NAPREDNI FIZIČKI PRAKTIKUM 1
SMJER: ISTRAŽIVAČKI STUDIJ FIZIKE

ELEKTRIČNI TITRAJNI KRUG

ZADACI

1. Odredite ovisnost impedancije o frekvenciji za

   - serijski RLC krug, za različite vrijednosti otpora, za različite vrijednosti kapaciteta, te za različite vrijednosti induktiviteta

   - paralelni RLC krug, za različite vrijednosti otpora, za različite vrijednosti kapaciteta, te za različite vrijednosti induktiviteta.

2. Za paralelni RLC krug izvedite izraz za impedanciju.

3. Q-faktor je definiran kao omjer rezonantne frekvencije ω_0 i širine pojasa propuštanja $\Delta\omega = \omega_2 - \omega_1$, gdje su ω_1 i ω_2 definirani tako da je $|Z(\omega_{1,2})| = \sqrt{2}R$. Odredite širinu pojasa propuštanja i Q-faktor iz izmjerenih rezultata, te ih usporedite s teorijskim vrijednostima navedenim u drugom dijelu pripreme.

4. Za serijski RLC krug izmjerite ovisnosti $I(t)$ i $U_2(t)$ za različite frekvencije, te potom iz Lissajousovih krivulja odredite fazni pomak. Nacrtajte graf ovisnosti faznog pomaka o frekvenciji.

5. Spojite sklop kao na slici 1. Na ulaz "Analog in 2" spojite napon s kon-
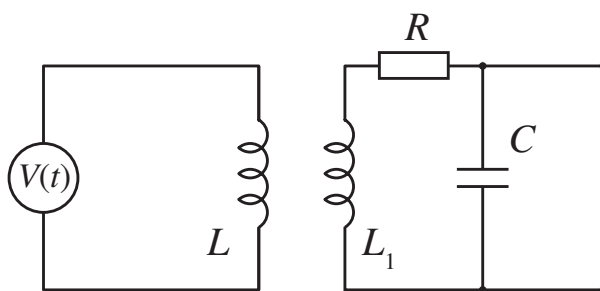
Slika 1. Shema za zadatak 5.

denzatora. Odredite period i frekvenciju prigušenog titranja za 3 različita otpora. Prigušeno titranje možete vidjeti ako je frekvencija titranja izvora mala i ako za oblik signala uzmete pravokutni.

6. Izmjerite amplitude za deset uzastopnih titraja (uvijek s iste strane, odnosno s istim predznakom). Ponoviti postupak za tri različita otpornika. Uz pomoć jednadžbe $V_n/V_{n+1} = \exp(\alpha t)$, gdje je $\alpha = R/2L$ faktor gušenja, prikažite V_n kao funkciju od V_0. Odredite logaritamski dekrement $\lambda = \ln(V_n/V_{n+1})$ za 3 različita otpora (prikažite podatke na istom grafu; obratite pažnju na pogodan odabir koordinatnih osi).

Related topics
Tuned circuit, series-tuned circuit, parallel-tuned circuit, resistance, capacitance, inductance, capacitor, coil, phase displacement, Q-factor, band-width ,impedance, loss resistance, damping.

Principle
The impedance of parallel and series tuned circuits is investigated as a function of frequency. The Q-factor and band-width of the circuits are investigated. The phase displacement between current and voltage is investigated for the series-tuned circuit.

Equipment
Cobra3 Basic Unit	12150.00	1
Power supply, 12 V-	12151.99	2
RS 232 data cable	14602.00	1
Cobra3 Power Graph software	14525.61	1
Cobra3 Universal writer software	14504.61	1
Cobra3 Function generator module	12111.00	1
Coil, 3600 turns	06516.01	1
Connection box	06030.23	1
PEK carbon resistor 1 W 5% 100 Ω	39104.63	1
PEK carbon resistor 1 W 5% 220 Ω	39104.64	1
PEK carbon resistor 1 W 5% 470 Ω	39104.15	1
PEK capacitor/case 2/1 µF/ 250 V	39113.01	1
PEK capacitor/case 1/2.2 µF/ 250 V	39113.02	1
PEK capacitor/case 1/4.7 µF/ 250 V	39113.03	1
Connecting plug	39170.00	2
Connecting cord, l = 250 mm, red	07360.01	2
Connecting cord, l = 250 mm, blue	07360.04	1
Connecting cord, l = 500 mm, red	07361.01	2
Connecting cord, l = 500 mm, blue	07361.04	2
PC, Windows® 95 or higher		

