

NAPREDNI FIZIČKI PRAKTIKUM 1  
studij Matematika i fizika; smjer nastavnički

# **MAXWELL–BOLTZMANNOVA RASPODJELA BRZINA**

## ZADACI

1. Sastavite uređaj prema uputama i provedite mjerenja za najmanje 1000 kuglica.
2. Izmjerite distribuciju brzina kuglica, i usporedite dobiveni rezultat sa Maxwell-Boltzmannovom raspodjelom.
3. Iz eksperimentalnih podataka nađite  $v_m$ ,  $\langle v \rangle$  i  $\langle v^2 \rangle$ . Provjerite njihove međusobne omjere. Komentirajte rezultat.
4. Izračunajte srednju kinetičku energiju jedne kuglice. Kad bi vibrirajuća podloga mirovala, kolika bi trebala biti temperatura da uslijed termičkog gibanja kuglice postignu tu energiju?

### Dodatne upute za rad

Eksperimentalna aparatura prikazana je na sl.1. Sastoji se od slijedećih dijelova:

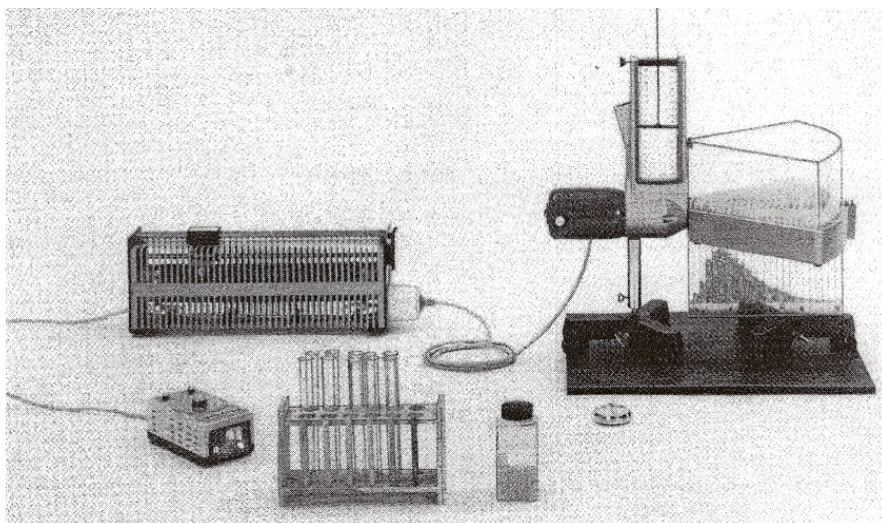
- Komora s vibrirajućim dnom i filterom za selekciju smjera kuglica
- Posuda u obliku kružnog isječka s 24 pregrade
- Stalak za posudu, koji ujedno omogućuje selekciju kuglica po pregradama
- Gumena podloga
- Promjenjivi otpornik
- Stroboskopska svjetiljka
- Epruvete sa stalkom
- Staklene kuglice

Uređaj treba složiti kao što je prikazano na sl.1. Komoru valja staviti u položaj u kojem je crvena linija na prednjoj strani komore u istoj ravnini kao vrh pregrada. Tad se može smatrati da je visinska razlika između otvora na filteru (sl.2) kroz koji izlaze kuglice i ravnine na koju kuglice padaju 8 cm.

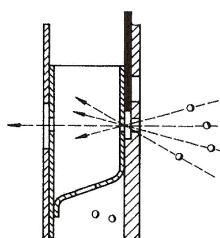
Potom je potrebno vijcima na nogama tronogih stalaka za komoru i posudu podesiti položaj u kojem su i jedan i drugi dio aparature u vodoravnom položaju, što se može očitati na malim libelama koje su ugrađene na iste.

Volumen u kojem će se kuglice gibati podešava se tako da visina bude 6 cm. Potom se na izlazni dio komore stavlja filter koji omogućuje selekciju smjera kuglica (sl.2).

