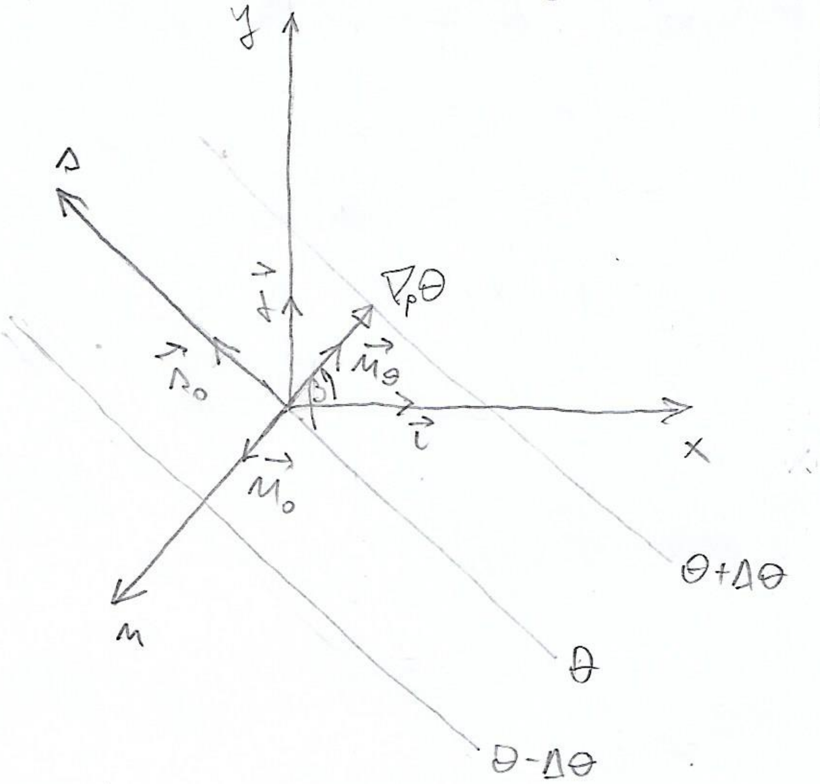
FRONTOGENETIČKI VEKTOR

- do sada smo promatrali frontogenetičku fju $F=\frac{d}{dt}|\nabla_H\theta|$ koja opisuje samo promjenu intenziteta kvazihorizontalnog gradijenta θ-e
- ako sada uključimo i promjenu njegovog smjera, dobije se frontogenetički vektor!

$$\boxed{\vec{F}=\frac{d}{dt}(\nabla_H\theta)}$$ ← FG vektor (za razliku od FG fje, nema aps. vrijednosti)

- odabiremo prirodni koordinatni sustav sn ⇒ s... smjer duž izentropa, n... smjer negativnog gradijenta pot. temperature

$$\boxed{\vec{F}=F_s\vec{s}_0+F_n\vec{n}_0} \leftarrow (*)$$



- raspis:

$$\vec{F}=\frac{d}{dt}(\nabla_H\theta)=\frac{\partial}{\partial t}(\nabla_H\theta)+u\frac{\partial}{\partial x}(\nabla_H\theta)+v\frac{\partial}{\partial y}(\nabla_H\theta)=\nabla_H\left(\frac{\partial\theta}{\partial t}\right)+u\nabla_H\left(\frac{\partial\theta}{\partial x}\right)+v\nabla_H\left(\frac{\partial\theta}{\partial y}\right)=$$

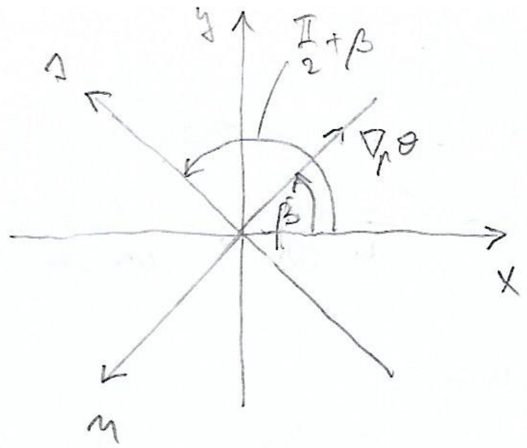
$$=\underbrace{\nabla_H\left(\frac{\partial\theta}{\partial t}\right)+\nabla_H\left(u\frac{\partial\theta}{\partial x}\right)+\nabla_H\left(v\frac{\partial\theta}{\partial y}\right)}_{\nabla_H\left(\frac{d\theta}{dt}\right)}-\frac{\partial\theta}{\partial x}\nabla_H u-\frac{\partial\theta}{\partial y}\nabla_H v$$

$$\Rightarrow \vec{F}=\nabla_H\left(\frac{d\theta}{dt}\right)-\left(\frac{\partial\theta}{\partial x}\nabla_H u+\frac{\partial\theta}{\partial y}\nabla_H v\right)$$