Tasks
– Determine the frequency dependence of the impedance of
  – a series tuned circuit with different damping resistors and different values of capacitance
  – a parallel tuned circuit with different damping resistors and different values of capacitance.
– Determine the Q-factor and the band-width from the obtained curves.
– Determine the frequency dependence of the phase shift between current and voltage in a series tuned circuit.

Set-up and procedure
– The experimental set up is as shown in Figs. 1, 2a and 2b.
– Connect the COBRA3 Basic Unit to the computer port COM1, COM2 or to USB port (for USB computer port use USB to RS232 Converter 14602.10).
– Start the "measure" program and select "Gauge" > "Cobra3 PowerGraph".
– Click the "Analog In2/S2" and select the "Module / Sensor" "Burst measurement" with the parameters seen in Fig. 3.
– Click the "Function Generator" symbol and set the parameters as in Fig. 4.
– Add a "Virtual device" by clicking the white triangle in the upper left of the "PowerGraph" window or by right-clicking the "Cobra3 Basic-Unit" symbol. Turn off all channels but the first and configure this one as seen in Fig. 5.

Fig. 1: Experimental set up for the measurement of the resonance frequency.
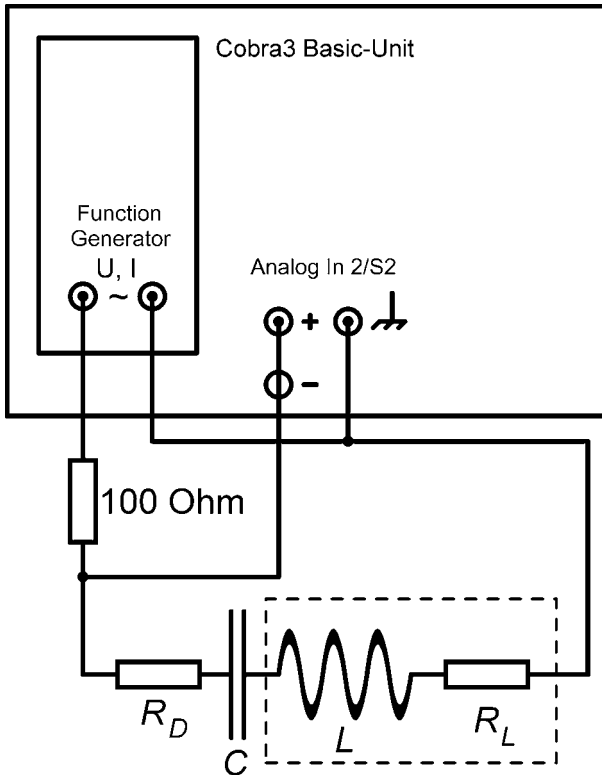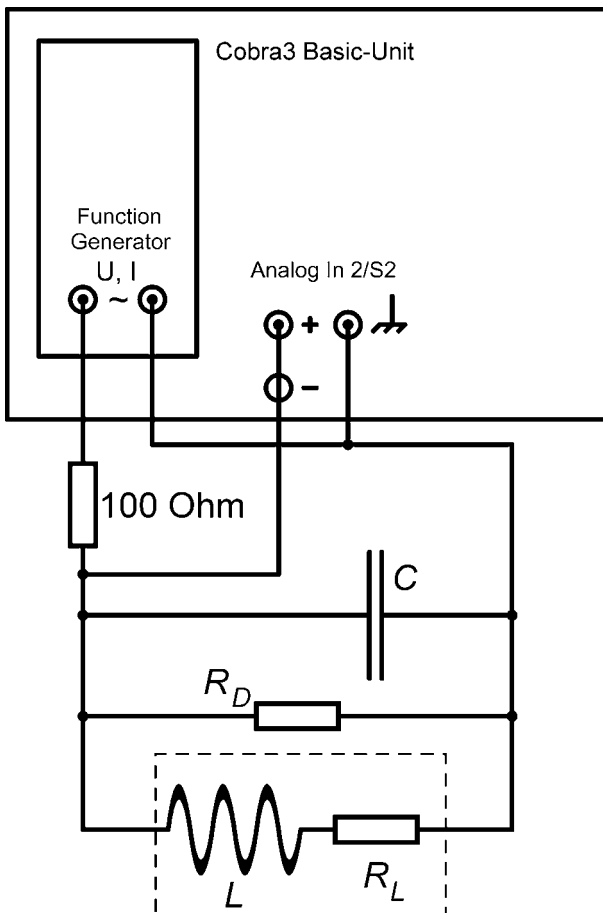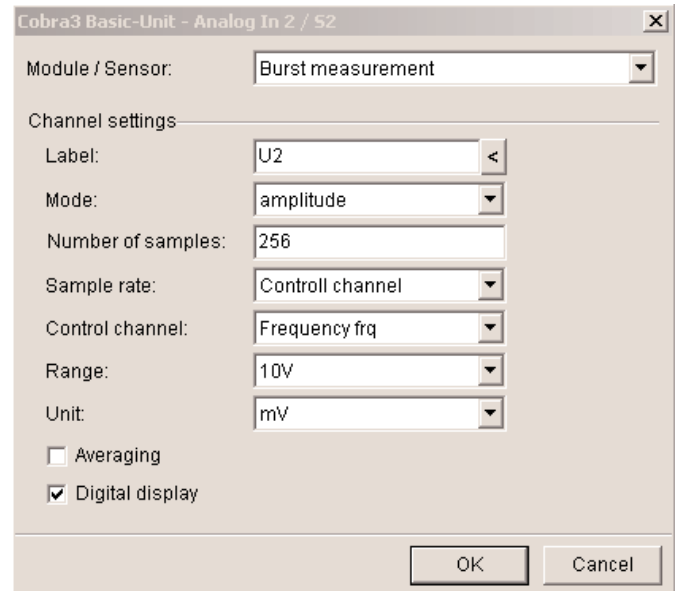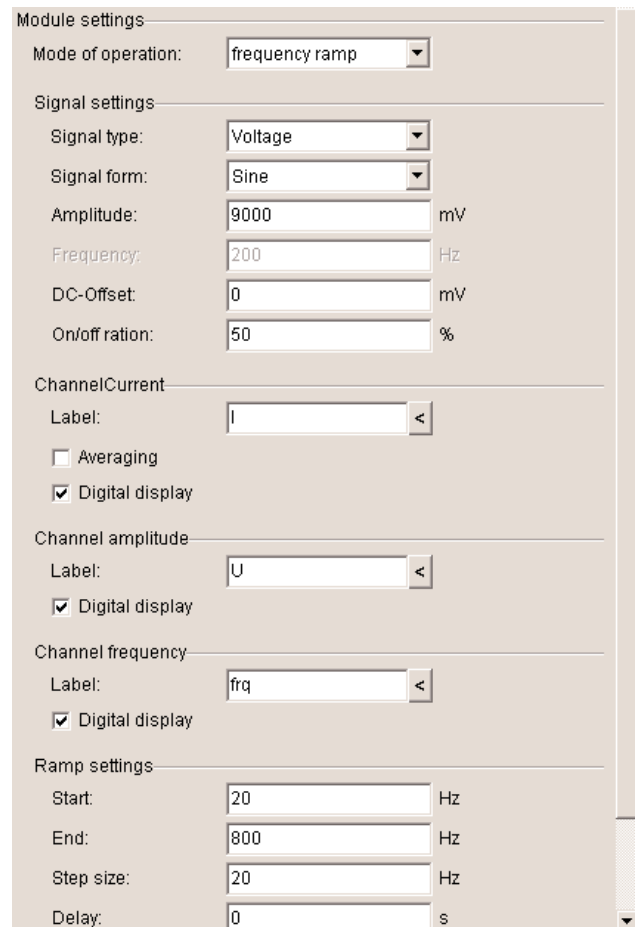
Fig. 2a: Series tuned RLC circuit.