Na ovom mjestu treba izmjeriti udaljenost pojedinih pregrada na posudi od okomite ravnine u kojoj leži otvor filtera. Taj će nam podatak koristiti za račun brzine izlazećih kuglica.



Slika 1.



Slika 2.

Motor koji pokreće vibrirajuću podlogu u komori spaja se na mrežu preko promjenjivog otpornika, koji služi za mijenjanje broja okretaja motora, a time i

brzine osciliranja podloge. Broj oscilacija u sekundi može se očitati korištenjem male stroboskopske svjetiljke: svjetiljka se uperi u podlogu i podešava se frekvencija titranja svjetiljke dok se ne dobije mirna slika podloge; tada se broj oscilacija u sekundi jednostavno očita na skali svjetiljke. U eksperimentu se preporuča korištenje brzine od oko  $3000 \text{ osc/min}$ .

Kad je sve podešeno na gore opisani način, uređaj je spreman za mjerenje.

Slijedeći korak je da se komora napuni kuglicama. Koristimo staklene kuglice diametra  $d = (2.0 \pm 0.2) \text{ mm}$ . Potrebno je odbrojati oko 400 kuglica za volumen koji je preporučen. To je moguće direktnim preciznim brojanjem kuglica ili vaganjem: izvaže se na analitičkoj vagi oko 100 kuglica i izračuna se prosječna masa jedne kuglice. Poznavanje tog podatka olakšava i kasnije prebrojavanje kuglica po pojedinim pregradama, a potrebno ga je znati i zbog računa srednje kinetičke energije jedne kuglice.

Iz komore u jednoj minuti izleti oko 45 kuglica. Gubitak kuglica je potrebno neprestano nadoknađivati novim kuglicama. Stoga epruvete treba napuniti sa po 45–50 kuglica, te svake minute dodavati u komoru sadržaj jedne epruvete.

U cilju dobivanja što pouzdanijih rezultata preporuča se da ukupan broj kuglica koje izlete iz komore bude bar 1000, odnosno da ukupno vrijeme trajanja eksperimenta bude bar dvadesetak minuta.

U svrhu ponovnog punjenja epruveta kuglicama eksperiment se može prekinuti pa nastaviti kad napunimo epruvete, ali se uvjeti mjerenja (geometrijski odnosi unutar aparature, brzina okretanja motora itd.) ne smiju nipošto mijenjati.

Posebnu je pažnju potrebno posvetiti pražnjenju pregrada u kojima se sakupe kuglice; taj postupak mora biti dobro isplaniran i vrlo pažljivo proveden.

Prvo je potrebno olovkom ili nekim štapićem sakupiti i u staklene odjeljke gurnuti one kuglice koje se nalaze u pregradama, ali nisu upale u odjeljke. Ne odvajajući posudu s pregradama i stalak s odjeljcima, potom se sve zajedno još malo potrese, kako bi i posljednja kuglica upala u svoj odjeljak. Kad smo sigurni da su sve kuglice upale u svoj odjeljak, skinemo posudu s pregradama. Od papira načinimo male čepove kojima *dobro* začepimo svaki odjeljak. Odčepljujući jedan po jedan odjeljak istresamo kuglice u pripremljene posude ili lađice od papira, izbrojimo ih i zabilježimo broj pregrade i pripadni broj kuglica. Zatim kuglice vratimo u bočicu. *Sve operacije s kuglicama potrebno je provoditi iznad kartonske kutije koja je priložena, kako bi se eventualno prosute kuglice mogle sakupiti.*

### Related topics

Kinetic theory of gases, temperature, gas, molecules, model kinetic energy, average velocity, velocity distribution.

### Principle

By means of the model apparatus for kinetic theory of gases the motion of gas molecules is simulated and the velocity is determined by registration of the throw distance of the glass balls. This velocity distribution is compared to the theoretical MAXWELL-BOLTZMANN equation.

