10 Transformacijama iz koordinatnog sustava sa vertikalnom θ-koordinatom u sustav s vertikalnom z-koordinatom, pokažite da Ertelova potencijalna vrtložnost P proporcionalna izrazu $FH_s^2 - S^4$, gdje su oznake preuzete iz Holton (2004).

Rj: Ertelova pot. vrtložnost u sustavu s θ-koord:

$$P = (\zeta_\theta + f)\left(-g\frac{\partial\theta}{\partial p}\right), \text{ gdje je } \zeta_\theta = -\left(\frac{\partial u_g}{\partial y}\right)_\theta$$

- digresija: ζ_θ ima samo komponentu promjene geostrofičkog vjetra po y-koordinati jer radimo u 2D ravnini (yz ili $y\theta$ ...)

$$\Rightarrow P = \left[-\left(\frac{\partial u_g}{\partial y}\right)_\theta + f\right]\left(-g\frac{\partial\theta}{\partial p}\right) \quad (*)$$

- općenito, za generaliziranu vert. koordinatu vrijedi:

$$\left(\frac{\partial a}{\partial x}\right)_\xi = \left(\frac{\partial a}{\partial x}\right)_\eta + \frac{\partial a}{\partial \eta}\left(\frac{\partial \eta}{\partial x}\right)_\xi$$

- ovdje:

$$\left.\begin{aligned} \xi &= z \\ \eta &= \theta \\ a &= u_g \\ x &= y \end{aligned}\right\} \quad \left(\frac{\partial u_g}{\partial y}\right)_z = \left(\frac{\partial u_g}{\partial y}\right)_\theta + \frac{\partial u_g}{\partial \theta}\left(\frac{\partial \theta}{\partial y}\right)_z$$

$$\Rightarrow -\left(\frac{\partial u_g}{\partial y}\right)_\theta = -\left(\frac{\partial u_g}{\partial y}\right)_z + \frac{\partial u_g}{\partial \theta}\left(\frac{\partial \theta}{\partial y}\right)_z \quad (**)$$

- kako vrijedi izraz (9.10) u Holtonu: $f\frac{\partial u_g}{\partial z} = -\frac{\partial b}{\partial y}$ gdje je $b = \frac{g\theta}{\theta_0}$ i vrijedi $\frac{\partial u_g}{\partial z} = \frac{\partial u_g}{\partial \theta}\frac{\partial \theta}{\partial z} \Rightarrow$

$$\Rightarrow \frac{\partial u_g}{\partial \theta} = \frac{\frac{\partial u_g}{\partial z}}{\frac{\partial \theta}{\partial z}} = \frac{-\frac{g}{f\theta_0}\frac{\partial \theta}{\partial y}}{\frac{\partial \theta}{\partial z}}$$

⇒ (**) sada poprima sljedeći oblik:

$$-\left(\frac{\partial u_g}{\partial y}\right)_\theta = \left(-\frac{\partial u_g}{\partial y}\right)_z + \left(-\frac{g}{f\theta_0}\right)\frac{\left(\frac{\partial \theta}{\partial y}\right)^2}{\frac{\partial \theta}{\partial z}}$$

- također, možemo pisati i sljedeću jednakost:

$$\frac{\partial \theta}{\partial p} = \frac{\partial \theta}{\partial z}\frac{\partial z}{\partial p} = -\frac{1}{\rho g}\frac{\partial \theta}{\partial z}$$

- to sve sada ubacimo u (*):

$$\Rightarrow P = \left[\left(-\frac{\partial u_g}{\partial y}\right)_z - \frac{g}{f\theta_0}\frac{\left(\frac{\partial \theta}{\partial y}\right)_z^2}{\frac{\partial \theta}{\partial z}} + f\right]\left(\cancel{-g}\,\frac{1}{\cancel{-}\rho \cancel{g}}\frac{\partial \theta}{\partial z}\right) =$$

$$= \frac{1}{\rho}\left[f - \left(\frac{\partial u_g}{\partial y}\right)_z\right]\frac{\partial \theta}{\partial z} - \frac{g}{\rho f \theta_0}\left(\frac{\partial \theta}{\partial y}\right)_z^2 =$$

$$= \frac{1}{\rho f}\underbrace{f\left[f - \left(\frac{\partial u_g}{\partial y}\right)_z\right]}_{F^2}\underbrace{\frac{g}{\theta_0}\frac{\partial \theta}{\partial z}}_{N_s^2}\frac{\theta_0}{g} - \frac{\theta_0}{\rho f g}\underbrace{\left(\frac{g}{\theta_0}\frac{\partial \theta}{\partial y}\right)_z^2}_{S^4} \Rightarrow$$

$$\Rightarrow P = \frac{\theta_0}{\rho f g}\left(F^2 N_s^2 - S^4\right)$$ ✓✓ OK

⇒ time je pokazana tražena proporcionalnost