Mase kvarkova i mezona u vakuumu i na konačnoj temperaturi

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QCD

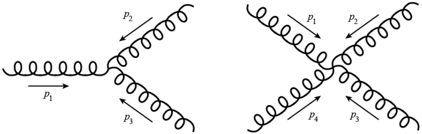
Teorija polja jake interakcije

- gustoća lagranžijana:

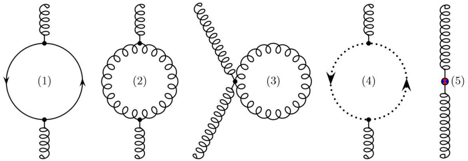
$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{\psi}_q (i\gamma^\mu D^\mu - m_q)\psi_q - \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu}$$
$$G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu(x) A_c^\nu \qquad (1)$$

  - $\mathrm{SU}(3)_c \to U = e^{-i\alpha_a t_a}$
  - $[t_a, t_b] = i f_{abc} t_c$, gdje $t_a = \frac{\lambda_a}{2}$
  - $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$
- neabelovska - baždarni bozoni nose naboj

QCD
Kubične i kvartične interakcije



- naboj se renormalizira
- gluon nosi naboj $\rightarrow$ asimptotska sloboda



QCD

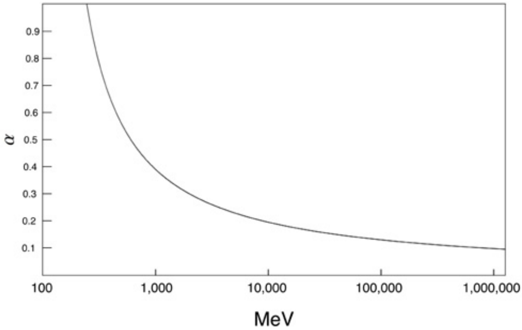
Klizna konstanta vezanja

- QED: abelovski ali $\alpha_{\mathrm{em}} = \frac{1}{137} \to \frac{1}{128}$
- QCD:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + B\alpha_s(Q_0^2)\ln\left(\frac{Q^2}{Q_0^2}\right)}$$
$$B = \frac{11N_c - 2N_f}{12\pi} \tag{2}$$

  - < 1 GeV $\to \alpha_{\mathrm{s}} \sim O(1)$

QCD
Klizna konstanta vezanja



- visoke energije - perturbativni režim
- niske energije - neperturbativni režim

QCD

Neperturbativni efekti

- $\widetilde{m}_u \approx \widetilde{m}_d$ reda veličine nekoliko MeV-a, $\mathcal{M}_p$ oko 1 GeV $\rightarrow$ zapravo posljedica neperturbativnosti
- konfinacija?

Funkcionalni integrali
Osnove

- formulacija teorije polja preko integrala po putevima
- generirajući funkcional za QCD

$$\mathcal{Z}[\eta,\bar{\eta},J] = \int \mathcal{D}[\bar{\psi}\psi A] \exp\left[i \int d^4x (\mathcal{L}_{\mathrm{QCD}} + \bar{\psi}\eta + \bar{\eta}\psi + J_\mu A_\mu)\right] \tag{3}$$

- nas zanima:

$$\begin{aligned}\langle 0|T\{\bar{\psi}(x_1)\psi(x_2)\}|0\rangle &= \frac{(-i)^2}{\mathcal{Z}[0,0,0]} \frac{\delta^2 \mathcal{Z}[\eta,\bar{\eta},0]}{\delta\bar{\eta}(x_1)\delta(-\eta(x_2))}\bigg|_{\bar{\eta}=\eta=0} \\ &= iS_F(x_1 - x_2)\end{aligned} \tag{4}$$

Funkcionalni integrali
DSE

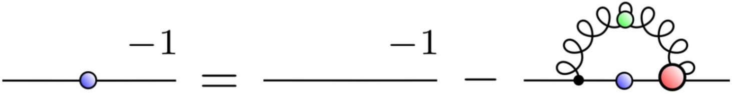
- Funkcionalni integrali imaju svojstvo:

$$\int \mathcal{D}[\phi] \frac{\delta}{\delta \phi} \equiv 0 \tag{5}$$

- pametno se izabere funkcija i derivira $\frac{\delta}{\delta \bar{\psi}_q} \frac{\delta}{\delta \psi_q}$
- prebacivanjem u impulsni prostor: Dyson - Schwinger jednadžba za $S_F(p)$

$$S_q(p)^{-1} = ip\cdot\gamma + \widetilde{m}_q + \int \frac{\mathrm{d}^4 l}{(2\pi)^4} g^2 D^{\mathrm{eff}}_{\mu\nu}(p-l)\gamma_\mu \frac{\lambda^a}{2} S_q(l)\Gamma^a_\nu(l,p) \tag{6}$$

