Samostalni seminar iz istraživanja u fizici

Analitički opis metastabilnosti u kritičnom ponašanju neuređenog sustava

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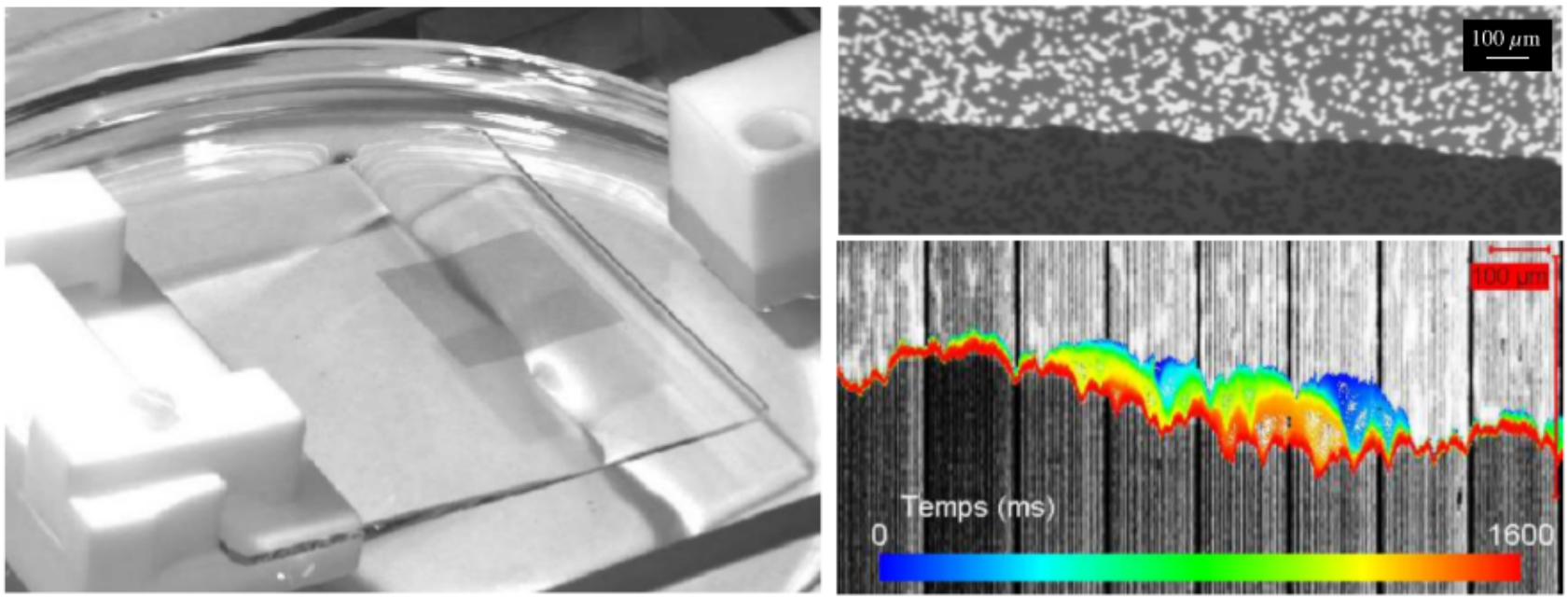
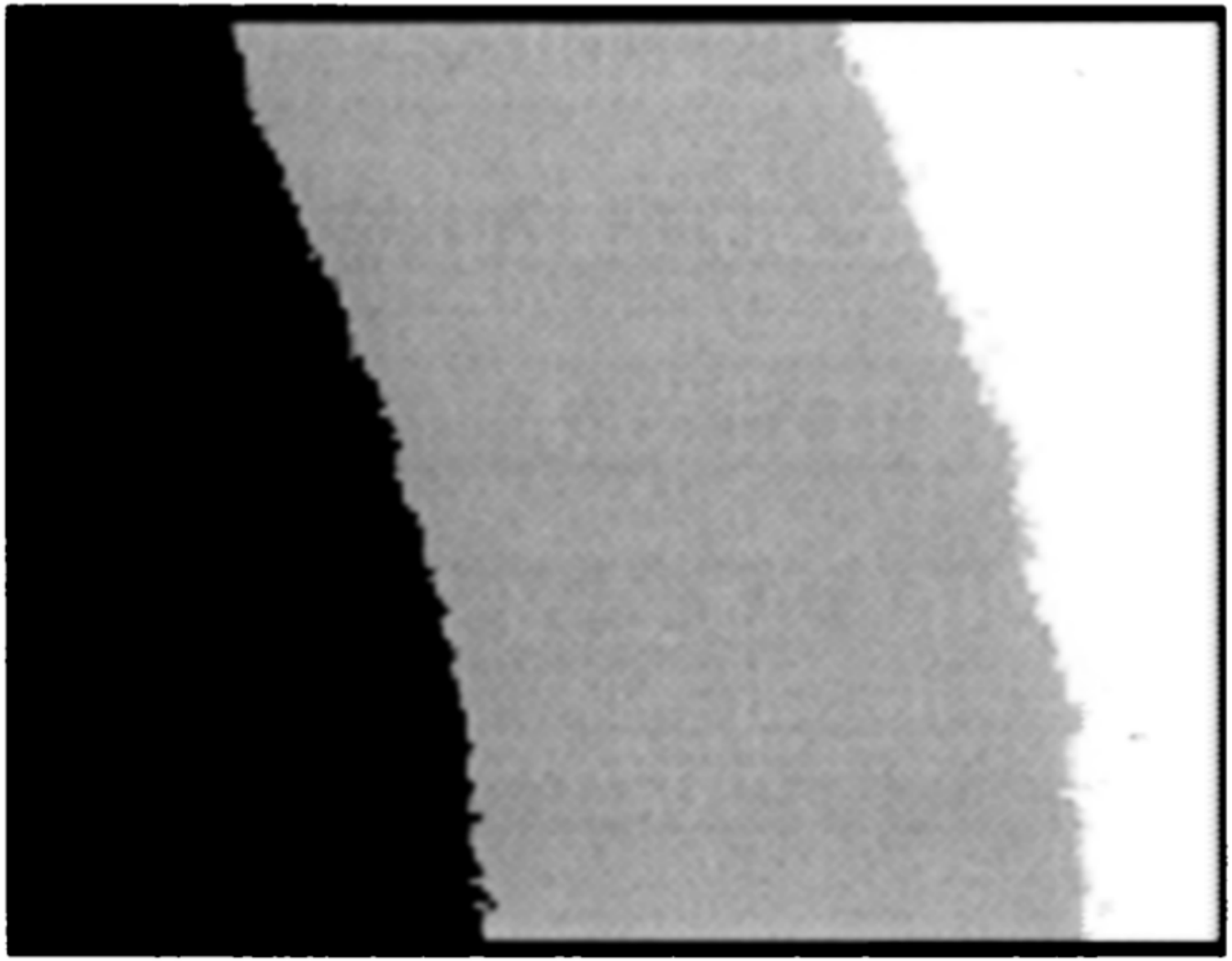
REM model i realizacije



