Diskretna 3D fotonička rešetka u sustavu 2D vezanih valovoda

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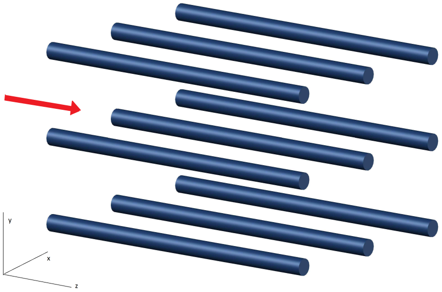
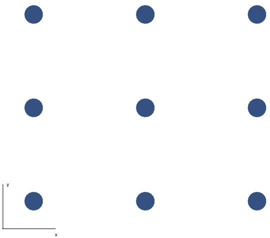
Problem

- Promatra se sustav slabo vezanih valovoda translacijski invarijantan u smjeru z te reducirane translacijske invarijantnosti u okomitoj (x-y) ravnini (fotonička rešetka)
- Rešetke i drugi diskretni sustavi mogu imati dimenziju veću od kontinuiranog prostora u kojem se nalaze pod određenim uvjetima $\Rightarrow$ traži se kako teoretski ostvariti te uvjete u sustavu 2D valovoda i dobiti 3D rešetku u presjeku sustava

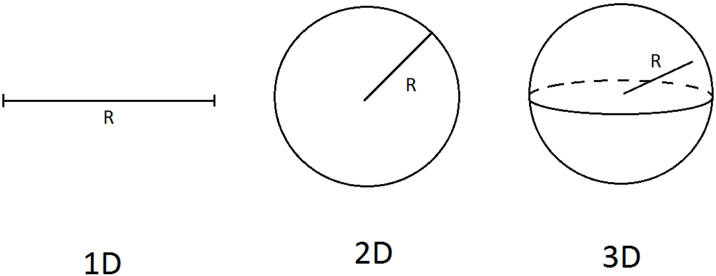


Promatrani sustav



Presjek sustava

Dimenzionalnost u jednostavnim kontinuiranim sustavima

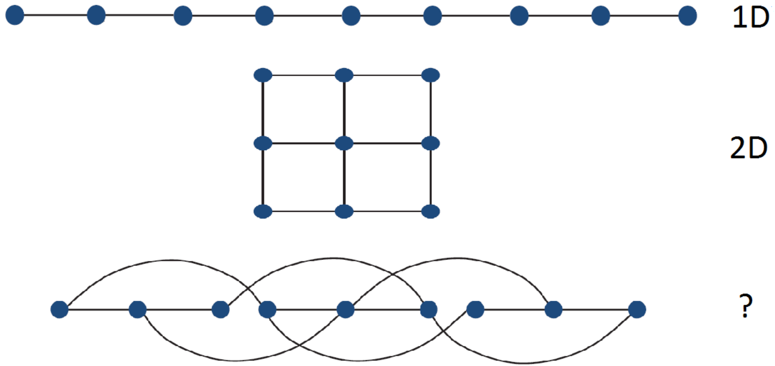


$V \sim R$ $\qquad$ $V \sim R^2$ $\qquad$ $V \sim R^3$

- Volumen u kontinutiranim sustavima:

