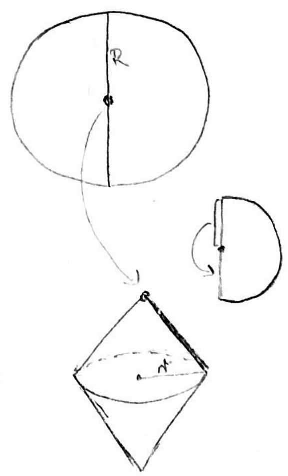
16. Krug površine $100\ cm^2$ razrezan je na dva polukruga koji su zatim savinuti u plašteve i spojeni tako da se dobije tijelo oblika <u>bove</u>. Izračunajte volumen tog tijela.

2 „korneta"



R... polumjer danog kruga

r ... polumjer baze stošca (sredina bove)

Tražimo volumen bove → 2 · volumen stošca

V... volumen bove

V_s... volumen stošca

$V = 2 \cdot V_s$

$V_s = \frac{1}{3} P_{BAZE} \cdot h_{STOŠCA}$

$P_{BAZE} = r^2\pi$

$P_{KRUGA} = 100\ cm^2 = R^2\pi$

$\Rightarrow \quad R = \sqrt{\frac{100}{\pi}} = \frac{10\sqrt{\pi}}{\pi}\ cm$

$O_{KRUGA} = 2R\pi = 2 \cdot \frac{10\sqrt{\pi}}{\pi} \cdot \pi\ cm = 20\sqrt{\pi}\ cm$

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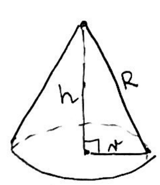
opseg baze stošca = poluopseg danog kruga $= \frac{1}{2} O_{KRUGA}$

$2r\pi = \frac{1}{2} \cdot 20\sqrt{\pi} \quad \Rightarrow \quad r = \frac{10\sqrt{\pi}}{2\pi} = \frac{5\sqrt{\pi}}{\pi}\ cm$

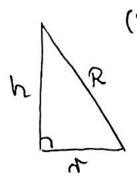


→ duljina svake izvodnice stošca je R

h... visina stošca

$\Rightarrow \quad h = \sqrt{R^2 - r^2} = \sqrt{\frac{100}{\pi} - \frac{25}{\pi}} = \sqrt{\frac{75}{\pi}} = \frac{5\sqrt{3}}{\sqrt{\pi}} = \frac{5\sqrt{3\pi}}{\pi}\ cm$

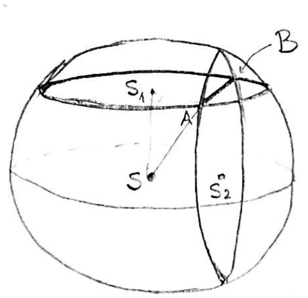
(Pitagora)



$V_s = \frac{1}{3} \cdot r^2\pi \cdot h = \frac{1}{3} \cdot \frac{25}{\cancel{\pi}} \cdot \cancel{\pi} \cdot \frac{5\sqrt{3\pi}}{\pi} = \frac{125\sqrt{3\pi}}{3\pi}\ cm^3$

$\Rightarrow \boxed{V = 2 \cdot V_s = \frac{250\sqrt{3\pi}}{3\pi}\ cm^3}$

(19) Dva međusobno okomita presjeka kugle, površina $185\pi\ cm^2$ i $320\pi\ cm^2$ sijeku se po tetivi duljine 16 cm. Koliki je polumjer te kugle?



S - središte kugle

r_1, r_2 - polumjeri presjeka (presjeci su krugovi)

S_1, S_2 - središta presjeka

A, B - krajevi tetive po kojoj se presjeci sijeku

R - polumjer kugle

$|SA| = |SB| = R$

$|S_1A| = |S_1B| = r_1$

$|S_2A| = |S_2B| = r_2$

$r_1^2\pi = 185\pi\ cm^2$

$r_2^2\pi = 320\pi\ cm^2$

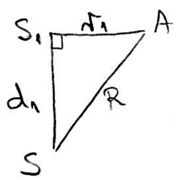
$|AB| = 16\ cm$

$R = ?$

Pokušat ćemo R izraziti preko $r_1, r_2, |AB|$.

Presjek kugle (tj. njegova površina) ovisi o udaljenosti od središta kugle pa uvodimo:

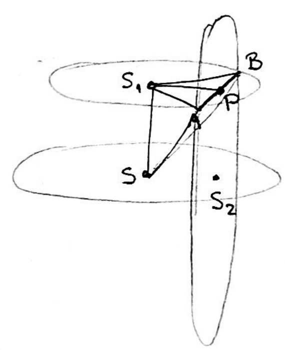
d_1, d_2 - udaljenosti ravnina presjeka od središta kugle S $(d_1 = |SS_1|,\ d_2 = |SS_2|)$

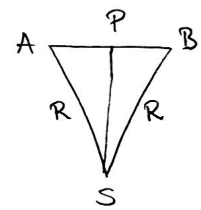


$d_1^2 + r_1^2 = R^2$

$d_2^2 + r_2^2 = R^2$ (analogno iz ΔSS_2A)

Neka je P polovište od $\overline{AB}$.



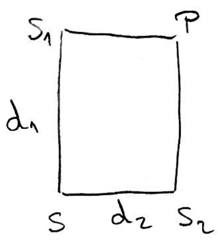


ΔABS jednakokr.

$\Rightarrow |SP|^2 + |PA|^2 = R^2$

$|SP|^2 = R^2 - \left(\frac{1}{2}|AB|\right)^2$

Uočimo da je SS_1PS_2 pravokutnik.



(paralelnost i okomitost odgovarajućih ravnina)

$\Rightarrow |SP|^2 = d_1^2 + d_2^2$

Spajamo:

$R^2 = d_1^2 + r_1^2 = d_2^2 + r_2^2$

$d_1^2 + d_2^2 = |SP|^2 = R^2 - \left(\frac{1}{2}|AB|\right)^2$

$d_1^2 + d_2^2 = (R^2 - r_1^2) + (R^2 - r_2^2)$

$R^2 = d_1^2 + 185 = d_2^2 + 320$

$\underbrace{d_1^2 + d_2^2 = R^2 - \frac{1}{4}\cdot 16^2 = 2R^2 - r_1^2 - r_2^2}$

$R^2 = r_1^2 + r_2^2 - 64 = 185 + 320 - 64$

$R^2 = 441$

$\boxed{R = 21}$