Dyson-Schwinger jednadžba
Fermionski propagator



$$S_q(p)^{-1} = ip\cdot\gamma + \widetilde{m}_q + \int \frac{\mathrm{d}^4 l}{(2\pi)^4} g^2 D^{\mathrm{eff}}_{\mu\nu}(p-l)\gamma_\mu \frac{\lambda^a}{2} S_q(l)\Gamma^a_\nu(l,p)$$
$$\Sigma_q \equiv \int \frac{\mathrm{d}^4 l}{(2\pi)^4} g^2 D^{\mathrm{eff}}_{\mu\nu}(p-l)\gamma_\mu \frac{\lambda^a}{2} S_q(l)\Gamma^a_\nu(l,p)$$
$$S_0(p)^{-1} = ip\cdot\gamma + \widetilde{m}_q \tag{7}$$

Dyson-Schwinger jednadžba

Fermionski propagator

- "obučeni" propagator
- jednadžba je egzaktna, generacija mase eksplicitno u Σ_q
- perturbativno:

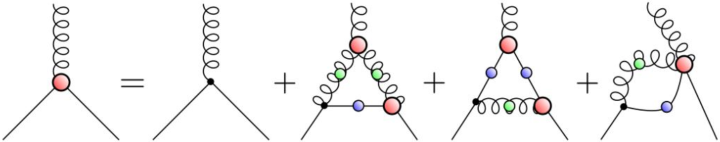
$$M_q(p^2)^{\mathrm{pert}} \sim \widetilde{m}_q \Big(1 - \frac{\alpha_s(p^2)}{\pi} \ln\Big[\frac{p^2}{\widetilde{m}_q^2}\Big] + \ldots\Big) \tag{8}$$

  odnosno, za $\widetilde{m}_q \to 0$ i $M_q \to 0$, nema smisla u kontekstu hardona
- DSE daje točno ponašanje (za $\widetilde{m}_q \to 0$ i $M_q \neq 0$), ali se poziva na obučeni verteks i obučeni gluonski propagator

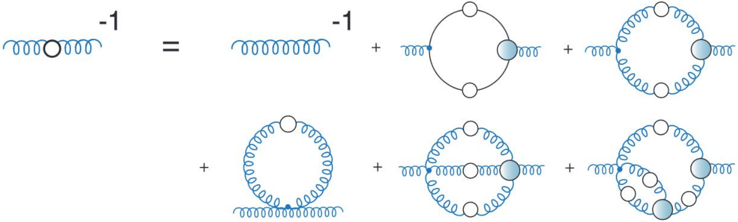
Dyson-Schwinger jednadžba

Verteks, gluonski propagator

- $D^{\text{eff}}_{\mu\nu}(p-l)$ i $\Gamma^{a}_{\nu}(l,p)$ također zadovoljavaju svoje DSE
- kao pravilo, DSE za n-točkastu traži (n+1)-točkastu Greenovu funkciju
- beskonačni toranj funkcionalnih diferencijalnih jednadžbi



Dyson-Schwinger jednadžba
Verteks, gluonski propagator



- moramo "rezati"
- rainbow ladder: $\Gamma^a_\nu(l,p) \rightarrow \frac{\lambda^a}{2}\gamma_\nu$
- funkcionalna ovisnost o impulsu $\rightarrow$ model gluonskog propagatora

Dyson-Schwinger jednadžba
Model gluonskog propagatora

- propagator se modelira
- konačne $T \rightarrow$ separabilni model
- $g^2 D^{\mathrm{eff}}_{\mu\nu}(p-l) \rightarrow \delta_{\mu\nu} D(p^2, l^2, p \cdot l)$
- $D(p^2, l^2, p \cdot l) = D_0 \mathcal{F}_0(p^2)\mathcal{F}_0(l^2) + D_1 \mathcal{F}_1(p^2)(p \cdot l)\mathcal{F}_1(l^2)$
- $\mathcal{F}_0(p^2) = \exp\left\{-\frac{p^2}{\Lambda_0^2}\right\}$ i $\mathcal{F}_1(p^2) = \exp\left\{-\frac{p^2}{\Lambda_1^2}\right\}$
- $\Lambda_0 = 0.638\mathrm{GeV}$ i $\Lambda_1 = 0.772\mathrm{GeV}$
- $D_0 = 638.8\mathrm{GeV}^{-2}$ i $D_1 = 784.6\mathrm{GeV}^{-4}$