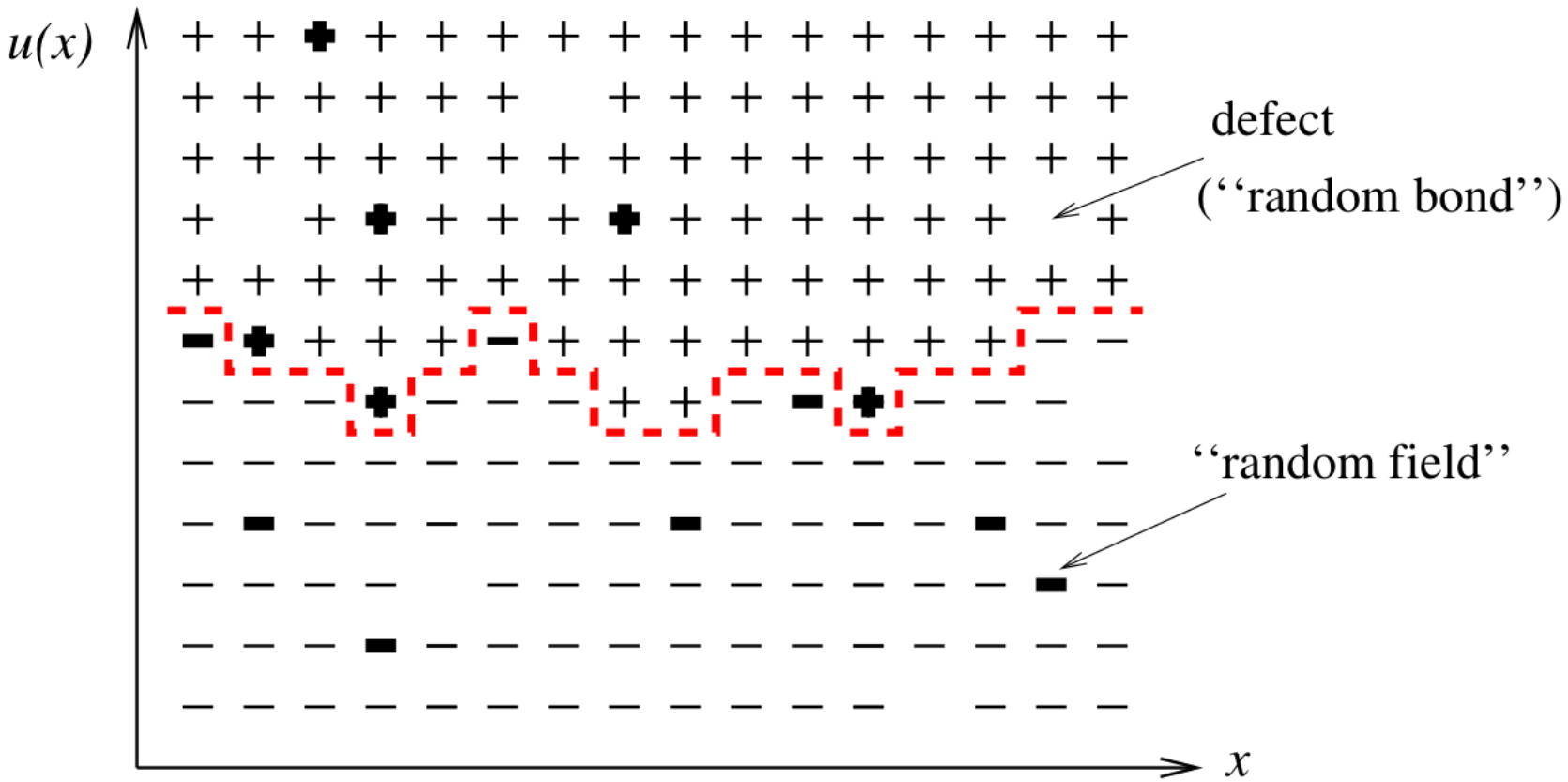
Slika : Kontaktna linija glicerina [1].