### Equipment

Kinetic gas theory apparatus	09060.00	1
Receiver with recording chamber	09061.00	1
Power supply 0-12 V DC/6 V, 12 V AC	13505.93	1
Digital stroboscope	21809.93	1
Stopwatch, digital, 1/100 sec.	03071.01	1
Tripod base -PASS-	02002.55	2
Connecting cord, $l = 750$ mm, red	07362.01	1
Connecting cord, $l = 750$ mm, blue	07362.04	1
Test tube, 160×16 mm, 100 pcs	37656.10	1
Test tube rack f. 12 tubes, wood	37686.00	1

### Tasks

1. Measure the velocity distribution of the “model gas”.
2. Compare the result to theoretical behaviour as described by the MAXWELL-BOLTZMANN distribution.
3. Discuss the results.

### Set-up and procedure

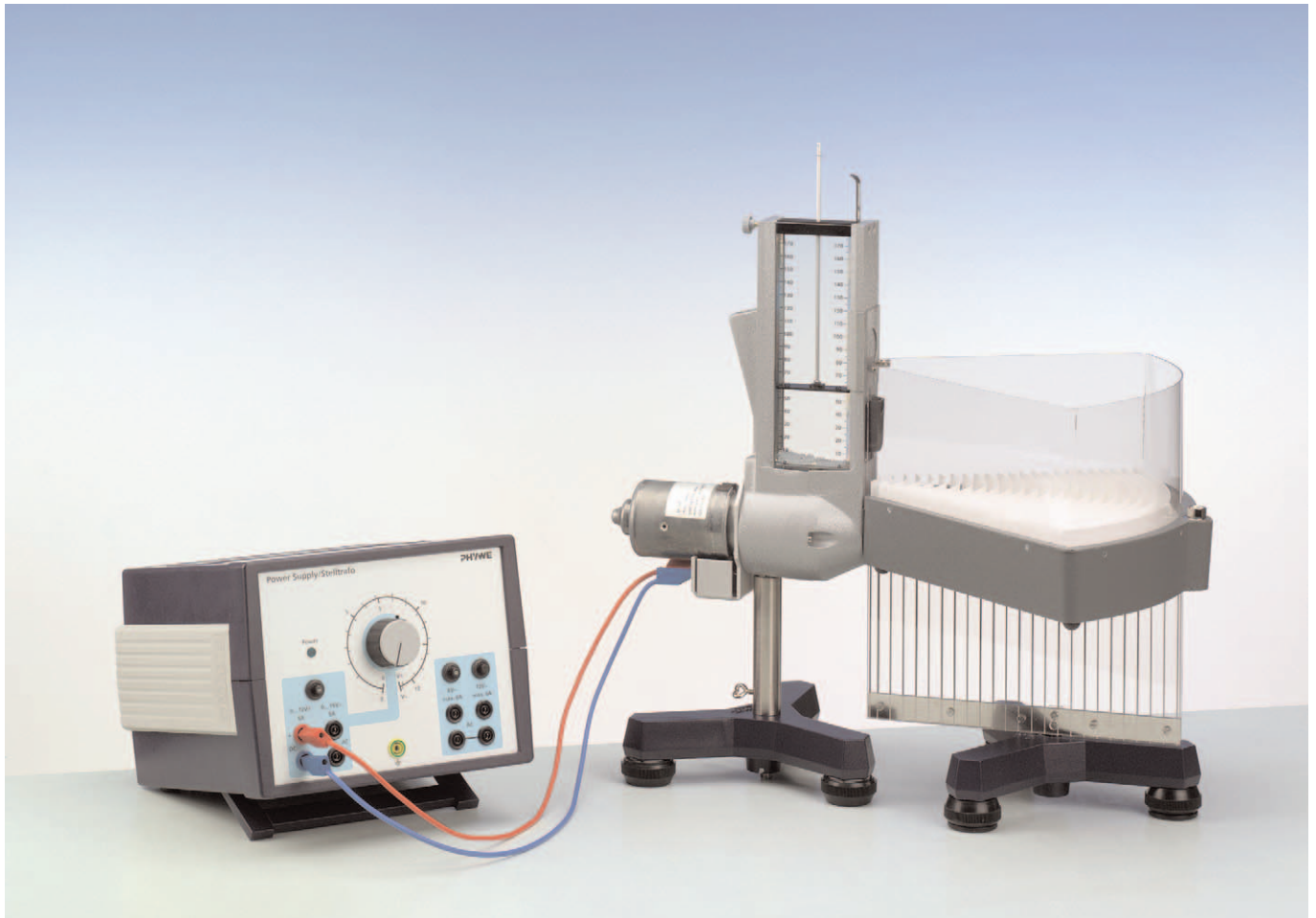
The experimental set-up is as shown in Figure 1.