Dyson-Schwinger jednadžba
Drugi modeli

U našem modelu:

$$\Sigma_q = \frac{4}{3}\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\left[D_0\mathcal{F}_0(p^2)\mathcal{F}_0(l^2)+D_1\mathcal{F}_1(p^2)(p\cdot l)\mathcal{F}_1(l^2)\right]\gamma_\mu S_q(l)\gamma_\mu \tag{9}$$

- Nambu-Jona-Lasinio $D(p^2, l^2, p\cdot l) \to \frac{1}{(2\pi)^2}\frac{C}{\Lambda^2}$, 2008. Yoichiro Nambu i Nobelova nagrada za DχSB
- Munczek-Nemirovsky $D(p^2, l^2, p\cdot l) \to \Lambda^2\delta^4(p)$

Kiralna simetrija lagranžijana
DχSB

- kiralna stanja

$$\begin{aligned} \psi = P_L\psi + P_R\psi = \psi_L + \psi_R \\ P_L = \frac{1-\gamma 5}{2} \quad P_R = \frac{1+\gamma 5}{2} \end{aligned} \tag{10}$$

- maseni dio lagranžijana kvari kiralnu simetriju

$$\mathcal{L}_m = m\bar{\psi}\psi \supset m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \tag{11}$$

- kako $\widetilde{m}_q \ll \mathcal{M}_p \rightarrow$ približna kiralna simetrija na skali hadrona
- slomljena *dinamički* ako je Σ_q dovoljno veliko

Mase kvarkova

Propagator

- Općeniti oblik propagatora

$$\begin{aligned} S_q(p)^{-1} &= i(p\cdot\gamma)A_q(p^2) + B_q(p^2) \\ \Rightarrow S_q(p) &= \frac{-i(p\cdot\gamma)A_q(p^2) + B_q(p^2)}{p^2 A_q^2(p^2) + B_q^2(p^2)} \end{aligned} \tag{12}$$

- $A_q(p^2) \approx 1$ pa usporedbom sa $S_0(p)^{-1} = ip\cdot\gamma + \widetilde{m}_q$ masom figurira $\mathcal{M}_q(p^2) = \frac{B_q(p^2)}{A_q(p^2)}$
- Ansatz za DSE

Mase kvarkova

Jednadžbe procjepa

$$ip\cdot\gamma A_q(p^2)+B_q(p^2)=ip\cdot\gamma+\widetilde{m}_q+\frac{4}{3}\int\frac{\mathrm{d}^4 l}{(2\pi)^4}D(p^2,l^2,p\cdot l)\times$$
$$\times\frac{2i(l\cdot\gamma)A_q(l^2)+4B_q(l^2)}{l^2A_q^2(l^2)+B_q^2(l^2)} \qquad (13)$$

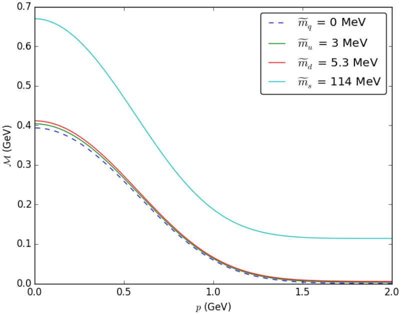
- jednadžbe procjepa $\leftrightarrow$ Tr[neparan broj γ_μ]=0 i $p_\mu p_\nu\int\mathrm{d}^4 l\, l_\mu l_\nu\to\delta_{\mu\nu}p_\mu p_\nu\int\mathrm{d}^4 l\,\frac{l^2}{4}=p^2\int\mathrm{d}^4 l\,\frac{l^2}{4}$