REM model i realizacije



Slika : Domenski zid u ultratankom sloju kobalta [2].

REM model i realizacije



Slika : Isingov magnet [1].

REM model i realizacije

- Navedene fizikalne sustave modeliramo elastičnom mnogostrukosti u neuređenom mediju.
- $T = 0K$: nema dinamike kojom bi se sustav relaksirao iz metastabilnog u osnovno stanje.
- Hamiltonijan REM modela i korelator nereda:

$$\begin{aligned} \mathcal{H}[u] = & \int d\vec{x} \frac{1}{2} \left[\vec{\nabla} \vec{u}\left(\vec{x}\right) \right]^2 \rightarrow \text{elastičnost} \\ & + \int d\vec{x} V\left[\vec{x}, \vec{u}\left(\vec{x}\right)\right] \rightarrow \text{nered}, \end{aligned} \tag{1}$$

$$\left\langle V\left(\vec{x}, \vec{u}\right) V\left(\vec{x}', \vec{u}'\right) \right\rangle := \delta^d\left(\vec{x} - \vec{x}'\right) R\left(\vec{u} - \vec{u}'\right). \tag{2}$$

Procjena doprinosa nereda i kritičnost

- Doprinos nereda procjenjujemo preko energetskih doprinosa.
- Larkin $\rightarrow$ gledamo kako se Hamiltonijan skalira $\Longrightarrow$
  Larkinova skala L_c, gornja kritična dimenzija $d = 4$:

$$E_{DO} = E_{el} \implies L_c = \left(\frac{c^2}{\bar{f}^2} \xi^d \right)^{\frac{1}{4-d}} . \tag{3}$$

- Termodinamička granica + $d < 4$ $\Longrightarrow$ nered dominantan.
- Kritično ponašanje [1]:

$$\left\langle \left[\vec{u}\left(\vec{x}\right) - \vec{u}\left(\vec{x}'\right) \right]^2 \right\rangle \propto \left| \vec{x} - \vec{x}' \right|^{2\zeta} ; \quad \zeta < 1. \tag{4}$$

Tok $\Delta_l(u) = -R''(u)$

$$\langle V(\vec{x},\vec{u})\, V(\vec{x}',\vec{u}')\rangle := \delta^d(\vec{x}-\vec{x}')\, R(\vec{u}-\vec{u}') \tag{5}$$

- Operacije renormalizacije [3]:
  "coarse-grain" $\rightarrow$ reskaliranje $\rightarrow$ renormalizacija.
- Početne veličine koje razmatramo (RG početni uvjet): analitičke funkcije.
- Ponašanje veličine $\Delta_l(u)$ pod ovim transformacijama opisano je sljedećom jednadžbom [4] uz $\epsilon = 4-d$:

$$\begin{aligned}\partial_l \Delta_l(u) = &(\epsilon - 3\zeta)\,\Delta_l(u) + \zeta\,[u\Delta_l(u)]' \\ &- \frac{1}{2}\left\{[\Delta_l(u) - \Delta_l(u=0)]^2\right\}''.\end{aligned} \tag{6}$$

- Fizikalni početni uvjet za nered nasumičnog polja [5]:

$$0 < \int du \Delta_{l=0}(u) < +\infty. \tag{7}$$

Fiksna točka $\partial_l \Delta_l = 0$

- Računamo $\zeta = \dfrac{\epsilon}{3} = \dfrac{4-d}{3}$:

$$\partial_l \int_{-\infty}^{\infty} du \Delta_l(u) = (\epsilon - 3\zeta) \int_{-\infty}^{\infty} du \Delta_l(u) + \zeta \left[u \Delta_l(u) \right] \Big|_{-\infty}^{\infty} - \Delta_l'(u) \left[\Delta_l(u) - \Delta_l(0) \right] \Big|_{-\infty}^{\infty}$$