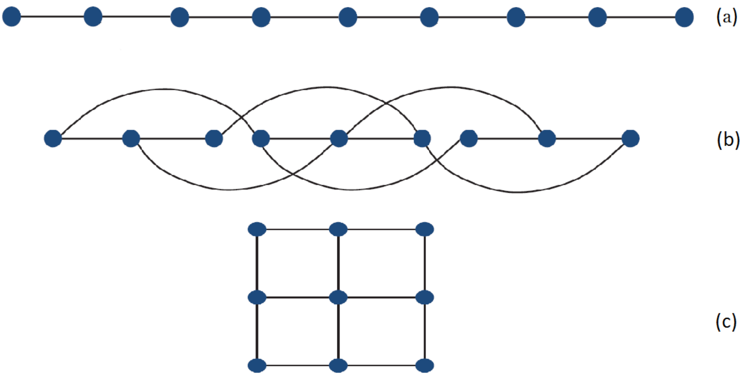
$$\text{dD} : V \sim R^d$$

Dimenzionalnost u složenim diskretnim sustavima



Matematička teorija

- Matematički: Rešetka $\rightarrow$ graf s prostornom grupom; graf - sustav točaka (vrhova) povezan crtama (bridovima)
- Volumen u diskretnim sustavima $\longrightarrow$ broj vrhova N unutar l koraka od jednog brida
  Analogno kontinuiranim sustavima:
  $$\text{dD} : N \sim l^d$$
- Ekvivalentna definicija preko metode brojanja kutija:
  $$M \sim l^{-d}$$
  M - minimalni broj kutija potrebnih za prekrivanje sustava
- d općenito nije jednak dimenziji kontinuiranog prostora u kojem se diskretni sustav nalazi $\Rightarrow$ govori o složenosti sustava



(a) 1D sustav u 1D prostoru, (b) 2D sustav u 1D prostoru, (c) 2D sustav u 2D prostoru

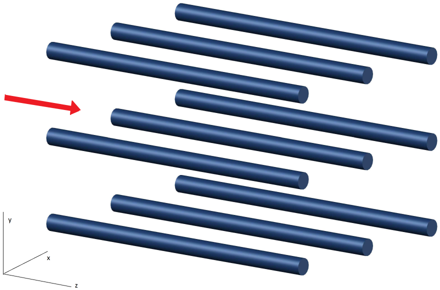
⇒ Treba prekinuti vezu između bliskih valovoda i uspostaviti je između dalekih

Svjetlost u fotoničkim sustavima

- Maxwellove jednadžbe u nemagnetskom linearnom materijalu bez slobodnih struja i naboja:

$$
\begin{aligned}
\vec{\nabla} \cdot (\epsilon(x, y)\vec{E}) &= 0 \\
\vec{\nabla} \cdot \vec{H} &= 0 \\
\vec{\nabla} \times \vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t} \\
\vec{\nabla} \times \vec{H} &= \epsilon(x, y) \frac{\partial \vec{E}}{\partial t}
\end{aligned}
$$

- Svođenje na dvije jednadžbe: jednu za $\vec{E}$ i drugu za određivanje $\vec{H}$ iz poznatog $\vec{E}$



- Rotacija Faradayevog zakona:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t} \vec{\nabla} \times \vec{H}$$

- Lijeva strana:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E}$$

- Desna strana prema Ampereovom zakonu:

$$-\mu_0 \frac{\partial}{\partial t} \vec{\nabla} \times \vec{H} = -\mu_0 \epsilon(x, y) \frac{\partial^2 \vec{E}}{\partial t^2}$$

- Ukupno, jednadžba za $\vec{E}$:

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E} = -\mu_0 \epsilon(x, y) \frac{\partial^2 \vec{E}}{\partial t^2}$$

Sređivanje jednadžbe za $\vec{E}$

- Prema Gaussovom zakonu:

$$\vec{\nabla} \cdot (\epsilon(x, y)\vec{E}) = \vec{E} \cdot \vec{\nabla}\epsilon(x, y) + \epsilon(x, y)\vec{\nabla} \cdot \vec{E} = 0$$

- Ako polje oscilira mnogo brže od dielektrične funkcije:

$$\epsilon(x, y)\vec{\nabla} \cdot \vec{E} \approx 0$$

  $\Longrightarrow$ polje je otprilike transverzalno

- U jednadžbi za $\vec{E}$ eliminira se prvi član:

$$\vec{\nabla}^2\vec{E} = \mu_0\epsilon(x, y)\frac{\partial^2\vec{E}}{\partial t^2}$$

- Ovo je poznata valna jednadžba

Vremenska ovisnost i jednadžba za $\vec{H}$

- Pretpostavi se oscilatorna vremenska ovisnost:

$$\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{-i\omega t}$$
$$\vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{-i\omega t}$$

- Uz $c^2 = \frac{1}{\mu_0 \epsilon_0}$:

$$\vec{\nabla}^2 \vec{E} = -\frac{\omega^2}{c^2}\epsilon_r(x,y)\vec{E} \tag{1}$$

- Uvrštavanjem ovisnosti polja $\vec{H}$ u Faradayev zakon i izražavanjem polja:

$$\vec{H} = \frac{1}{i\omega\mu_0}\vec{\nabla} \times \vec{E} \tag{2}$$

- Jednadžbe (1) i (2) su tražene jednadžbe.