Mase kvarkova
Jednadžbe procjepa

$$B_q(p^2) - \widetilde{m}_q = \mathcal{F}_0(p^2)\frac{16D_0}{3}\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{\mathcal{F}_0(l^2)B_q(l^2)}{l^2A_q^2(l^2)+B_q^2(l^2)} \equiv b_q\mathcal{F}_0(p^2)$$
$$A_q(p^2) - 1 = \mathcal{F}_1(p^2)\frac{2D_1}{3}\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{l^2\mathcal{F}_1(l^2)A_q(l^2)}{l^2A_q^2(l^2)+B_q^2(l^2)} \equiv a_q\mathcal{F}_1(p^2) \tag{14}$$

- vezane jednadžbe
- iterativno rješavanje

Mase kvarkova
Jednadžbe procjepa



- uspjeh $\rightarrow$ mase veličine $\mathcal{M}_p/3$
- $a_q = 0.941$ i $b_q = 0.794$

Mase mezona
Bethe-Salpeter jednadžba

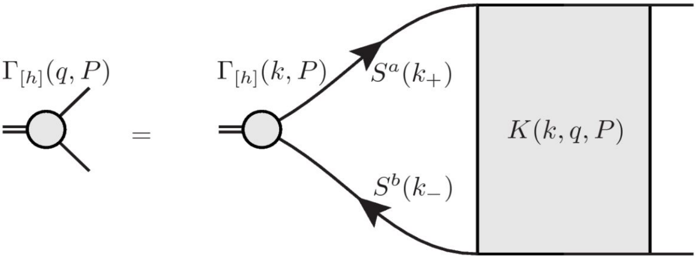
- za vektorske mezone $\mathcal{M}_V/2 \approx \mathcal{M}_p/3 \rightarrow$ ništa čudno
- međutim $\mathcal{M}_\pi/2 \ll \mathcal{M}_p/3$
- vezana stanja: Bethe-Salpeter jednadžba

$$\Gamma_{q\bar{q}'}(p,P) = \int \frac{\mathrm{d}^4 l}{(2\pi)^4} S_q(l_+)\Gamma_{q\bar{q}'}(l,P)S_{q'}(l_-)K(l,p;P) \tag{15}$$

- gdje $l_\pm \equiv l \pm \frac{P}{2} \rightarrow$ *ladder* aproksimacija

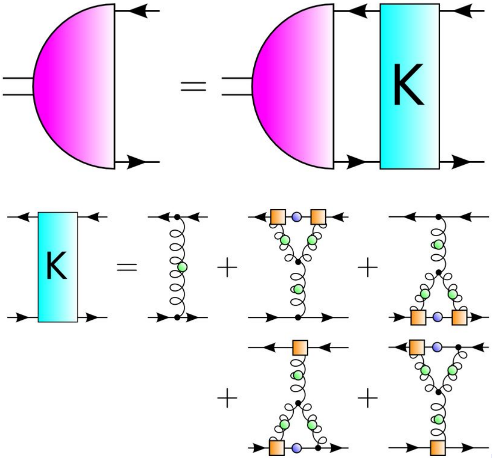
$$-\Gamma_{q\bar{q}'}(p,P) = \frac{4}{3}\int \frac{\mathrm{d}^4 l}{(2\pi)^4} g^2 D^{\mathrm{eff}}_{\mu\nu}(p-l)\gamma_\mu S_q(l_+)\Gamma_{q\bar{q}'}(l,P)S_{q'}(l_-)\gamma_\nu \tag{16}$$

Mase mezona
Bethe-Salpeter jednadžba



Mase mezona

Bethe-Salpeter jednadžba



Mase mezona
Bethe-Salpeter jednadžba

- simetrije (skalar rezan):

$$\Gamma_P(l,P) = \gamma_5\big(iE_P(P^2) + (P\cdot\gamma)F_P(P^2)\big)\mathcal{F}_0(l^2)$$
$$\Gamma_S(l,P) = E_S(P^2)\mathcal{F}_0(l^2)$$

- fiktivni problem svojstvenih vrijednosti

$$-\lambda(P^2)\Gamma_{q\bar{q}'}(p,P) = \frac{4}{3}\int\frac{\mathrm{d}^4 l}{(2\pi)^4} g^2 D^{\mathrm{eff}}_{\mu\nu}(p-l)\times \times\gamma_\mu S_q(l_+)\Gamma_{q\bar{q}'}(l,P)S_{q'}(l_-)\gamma_\nu \tag{17}$$