$$\xrightarrow[R(-u)=R(u), \quad (7)]{\partial_l \Delta_l = 0}$$

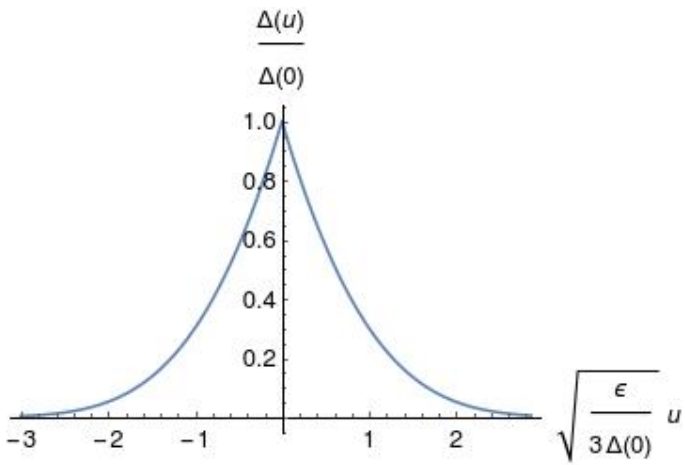
$$\partial_l \left[\ln \int_{-\infty}^{\infty} du \Delta_l(u) \right] = (\epsilon - 3\zeta) = 0. \tag{8}$$

Fiksna točka $\partial_l \Delta_l = 0$

- Implicitno rješenje uz $\partial_l \Delta_l = 0$ i $\epsilon - 3\zeta = 0$:

$$\begin{aligned} &\left\{ \frac{\epsilon}{3} u \Delta(u) - \frac{1}{2} \left[(\Delta(u) - \Delta(0))^2 \right]' \right\}' = 0 \\ &\overset{\int\int}{\Longrightarrow} \frac{\epsilon}{6\Delta(0)} u^2 = \frac{\Delta(u)}{\Delta(0)} - 1 - \ln\left[\frac{\Delta(u)}{\Delta(0)} \right]. \end{aligned} \tag{9}$$

- Reskaliranjem se u fiksnoj točki razvija neanalitičnost u obliku šiljka:



Slika : Funkcija $\Delta(u)$ u fiksnoj točki.

Neanaliticnost korelatora nereda

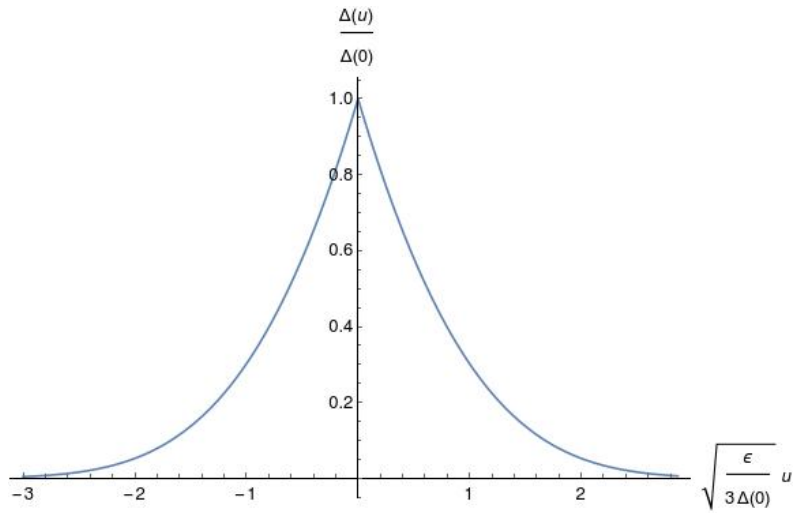
Pojava neanaliticnosti reskaliranjem

$$\Delta_l''(0) = \frac{\Delta_0''(0)\, e^{\epsilon l}}{1 + \frac{3}{\epsilon}\Delta_0''(0)\left(e^{\epsilon l} - 1\right)} \tag{10}$$

$$\Longrightarrow \; l_c = \frac{1}{\epsilon} \ln\left[1 - \frac{\epsilon}{3\Delta_0''(0)}\right] \tag{11}$$

REM model rekapitulacija

- Analitično $\xrightarrow{FRG}$ neanalitično.
- Skala l_c odgovara heurističnoj Larkinovoj L_c.



Slika : Funkcija $\Delta(u)$ u fiksnoj točki.

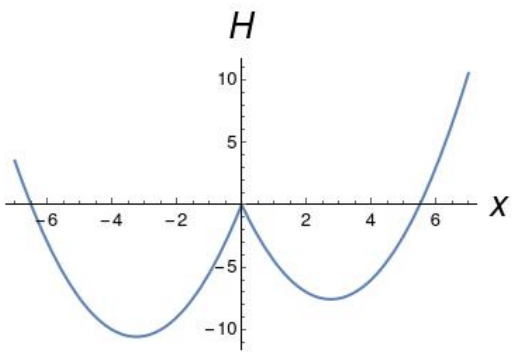
- Singularitet-šiljak Δ-e u fiksnoj točki je potpis metastabilnosti.
- $\Delta_l''(0)$ divergira u l_c točki skale.

