n = 1,2,3,\ldots$$

$$\sum_{n=1}^{\infty} A_n \cos\frac{n\pi}{L} x = \frac{\tau_x}{g\rho D}\left(x - \frac{L}{2}\right)$$

Rješenje (2)

$$\zeta = \frac{\tau_x}{g\rho D}\frac{4L}{\pi^2}\left(-\cos\frac{\pi}{L}x\cos\sigma_1 t - \frac{1}{9}\cos\frac{3\pi}{L}x\cos\sigma_3 t - \ldots\right)$$

$$U = -\frac{\tau_x}{g\rho D^2}\frac{4L^2}{\pi^3}\left(\sigma_1\sin\frac{\pi}{L}x\sin\sigma_1 t + \frac{\sigma_3}{27}\sin\frac{3\pi}{L}x\sin\sigma_3 t + \ldots\right)$$