Mase mezona
Bethe-Salpeter jednadžba

Za skalar lako (amplituda proporcionalna jedinici):

$$\lambda(P^2) = -\frac{4}{3} D_0 tr_s \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \mathcal{F}_0^2(l^2) \left[S_q(l_+) S_{q'}(l_-) \right] \tag{18}$$

Za pseudoskalar:

$$\mathcal{K}(P^2) f = \lambda(P^2) f \tag{19}$$

gdje $f = \begin{pmatrix} E_P(P^2) \\ F_P(P^2) \end{pmatrix}$, a matrica $\mathcal{K}$:

$$\mathcal{K}_{ij}(P^2) = -\frac{4 D_0}{3} tr_s \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \mathcal{F}_0^2(q^2) \left[\hat{t}_i S(l_+) t_j S(q_-) \right] \tag{20}$$

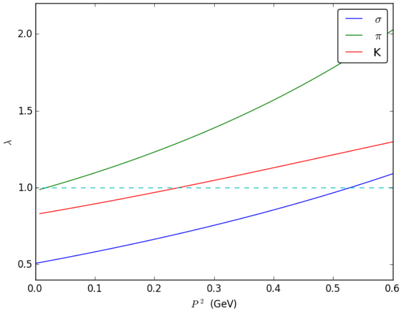
Mase mezona

Bethe-Salpeter jednadžba

vektor-stupci $\hat{t} = \begin{pmatrix} i\gamma_5 \\ -\gamma_5 \frac{p\cdot\gamma}{2P^2} \end{pmatrix}$ i $t = \begin{pmatrix} i\gamma_5 \\ \gamma_5 (p\cdot\gamma) \end{pmatrix}$. Od interesa je:

$$\lambda(P^2) = \frac{\mathcal{K}_{11} + \mathcal{K}_{22} + \sqrt{\mathcal{K}_{11}^2 + \mathcal{K}_{22}^2 - 2\mathcal{K}_{11}\mathcal{K}_{22} + 4\mathcal{K}_{12}\mathcal{K}_{21}}}{2} \tag{21}$$

Mase mezona
Bethe-Salpeter jednadžba



	M_σ	M_π	M_K
rezultat (MeV)	727	138	495
eksperiment (MeV)	600	138	495

Konačne temperature

Matsubara formalizam

- korespodencijom $e^{-\beta\hat{H}} \leftrightarrow e^{i\hat{H}t}$ gdje $t = i\beta$ prirodno
- KMS uvjet periodičnosti $\rightarrow$

$$\begin{aligned} \text{gluon } &\rightarrow A_\mu(\vec{x}, \tau + \beta) = A_\mu(\vec{x}, \tau) \\ \text{kvark } &\rightarrow \psi(\vec{x}, \tau + \beta) = -\psi(\vec{x}, \tau) \end{aligned} \tag{22}$$

- četvrta komponenta impulsa diskretizirana:

$$\begin{aligned} \text{gluon} : p_4 &\rightarrow \omega_n = (2n+1)\pi T \\ \text{kvark} : p_4 &\rightarrow \omega_n = 2n\pi T \\ p &\rightarrow p_n = (\omega_n, \vec{p}) \\ \int_{-\infty}^{+\infty} \frac{\mathrm{d}p_4}{2\pi} &\rightarrow T \sum_{n=-\infty}^{\infty} \end{aligned} \tag{23}$$

Mase kvarkova
Propagator

- nakon ekstenzije, zbog simetrije općenito:

$$S_q^{-1}(p_n, T) = i(\vec{\gamma} \cdot \vec{p})A_q(p_n^2, T) + i(\gamma_4 \omega_n)C_q(p_n^2, T) + B_q(p_n^2, T)$$
$$\Rightarrow S_q(p_n, T) = \frac{B_q(p_n^2, T) - i(\vec{\gamma} \cdot \vec{p})A_q(p_n^2, T) - i(\gamma_4 \omega_n)C_q(p_n^2, T)}{d_q(p_n^2, T)}$$
$$d_q(p_n, T) \equiv \vec{p}^2 A_q^2(p_n^2, T) + \omega_n^2 C_q^2(p_n^2, T) + B_q^2(p_n^2, T) \tag{24}$$