Model-igračka

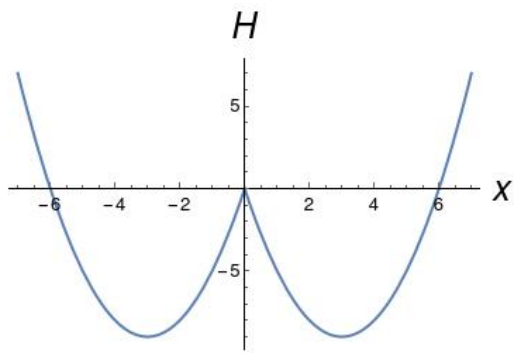
- Uvodimo model-igračku s nametnutim metastabilnim stanjima $\rightarrow$ tražimo analognu neanalitičnost.
- Model:

$$H = -a|x| + bx^2, \quad a, b > 0; \tag{12}$$

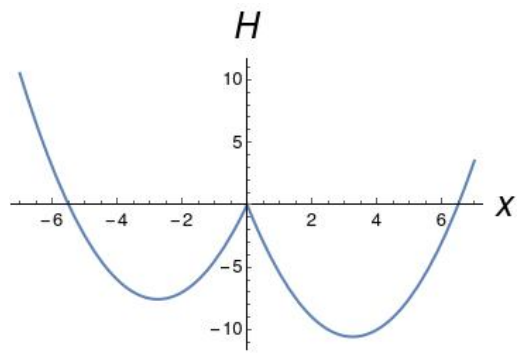
$$\text{nered: } P(h) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{h^2}{2\sigma}\right). \tag{13}$$



(a) $h < 0$



(b) $h = 0$

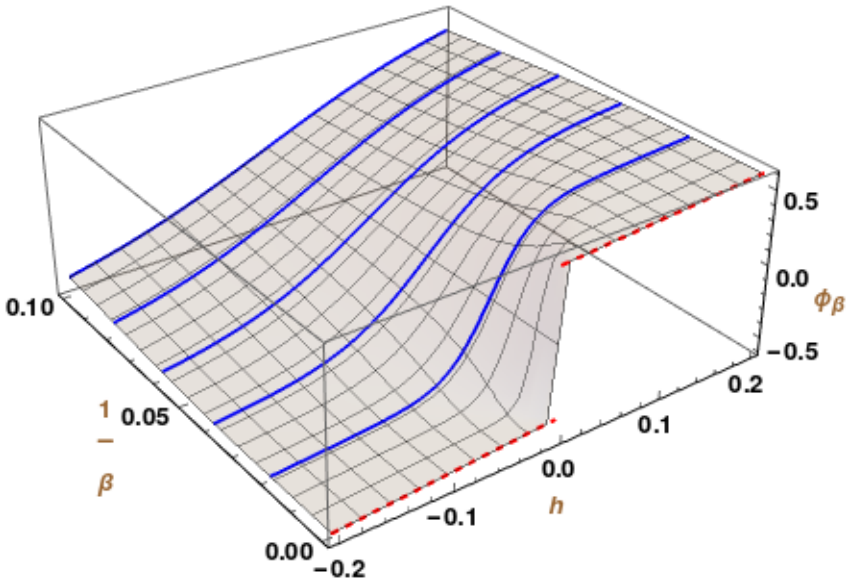


(c) $h > 0$

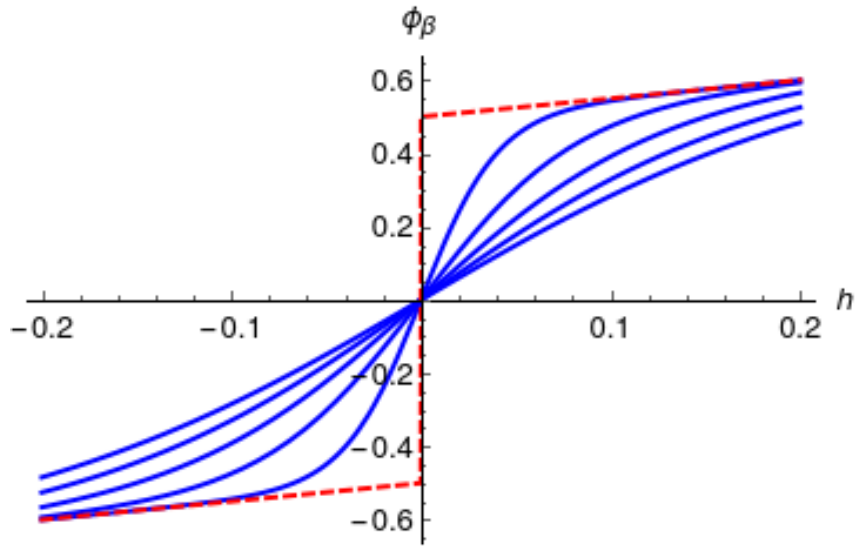
Slika : Hamiltonijan modela-igračke s doprinosom nereda $-hx$.

Model-igračka

Parametar uređenja



(a) Puni dijagram s izdvojenim krivuljama.



(b) Izdvojene ovisnosti s a) usporedno.

Slika : Ovisnost parametra uređenja o temperaturi $1/\beta$ i h.