- uz ~~pp~~ adijabatičnost $\left(\frac{d\theta}{dt}=0\right)$ ⇒ $\boxed{\vec{F}=-\left[\frac{\partial\theta}{\partial x}\left(\frac{\partial u}{\partial x}\vec{i}+\frac{\partial u}{\partial y}\vec{j}\right)+\frac{\partial\theta}{\partial y}\left(\frac{\partial v}{\partial x}\vec{i}+\frac{\partial v}{\partial y}\vec{j}\right)\right]}$

- napravimo (*) skalarno sa $\vec{s}_0$, a zatim sa $\vec{n}_0$:

$$\vec{F}=F_s\vec{s}_0+F_n\vec{n}_0 \,/\, \vec{s}_0, \vec{n}_0 \Rightarrow F_s=\vec{F}\vec{s}_0 \quad ; \quad F_n=\vec{F}\vec{n}_0$$



$$\vec{s}_0 = \vec{i}\cos\left(\frac{\pi}{2}+\beta\right) + \vec{j}\sin\left(\frac{\pi}{2}+\beta\right) = -\sin\beta\,\vec{i} + \cos\beta\,\vec{j}$$

$$\vec{n}_0 = -\vec{i}\sin\left(\frac{\pi}{2}+\beta\right) + \vec{j}\cos\left(\frac{\pi}{2}+\beta\right) = -\cos\beta\,\vec{i} - \sin\beta\,\vec{j}$$

$$\frac{\partial\theta}{\partial x} = |\nabla_H\theta|\cos\beta \;;\quad \frac{\partial\theta}{\partial y} = |\nabla_H\theta|\sin\beta$$

$$F_s = \vec{F}\cdot\vec{s}_0 = +\left[|\nabla_H\theta|\cos\beta\left(\frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j}\right) + |\nabla_H\theta|\sin\beta\left(\frac{\partial v}{\partial x}\vec{i} + \frac{\partial v}{\partial y}\vec{j}\right)\right]\left[+\left(\sin\beta\,\vec{i} - \cos\beta\,\vec{j}\right)\right]$$

$$= |\nabla_H\theta|\cos\beta\frac{\partial u}{\partial x}\sin\beta + |\nabla_H\theta|\sin\beta\frac{\partial v}{\partial x}\sin\beta - |\nabla_H\theta|\cos\beta\frac{\partial u}{\partial y}\cos\beta - |\nabla_H\theta|\sin\beta\frac{\partial v}{\partial y}\cos\beta =$$

$$= |\nabla_H\theta|\sin\beta\cos\beta\,\frac{1}{2}(\delta+D) + |\nabla_H\theta|\sin^2\beta\,\frac{1}{2}\zeta - |\nabla_H\theta|\cos^2\beta\left(-\frac{1}{2}\zeta\right) - |\nabla_H\theta|\sin\beta\cos\beta\,\frac{1}{2}(\delta-D) =$$

$$= |\nabla_H\theta|\left[\frac{1}{2}\sin\beta\cos\beta\left(\cancel{\delta}+D-\cancel{\delta}+D\right) + \frac{1}{2}\zeta\underbrace{\left(\sin^2\beta+\cos^2\beta\right)}_{1}\right] =$$

$$= |\nabla_H\theta|\left(D\sin\beta\cos\beta + \frac{1}{2}\zeta\right)$$

$$\Rightarrow \boxed{F_s = \frac{1}{2}|\nabla_H\theta|\left(D\sin 2\beta + \zeta\right)}$$

$$F_n = \vec{F}\cdot\vec{n}_0 = +|\nabla_H\theta|\left[\cos\beta\left(\frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j}\right) + \sin\beta\left(\frac{\partial v}{\partial x}\vec{i} + \frac{\partial v}{\partial y}\vec{j}\right)\right]\left[+\left(\cos\beta\,\vec{i} + \sin\beta\,\vec{j}\right)\right]$$

$$= |\nabla_H\theta|\left(\cos^2\beta\frac{\partial u}{\partial x} + \sin\beta\cos\beta\frac{\partial v}{\partial x} + \sin\beta\cos\beta\frac{\partial u}{\partial y} + \sin^2\beta\frac{\partial v}{\partial y}\right) =$$

$$= |\nabla_H\theta|\left[\frac{1}{2}(\delta+D)\cos^2\beta + \cancel{\frac{1}{2}\zeta\sin\beta\cos\beta} - \cancel{\frac{1}{2}\zeta\sin\beta\cos\beta} + \frac{1}{2}(\delta-D)\sin^2\beta\right] =$$

$$= \frac{1}{2}|\nabla_H\theta|\left[\delta\underbrace{\left(\sin^2\beta+\cos^2\beta\right)}_{1} + D\underbrace{\left(\cos^2\beta-\sin^2\beta\right)}_{\cos 2\beta}\right]$$

$$\Rightarrow \boxed{F_n = \frac{1}{2}|\nabla_H\theta|\left(D\cos 2\beta + \delta\right)}$$