Mase kvarkova

Jednadžbe procjepa

- opet, tragiranjem, sada 3 *gap* jednadžbe

$$
\begin{aligned}
a_q(T) &= \frac{8D_1}{9} T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \vec{p}^2 \mathcal{F}_1(p_n^2) \frac{1+\mathcal{F}_1(p_n^2) a_q(T)}{d_q(p_n^2, T)} \\
b_q(T) &= \frac{16 D_0}{3} T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathcal{F}_0(p_n^2) \frac{\tilde{m}_q + b_q(T)\mathcal{F}_0(p_n^2)}{d_q(p_n^2, T)} \\
c_q(T) &= \frac{8D_1}{3} T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \omega_n^2 \mathcal{F}_1(p_n^2) \frac{1+\mathcal{F}_1(p_n^2) c_q(T)}{d_q(p_n^2, T)}
\end{aligned}
\tag{25}
$$

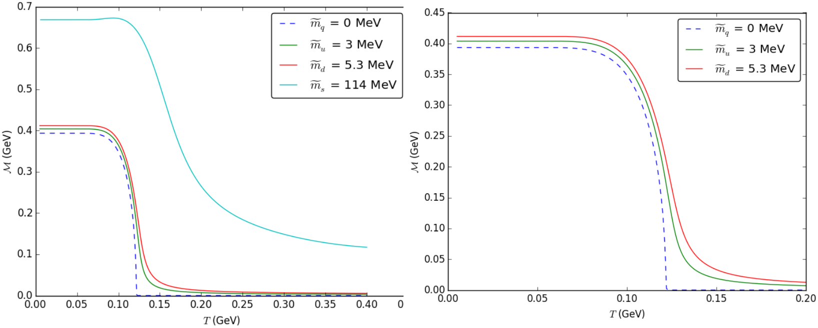
Mase kvarkova
Jednadžbe procjepa

- *gap* koeficijenti

$$\begin{aligned} B_q(p_n^2, T) &= \tilde{m}_q + b_q(T)\mathcal{F}_0(p_n^2) \\ A_q(p_n^2, T) &= 1 + a_q(T)\mathcal{F}_1(p_n^2) \\ C_q(p_n^2, T) &= 1 + c_q(T)\mathcal{F}_1(p_n^2) \end{aligned} \tag{26}$$

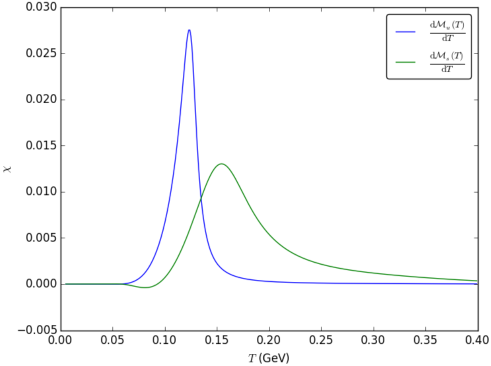
- Efektivna masa predstavljena odnosom $\mathcal{M}_q = \frac{B_q(p_n^2, T)}{A_q(p_n^2, T)}$

Mase kvarkova
Jednadžbe procjepa



Mase kvarkova
Susceptibilnosti

- ispod kritične temperature - mase naglo počinju rasti
- generalizirane susceptibilnosti $\chi_q = \frac{\mathrm{d}\mathcal{M}_q}{\mathrm{d}T}$



Mase kvarkova
Termodinamika

- $T_s = 155$ GeV, $T_u = 123$ GeV, $T_\chi = 119$ GeV $\rightarrow$ nerealno
- sinkronizacija - petlja Poylakova
- neka druga termodinamička svojstva uključenjem pozadinskog gluonskog polja pomoću Landau potencijala sa parametrom uređenja

$$\begin{aligned}\langle 0|\bar{\psi}\psi|0\rangle_{\widetilde{m}_q=0} &= -N_c \mathrm{tr}_\mathrm{s} S_0(x,x) \\ &= -4N_c T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{b_0(T)\mathcal{F}_0(p_n^2)}{d_0(p_n^2, T)}\end{aligned}$$

Mase mezona

Bethe-Salpeter jednadžba

- analogno, polovi u korelacijskog funkciji za $\lambda(P^2) \to \lambda(\nu_m^2, \vec{P}^2; T)$, imaginarna energetska os diskretizirana sa $P = (\nu_m^2, \vec{P})$
- nule funkcije $1 - \widetilde{\lambda}(\nu_m^2, 0; T)$ karakterizirat će temporalne mase, a nule funkcije $1 - \widetilde{\lambda}(0, \vec{P}^2; T)$ spacijalne
- približno slomljena O(4) simetrija $\to$ gotovo degenerirane, barem do temperature $\sim$ 100 MeV. Iznad, opisuju druge aspekte modova vezanih stanja