Model-igračka

Analitički račun kumulanata

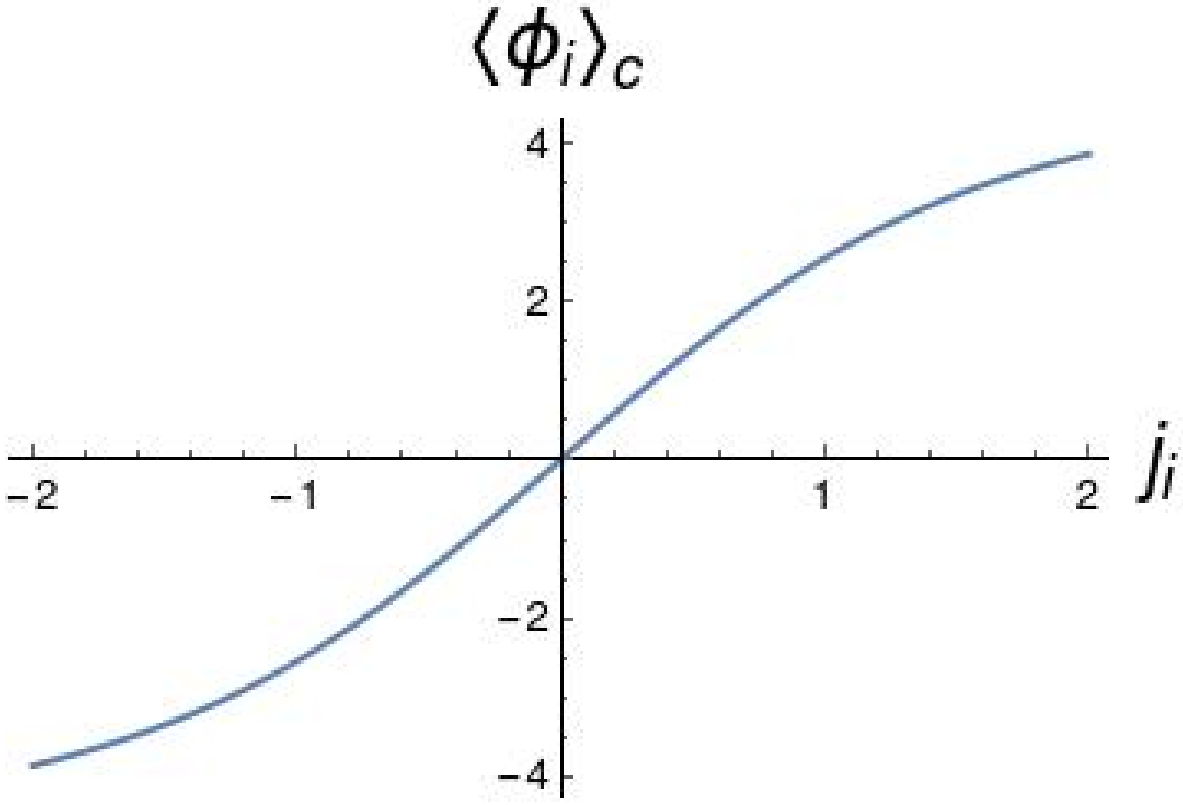
- Prvi kumulant:

$$\langle \phi_i \rangle_c = \langle \phi_i \rangle = \int_{-\infty}^{\infty} dh P(h) \phi[h + j_i]. \tag{14}$$

- Drugi kumulant:

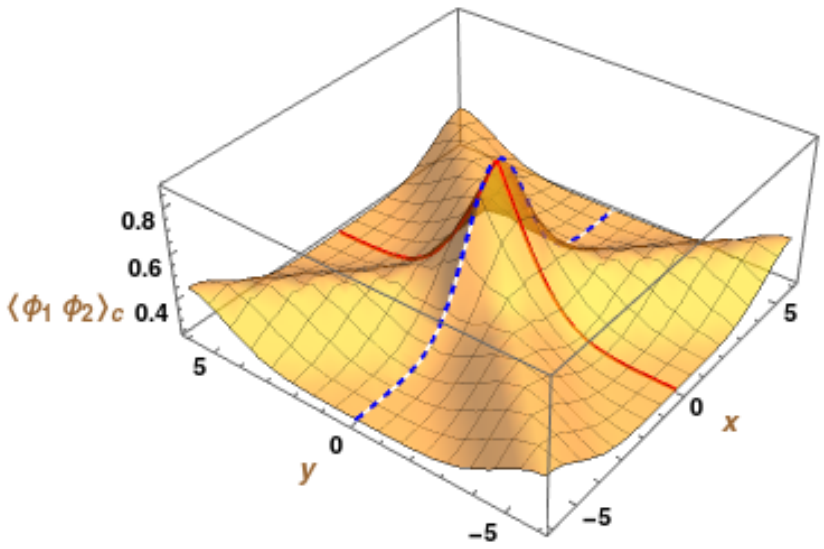
$$\begin{aligned} \langle \phi_1 \phi_2 \rangle_c =& \langle \phi_1 \phi_2 \rangle - \langle \phi_1 \rangle \langle \phi_2 \rangle \\ =& \int_{-\infty}^{\infty} dh P(h) \phi[h + j_1] \phi[h + j_2] - \langle \phi_1 \rangle \langle \phi_2 \rangle. \end{aligned} \tag{15}$$