Sređivanje jednadžbe za $\vec{E}$

- Pretpostavka oblika za $\vec{E}$:

$$\vec{E} = \psi(x, y, z)e^{ikz}\hat{e}$$

- Uvrštavanjem u (1) i sređivanjem:

$$\vec{\nabla}^2\psi + 2ik\hat{z}\cdot\vec{\nabla}\psi - k^2\psi = -\frac{\omega^2}{c^2}\epsilon_r(x, y)\psi$$

- Indeks loma: $n(x, y) = \sqrt{\epsilon_r(x, y)} = n_0 + \delta n(x, y)$
Pretpostavimo $n_0 >> \delta n(x, y) \Longrightarrow$ ostavljaju se samo doprinosi prvog reda u $\delta n \Longrightarrow \epsilon_r(x, y) \approx n_0^2 + 2n_0\delta n(x, y)$
- ω i k se mogu povezati disperzijskom relacijom za homogeni dielektrik:

$$\omega = c\frac{k}{n_0}$$

- Najniži red varijacije omjera $\frac{k}{n}$ je najmanje drugi pa se odbacuje

Paraaksijalna aproksimacija

- Eliminacijom ω i ϵ_r te sređivanjem:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + 2ik\frac{\partial\psi}{\partial z} = -2k^2\frac{\delta n}{n_0}\psi$$

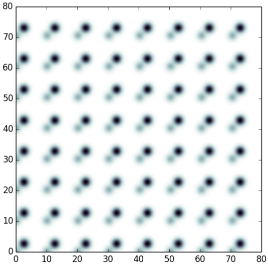
- Paraaksijalna aproksimacija: $\left|\frac{\partial^2\psi}{\partial z^2}\right| << \left|2ik\frac{\partial\psi}{\partial z}\right|$

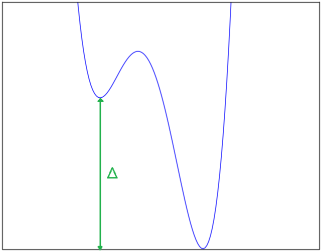
$$\Rightarrow i\frac{\partial\psi}{\partial z} = -\frac{1}{2k}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi - k\frac{\delta n}{n_0}\psi$$

- Jednadžba za ψ, a time i $\vec{E}$ svodi se na 2D vremenski ovisnu Schrödingerovu jednadžbu $\rightarrow$ mogućnost modeliranja QM sustava

Presjek sustava valovoda

$$\delta n_L = \delta n_{L0} \sum_{mn} e^{-\frac{(\vec{r}-\vec{R}_{mn})^2}{2\sigma^2}} + (\delta n_{L0} + \Delta) \sum_{mn} e^{-\frac{(\vec{r}-\vec{R}_{mn}-\vec{b})^2}{2\sigma^2}}$$

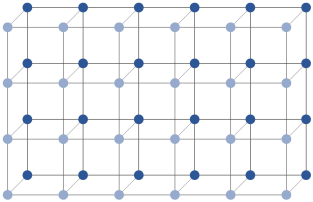




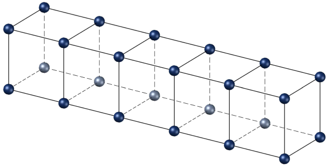
- Jačina vezanja mjesta *mn* i *m′n′* u aproksimaciji slabog vezanja valovoda: $J \sim \sum_{jk} \int \chi_j^{(mn)} \delta n \chi_k^{(m'n')*} \mathrm{d}V$

Ideja

- Treba pronaći konstante potencijala takve da je veza jaka samo između elemenata baze te između elemenata baze na istim mjestima u različitim ćelijama → rešetka je ekvivalentna sloju tetragonskih ćelija
- Veza slabi s udaljenošću (manji preklop) i razlikom u indeksu loma (izlazak iz rezonancije)