Fig. 3: Analog In 2 S2 – settings

– The "Settings" chart of PowerGraph should look like Fig. 6.
– Configure a diagram to be seen during the measurement on the "Displays" chart of PowerGraph as in Fig. 7 and turn on some Displays for the frequency, the voltages and the current.

Fig. 2b: Parallel tuned RLC circuit.

Fig. 4: Function Generator settings

– Set up a series tuned circuit as seen in Fig. 2a. Start a measurement with the "Continue" button. After the measurement has stopped, the recorded curves are visible in the "measure" program main menu.
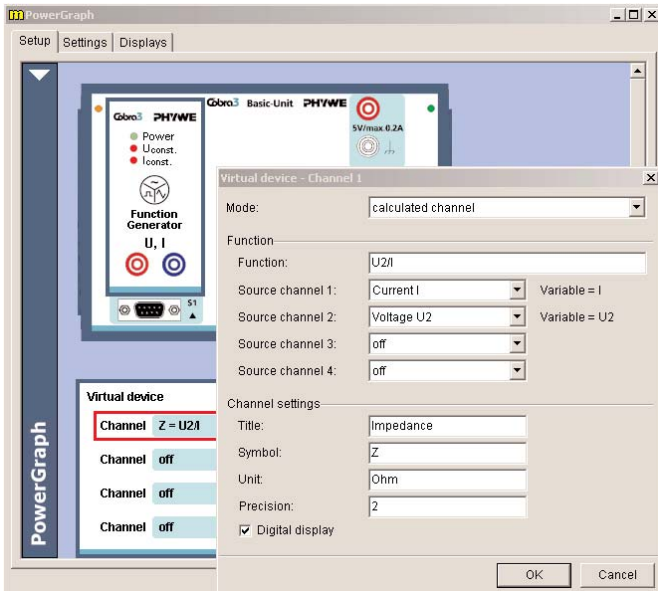
Fig. 5: Virtual device settings.
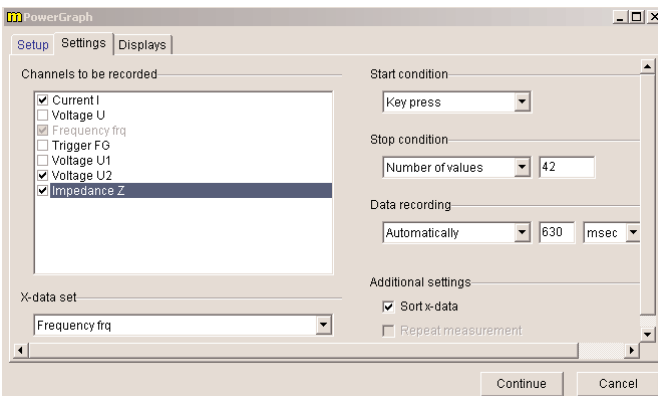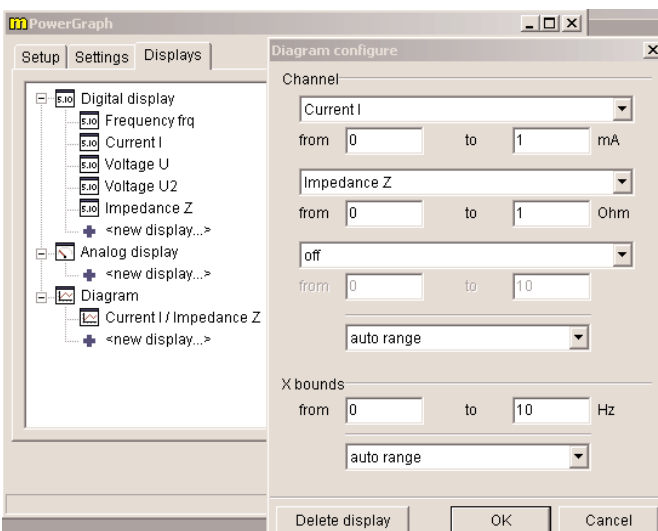

Fig. 6: "Settings" chart of PowerGraph.