Model-igračka

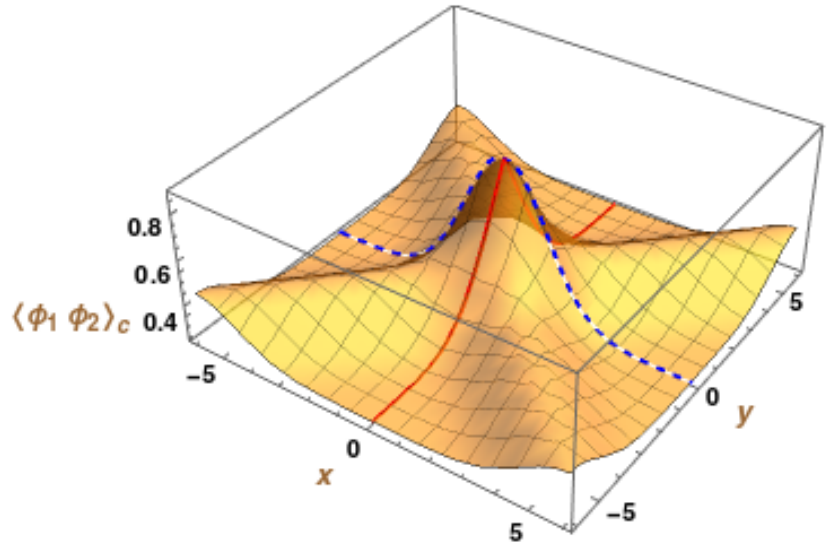


Slika : Prvi kumulant $\langle \phi_i \rangle_c$.

Model-igračka



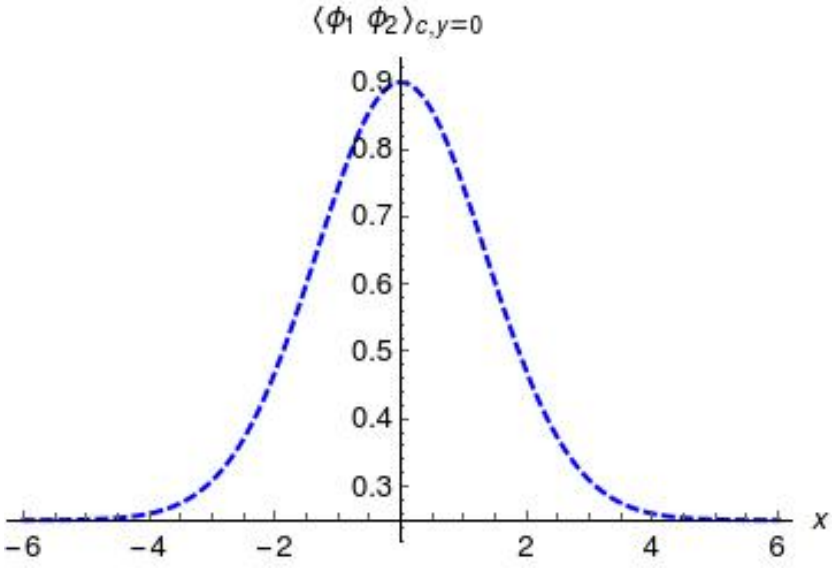
(a) Prikaz drugog kumulanta s naglašenim $x = 0$ i $y = 0$ ovisnostima.



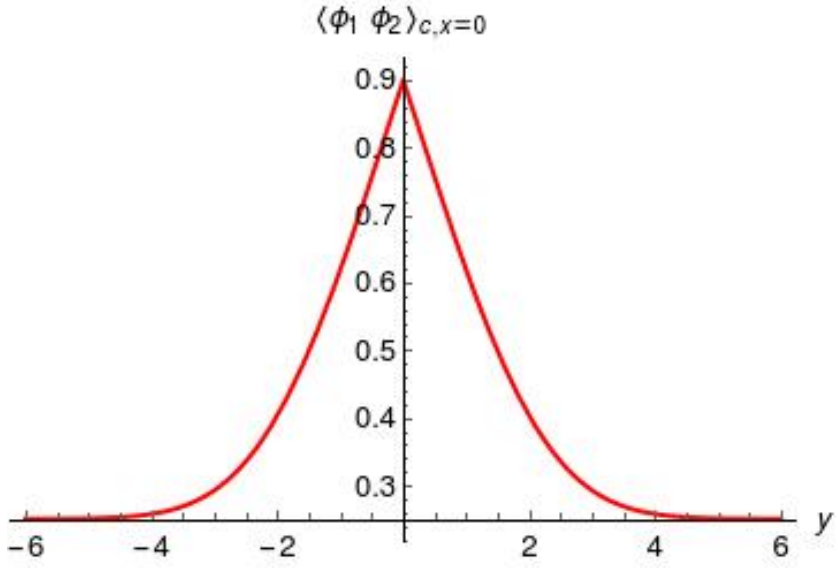
(b) Prikaz *a*) s pogledom iz drugog smjera.

Slika : Ovisnost drugog kumulanta o $x = \frac{j_1 + j_2}{\sqrt{2}}$ i $y = \frac{j_1 - j_2}{\sqrt{2}}$.

Model-igračka



(a) Izdvojena analitička ovisnost o x za $y = 0$.