Shema traženog presjeka rešetke



Ekvivalentni sloj tetragonskih ćelija

Konstante

- Izaberu se konstante i prati intenzitet $I \sim \vec{E}^2 \sim |\psi|^2$ u ovisnosti o z:
  - Valna duljina:
    $\lambda = 500 \times 10^{-3} \mu\text{m} \Rightarrow k = \frac{2\pi n_0}{\lambda} \approx 28.9 \mu\text{m}^{-1}$
  - $n_0 = 2.3$ (indeks loma stroncij-barij niobata $Sr_x Ba_{1-x} Nb_2 O_6$)
  - $\delta n_{L0} = 2 \times 10^{-3}$
  - $a = 10 \mu\text{m}$
  - $\sigma = 0.1a$
  - $b_x = b_y = 0.25a$
  - $z_0 = 0$
  - $z_{max} = 20\text{mm}$
  - $\Delta z = 1 \mu\text{m}$
- Evolucija se računa numerički u Pythonu implementacijom *Split step FFT* metode

Split-step FFT metoda

- Operator evolucije:

$$
\begin{aligned}
i\frac{\partial}{\partial z}\psi(x,y,z) &= \hat{H}\psi(x,y,z) \\
\psi(x,y,z_0+\Delta z) &= e^{-i\hat{H}\Delta z}\psi(x,y,z_0)
\end{aligned}
$$

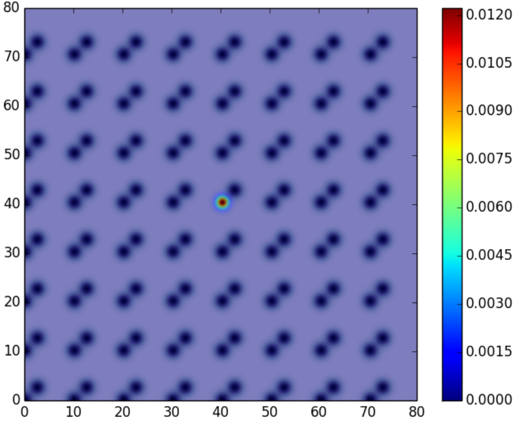
$\hat{H} = -\frac{1}{2k}\vec{\nabla}^2 - \frac{k\delta n}{n_0}$

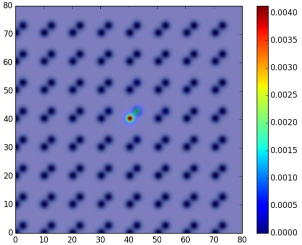
- Prvi član operatora dijagonalan je u recipročnom prostoru, dok je drugi dijagonalan u direktnom prostoru
- Split step metoda:
  1. Pola koraka u direktnom prostoru
  2. FFT
  3. Cijeli korak u recipročnom prostoru
  4. IFFT
  5. Pola koraka u direktnom prostoru
- Metoda je stabilna i unitarna; rastavljanje operatora daje pogrešku reda veličine Δz^3

Evolucija za $\Delta = 0$

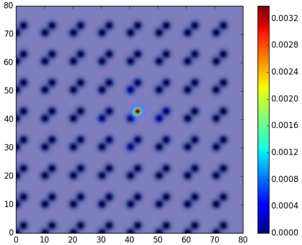
- Početni uvjet:

$$\psi(x, y, z = 0) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-4a)^2}{2\sigma^2} - \frac{(y-4a)^2}{2\sigma^2}}$$