According to the operating instructions for the determination of particle velocities the apparatus is equipped with the corresponding receiver with recording chamber.

At first determine the average weight of one glass ball by weighing out a known number (100) of balls to avoid the time consuming counting of glass balls during the experiments. In a next experiment the average number of glass balls pushed out from the apparatus during 1 minute is determined. Therefore fill the apparatus with 400 glass balls and set it to the following conditions:

- height of the upper piston: 6 cm
- frequency of the oscillator:  $50 \text{ s}^{-1}$  (controlled by the voltage and the stroboscope).

Fig. 1: Experimental set-up: Maxwellian velocity distribution.



Then open the outlet for 1 minute and determine the number of pushed out balls by weighing. Afterwards the apparatus is refilled with these balls and the same experiment is repeated. For preparing the simulation experiment four test tubes were filled each with the determined average number of lost balls per minute. Then make the following settings at the apparatus:

- height of the upper piston: 6 cm
- height difference between outlet and receiver: 8 cm
- number of balls: 400
- frequency of oscillator: 50 s<sup>-1</sup>

When the frequency is stable open the outlet for 5 minutes. Every one minute fill up the apparatus with the balls in a test tube to maintain a constant "particle density". Afterwards the test tubes are refilled.

This experiment is repeated four times.

The number of glass balls in each of the 24 compartments of the receiver is determined by weighing.

### Theory and evaluation

The molecules of an ideal gas possesses per definition only kinetic energy which is defined by

$$\overline{E}_k = \frac{m}{2} \cdot \overline{c}^2 \quad (1)$$

$\overline{E}_k$  Average kinetic energy

$m$  Mass of the molecule

$\overline{c}$  Average velocity of the molecule

Based on the kinetic theory the pressure of an ideal gas can be described by

$$p = \frac{1}{3} \cdot \rho \cdot \overline{c}^2 \quad (2)$$

$p$  Pressure

$\rho$  Density

Combining of the equations (1), (2) and the equation of state for ideal gases

$$p \cdot V_{\text{mol}} = R \cdot T \quad (3)$$

$V_{\text{mol}}$  Molar volume

$R$  Gas constant

$T$  Absolute temperature

leads to the following expression for  $\overline{c}$ :

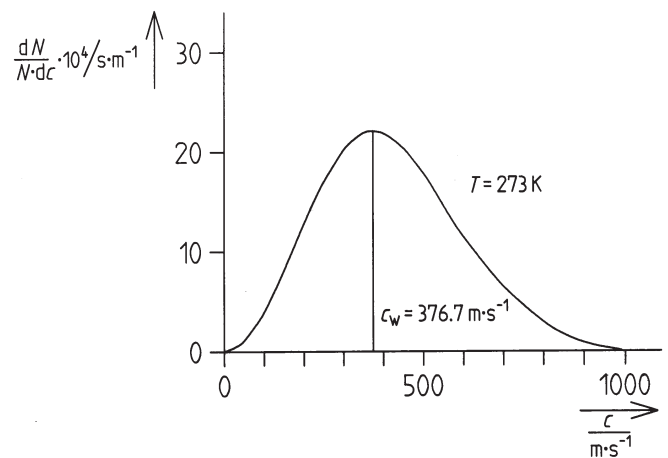
$$\overline{c} = \left( \frac{3 \cdot R \cdot T}{M} \right)^{1/2}$$

$$\overline{c} = \left( \frac{3 \cdot k \cdot T}{m} \right)^{1/2} \quad (4)$$

$k$  BOLTZMANN constant

This means that the average kinetic energy is directly proportional to the absolute temperature of the gas which is the interpretation of temperature on the molecular level.