$\Rightarrow$ vidimo da je F_n jednak negativnoj vrijednosti frontogenetičke fje:
$\boxed{F_n = -F}$ $\Rightarrow$ F_n označava promjenu iznosa gradijenta pot. temperature

- F_s označava promjenu smjera gradijenta pot. temperature $\Rightarrow$ ζ ne utječe na samu frontogenetičku fju, ali doprinosi frontogenezi

FRONTOGENEZA i $\vec{Q}$ VEKTOR

- neka je polarni koord. sustav ujedno i prirodni (ns) ⇒ tada je:

$$\vec{F} = -\left[\underbrace{\cancel{\frac{\partial\theta}{\partial s}}}_{0}\left(\frac{\partial u_g}{\partial s}\vec{s}_0 + \frac{\partial u_g}{\partial n}\vec{n}_0\right) + \frac{\partial\theta}{\partial n}\left(\frac{\partial v_g}{\partial s}\vec{s}_0 + \frac{\partial v_g}{\partial n}\vec{n}_0\right)\right] = -\frac{\partial\theta}{\partial n}\frac{\partial v_g}{\partial s}\vec{s}_0 - \frac{\partial\theta}{\partial n}\frac{\partial v_g}{\partial n}\vec{n}_0$$

- definicija $\vec{Q}$ vektora u prirodnim koordinatama:

$$\vec{Q} = \frac{R}{\sigma p}\left(\frac{p}{p_0}\right)^{\frac{R}{c_p}}\left[\left(-\frac{\partial v_g}{\partial s}\frac{\partial\theta}{\partial n}\right)\vec{s}_0 + \left(-\frac{\partial v_g}{\partial n}\frac{\partial\theta}{\partial n}\right)\vec{n}_0\right] = Q_s\vec{s}_0 + Q_n\vec{n}_0$$

$$\Rightarrow \boxed{\vec{F} = \frac{\sigma p}{R}\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}}\vec{Q}}$$

- budući da vrijedi $\vec{n}_0 = -\frac{\nabla_H\theta}{|\nabla_H\theta|}$ te da je $F_n = \vec{F}\cdot\vec{n}_0$ i $F = -F_n$

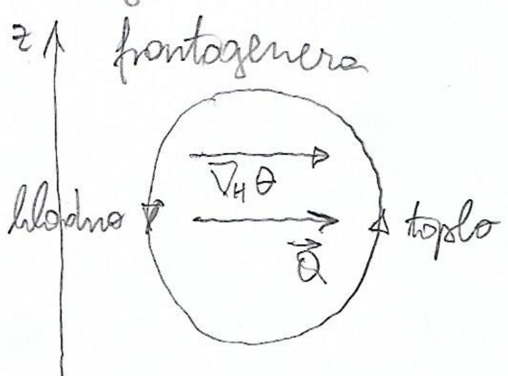
$$\Rightarrow F_n = \frac{\sigma p}{R}\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}}\vec{Q}\cdot\left(-\frac{\nabla_H\theta}{|\nabla_H\theta|}\right) \Rightarrow \boxed{F = \frac{\sigma p}{R|\nabla_H\theta|}\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}}(\nabla_H\theta\cdot\vec{Q})}$$

- u statički stabilnoj atmosferi ($\sigma > 0$), uz uvjet da je $\nabla_H\theta\cdot\vec{Q} > 0$ će se odvijati frontogeneza, a za $\nabla_H\theta\cdot\vec{Q} < 0$ frontoliza
- smjer $\vec{Q}$ vektora nam ukazuje na područja uzlaznog/silaznog gibanja:

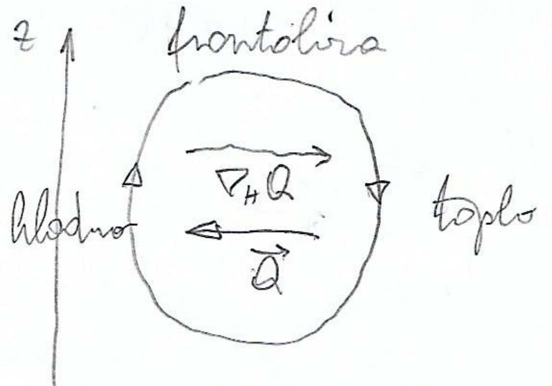
$$\otimes \xrightarrow{\vec{Q}} \odot$$

⇒ područje na koje $\vec{Q}$ pokazuje je područje uzlaznog gibanja, a područje iz kojeg dolazi je područje silaznog gibanja

- frontogeneza je prisutna kod direktne termalne cirkulacije kada se topli zrak diže, a hladniji spušta, dok je frontoliza prisutna kod indirektne termalne cirkulacije ⇒ topliji zrak se spušta, a hladniji diže



$\nabla_H\theta\cdot\vec{Q} > 0$



$\nabla_H\theta\cdot\vec{Q} < 0$