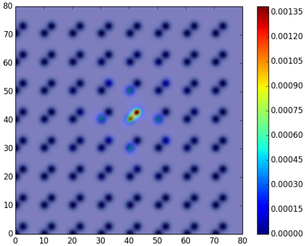




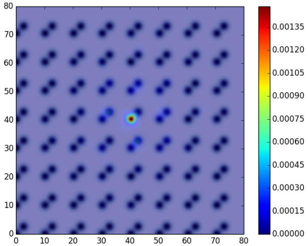
$z = 2.5$mm



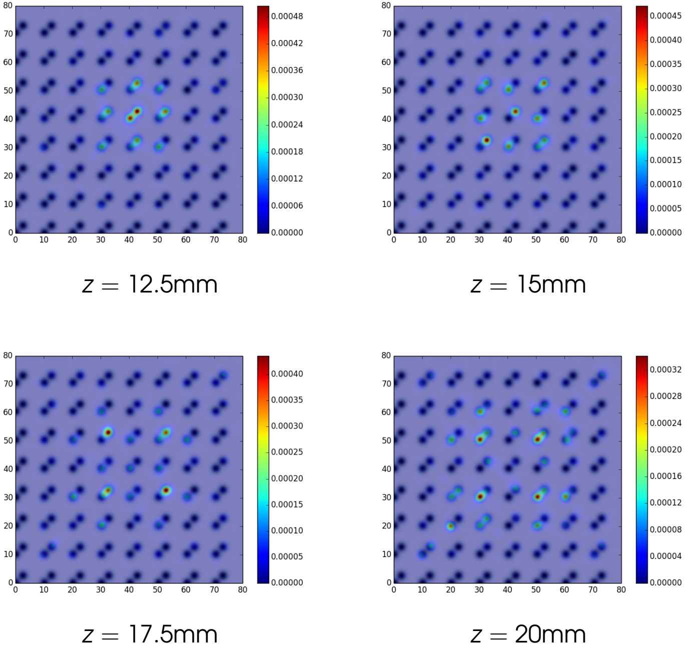
$z = 5$mm



$z = 7.5$mm



$z = 10$mm



80
70
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0 10 20 30 40 50 60 70 80
0.00048
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$z = 12.5$mm
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$z = 15$mm
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$z = 17.5$mm
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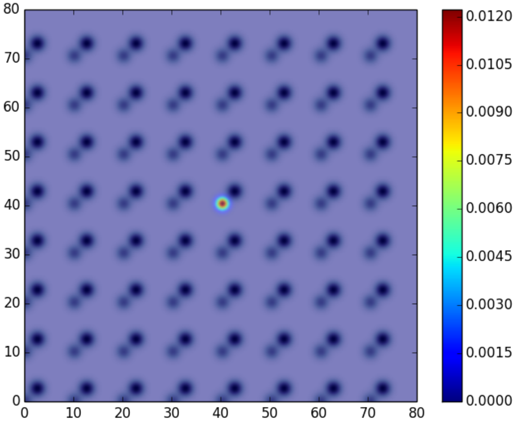
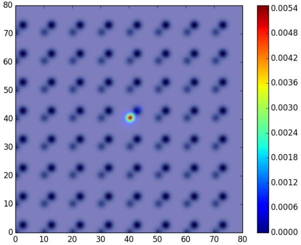
0.00012
0.00008
0.00004
0.00000
$z = 20$mm

Evolucija za $\Delta = n_{L0}$

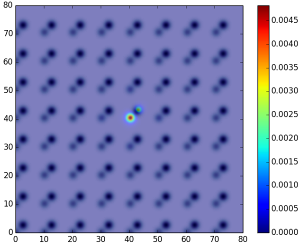
- Početni uvjet:

$$\psi(x, y, z = 0) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-4a)^2}{2\sigma^2} - \frac{(y-4a)^2}{2\sigma^2}}$$

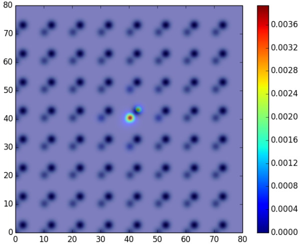




$z = 2.5\text{mm}$



$z = 5\text{mm}$



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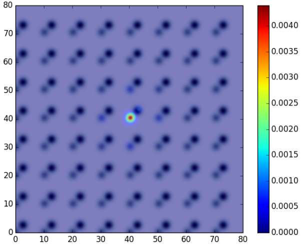
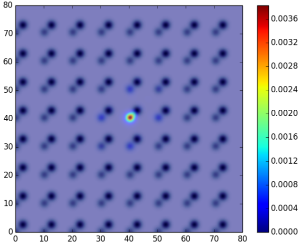
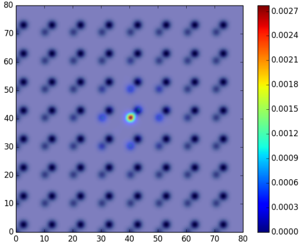