Fig. 2: Distribution of molecule velocities of oxygen at 273 K.



The direct determination of the velocity of a certain molecule is impossible because it changes incessantly caused by collisions with other molecules. For a great number of molecules one can derive a distribution function for molecule velocities by means of statistical methods. This was done by MAXWELL and BOLTZMANN with the following result:

$$\frac{dN}{N} = \sqrt{\frac{2}{\pi}} \cdot \left( \frac{m}{k \cdot T} \right)^{3/2} \cdot c^2 \cdot \exp\left( -\frac{m \cdot c^2}{2 k T} \right) \cdot dc \quad (5)$$

This equation describes the probability that the velocity of a molecule is within the interval  $(c, c+dc)$ . As an example the corresponding distribution function for oxygen at 273 K is shown in Figure 2.

For the velocity at the maximum  $c_w$  of the curve (velocity with highest probability) the following relation can be derived:

$$c_w = \left( \frac{2 \cdot k \cdot T}{m} \right)^{1/2} \quad (6)$$

Introducing of equation (6) into equation (5) leads to

$$\frac{dN}{N} = \frac{4}{\sqrt{\pi}} \cdot \left( \frac{1}{c_w^2} \right)^{3/2} \cdot c^2 \cdot \exp\left( -\frac{c^2}{c_w^2} \right) \cdot dc \quad (7)$$

Note that  $c_w \neq \overline{c}$   
and  $c_w : \overline{c} : \sqrt{\overline{c}^2} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3} = 1 : 1.13 : 1.22$ .

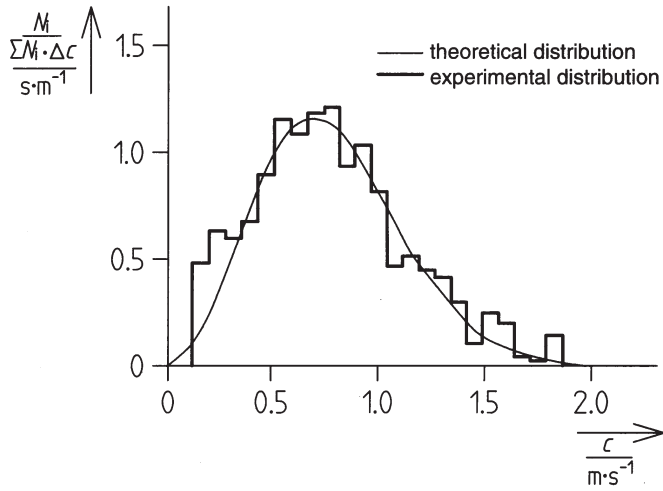
Concerning the model experiment with glass balls the velocity of the balls can be determined by means of the throw distance  $s$ :

$$c = s \cdot \left( \frac{g}{2 \cdot h} \right)^{1/2} = K \cdot s \quad (8)$$

$g$  Acceleration at the earth surface (= 9.81 ms<sup>-2</sup>)

$h$  Height difference between outlet and receiver

Fig. 3: Experimental and theoretical velocity distribution in the model experiment



Now the experimental results (number of balls per throw distance interval) can be displayed graphically in the form

$$\frac{1}{\sum N_i} \cdot \frac{N_i}{\Delta c} = f(c) \quad (9)$$

- $N_i$  = Number of balls in the interval  $i, i = 1 \dots 23$
- $\Delta c$  = Velocity interval corresponding to  $\Delta s$   
= 1 cm ( $0.078 \text{ ms}^{-1}$ )

as shown in Figure 3.

The theoretical distribution function can be evaluated by means of equation (7) using the velocity at the maximum of the experimental distribution as  $c_w$ . The result for the example in Figure 3 is also shown in the diagram. The agreement between the two curves is reasonably good taking into account the model character of the experiment.

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## Maxwellian velocity distribution