Mase mezona

Bethe-Salpeter jednadžba

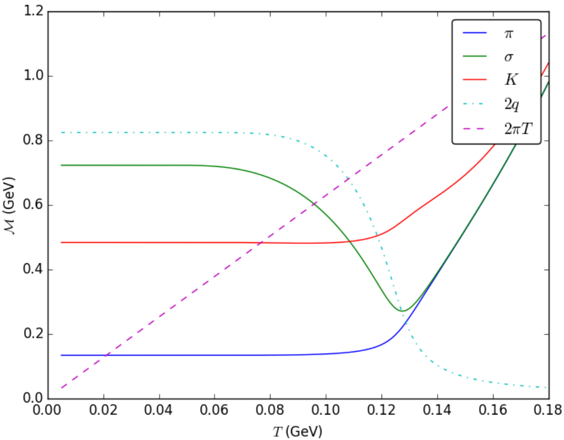
$$-\widetilde{\lambda}(0,\vec{P}^2;T)\Gamma_{qq'}(p_m,\vec{P}) = \frac{4}{3}T\sum_n \int \frac{\mathrm{d}^3 l}{(2\pi)^3} g^2 D^{\mathrm{eff}}_{\mu\nu}(\omega_\mu-\omega_\nu,\vec{p}-\vec{l})\times \\ \times\, \gamma_\mu S_q((l_n)_+)\Gamma_{qq'}(l_m,\vec{P})S_{q'}((l_n)_-)\gamma_\nu \tag{27}$$

- Ansatz

$$\begin{aligned}\Gamma_P(l_n,\vec{P}) &= \gamma_5\big(iE_P(\vec{P}^2)+(\vec{P}\cdot\vec{\gamma})F_P(\vec{P}^2)\big)\mathcal{F}_0(l_n^2)\\ \Gamma_S(l_n,\vec{P}) &= E_S(\vec{P}^2)\mathcal{F}_0(l_n^2)\end{aligned} \tag{28}$$

Mase mezona

Rezultati



Mase mezona

Rezultati

- iznad T_u susreću se π, σ i dva kvarka, te σ i π postaju degenerirani $\rightarrow$ dekonfinacija
- kritična temperatura preniska, može se dići Polyakov petljom i popraviti nejednakost temperatura
- $2\pi T$ limes $\rightarrow$ dinamički generirana masa zanemariva $\rightarrow$ prostorna masa se priblizava termalnoj masi para bezmasenih kvarkova

Mase kvarkova i mezona
Zaključak

- DSE i BSE - egzaktne jednadžbe, modelirane i riješene numerički
- generiranje mase demonstrirano jednostavnim separabilnim modelom gluonskog propagatora
- objašnjena niska masa lakih pseudoskalarnih mezona (u usporedbi s efektivnim konstituentima)
- konačne temperature - treba popraviti termodinamiku

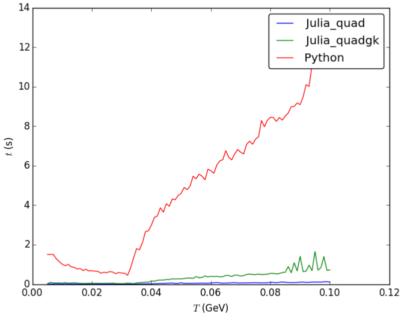
Numeričke metode

Integrali

- optimalno - Gaussova kvadratura
- kut Gauss-Legendre kvadratura: $\int_{-1}^{1} f(x)\mathrm{d}x \approx \sum_{i=1}^{n} w_i f(x_i)$ (implementirano generiranje korijena)
- impuls, Gauss-Laguerre kvadratura: $\int_0^\infty e^{-x} g(x)\mathrm{d}x \approx \sum_{i=1}^{n} w_i' g(x_i)$ (korijene generira "GaussQuadrature.jl")
- traženje nultočki $1 - \widetilde{\lambda}(0, \vec{P}^2; T)$, pomoću "Roots.jl", sporo konvergira za velike $\vec{P}^2$

Numeričke metode

Julia



- "quad" iz "integrate" knjižnice "scipy" - 7 min 42 sec, Julia (*core* quadgk) 33.3 sec, ručno kvadratura 5.5 sec