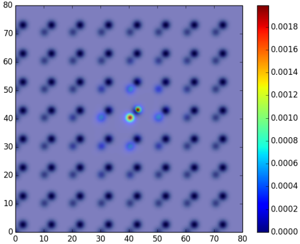
$z = 10\text{mm}$



$z = 12.5\text{mm}$



$z = 15\text{mm}$



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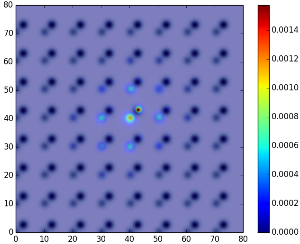
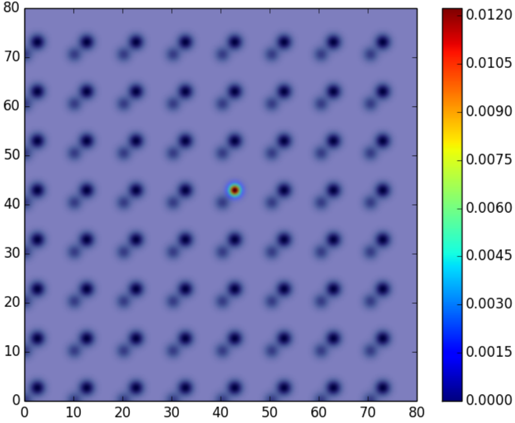


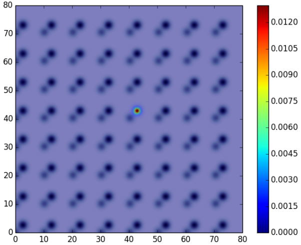
$z = 20\text{mm}$

Evolucija za $\Delta = n_{L0}$

- Početni uvjet:

$$\psi(x, y, z = 0) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-4a-b_x)^2}{2\sigma^2} - \frac{(y-4a-b_y)^2}{2\sigma^2}}$$





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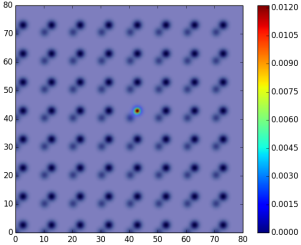
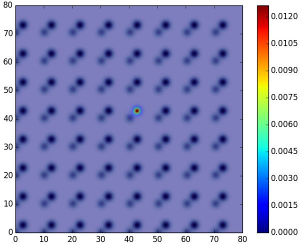


$z = 5\text{mm}$



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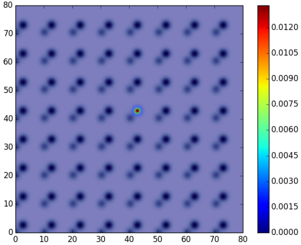
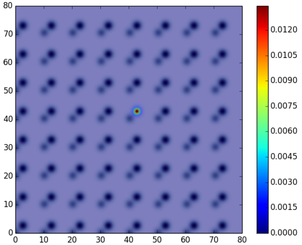
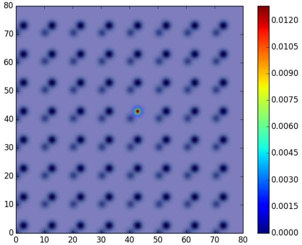


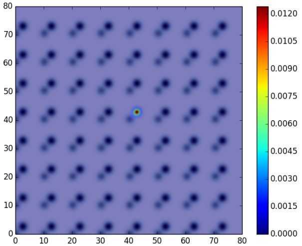
$z = 10\text{mm}$



$z = 12.5\text{mm}$



$z = 15\text{mm}$



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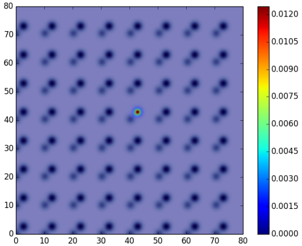
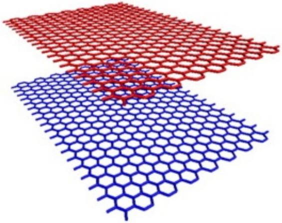


$z = 20\text{mm}$

Komentar

- Podizanje indeksa loma jednog od valovoda daje dobro tuneliranje između valovoda s nižim indeksom loma
- Tuneliranje između valovoda s višim indeksom loma ne postoji za $\Delta = \delta n_{L0}$
- Za $\Delta = 0.5n_{L0}, 0.4n_{L0}, 0.25n_{L0}$ tuneliranje se bitno pojačava samo između elemenata baze
- Moguće riješenje promoću *grating assissted tunnelinga* $\rightarrow$ rešetka periodična u z smjeru vraća tuneliranje analogno laserski potpomognutom tuneliranju u sustavu hladnih atoma
- Ako se postigne željeno tuneliranje, u sustavu 2D vezanih valovoda mogla bi se konstruirati rešekta bigrafena (*bilayer graphene*) i pročavati njena svostva ili pak proučavati solitone u 3D



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