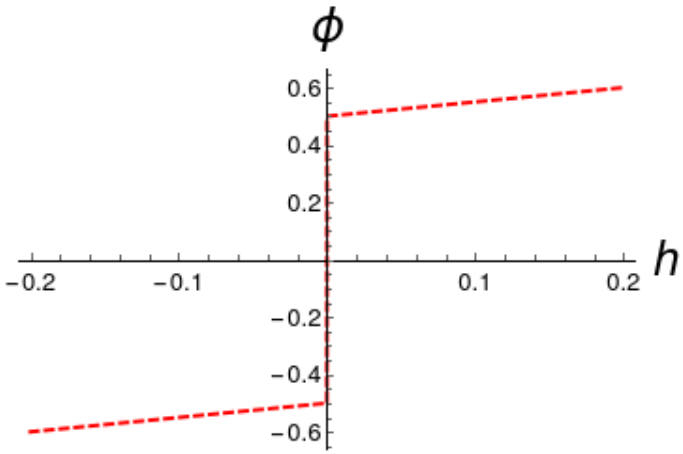


(b) Izdvojena neanalitička ovisnost o y za $x = 0$ s razvidnim singularitetom-šiljkom.

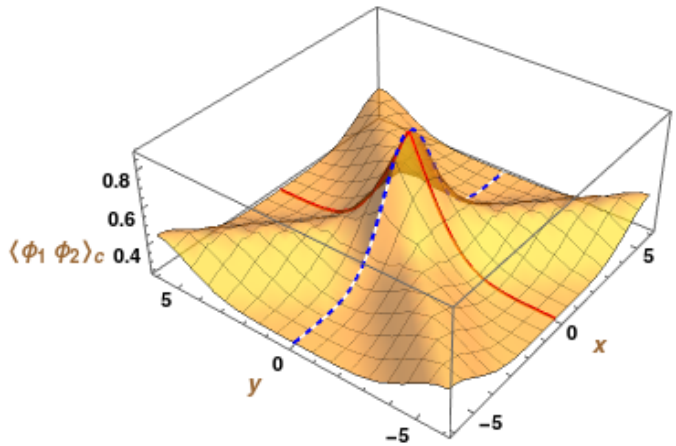
Slika : Ovisnost drugog kumulanta o $x = \frac{j_1 + j_2}{\sqrt{2}}$ i $y = \frac{j_1 - j_2}{\sqrt{2}}$.

Model-igračka rekapitulacija

- $T = 0K$ metastabilnost producira $\langle\phi_1\phi_2\rangle - \langle\phi_1\rangle\langle\phi_2\rangle$ šiljak:



(a) Parametar uređenja.

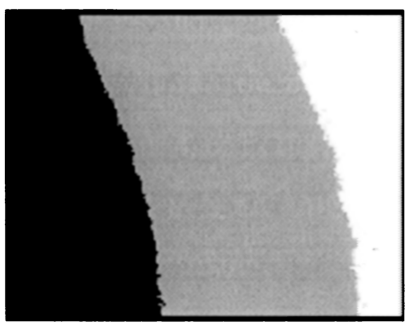


(b) $\langle\phi_1\phi_2\rangle - \langle\phi_1\rangle\langle\phi_2\rangle$.

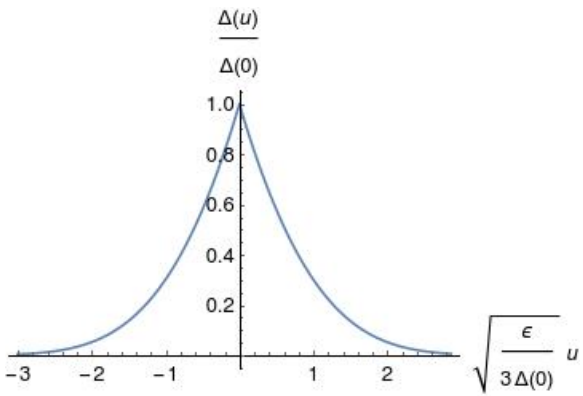
Slika : Grafovi $T = 0K$ veličina interesantnih za model-igračku.

Zaključak

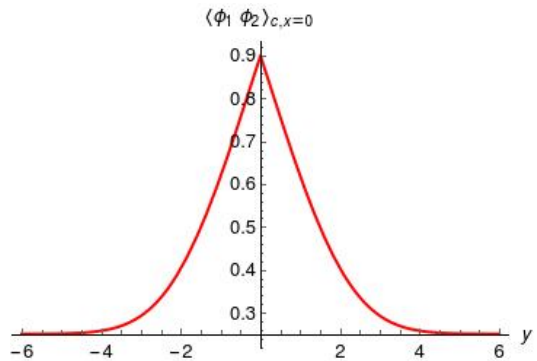
- REM model i model-igračka:
  neuređeni sustavi s metastabilnosti.
- Analitički $T = 0K$ račun:
  singularitet-šiljak.



(a) Domenski zid u ultratankom sloju [2]: $d = 2 \implies \zeta = \frac{2}{3}$.



(b) REM $\Delta(u)$ u $T = 0K$ fiksnoj točki.



(c) $T = 0K$ korelator drugog modela ($x = 0$).

Literatura

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