Fig. 7: "Displays" chart of PowerGraph.

– Record curves for R_D = 0 Ω, 220 Ω, 470 Ω with the 2.2 µF capacitor.
– Record curves for R_D = 0 Ω with the 1 µF capacitor and the 4.7 µF capacitor.
– Use "Measurement" > "Assume channel…" and "Measurement" > "Channel manager…" to display the three impedance curves with the damping resistor values R_D = 0 Ω, 220 Ω, 470 Ω for the 2.2 µF capacitor in a single plot. Scale the impedance curves to the same value either using the "Scale curves" tool with the option "set to values" or using "Measurement" > "Display options…" filling appropriate values into the field "Displayed area" on the "Channels" chart. The result may look like Fig. 8.
– In a similar way produce a plot of the impedance over the frequency for the series tuned circuit with no additional damping resistor and the three capacitance values C = 1 µF, 2.2 µF, 4.7 µF. Fig. 9 shows a possible result.

Fig. 8: Impedance in dependence on frequency for different damping resistors in a series tuned circuit.
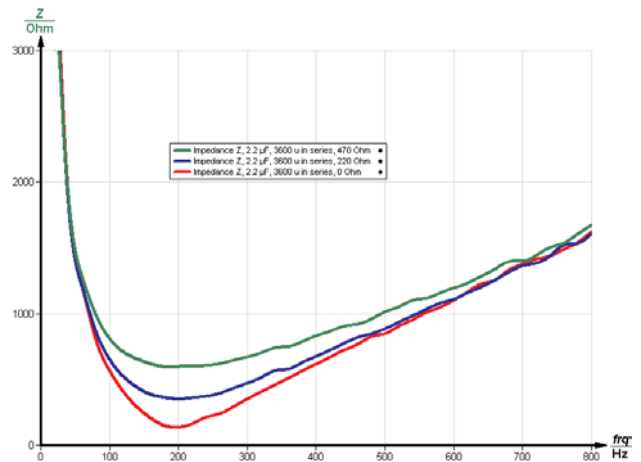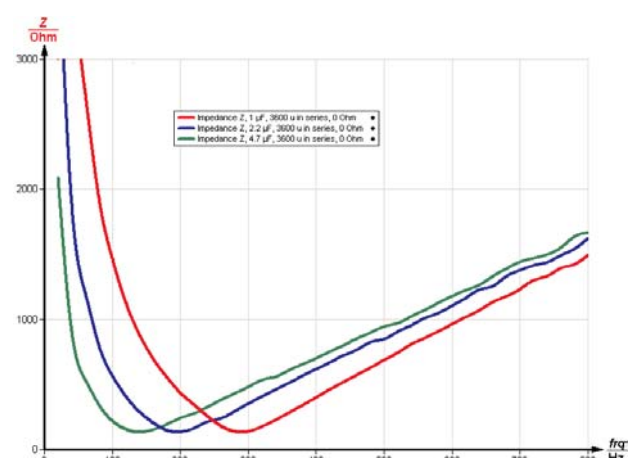

Fig. 9: Impedance in dependence on frequency for different capacitors in a series tuned circuit.

– Set up a parallel tuned circuit as in Fig. 2b.
– Record curves with the 2.2 µF capacitor and different damping resistors $R_D = \infty$ Ω, 470 Ω, 220 Ω.
– Record curves with the damping resistor $R_D = \infty$ Ω (i.e. no resistor) and $C = 1$ µF, 4.7 µF.
– Plot the impedance in dependence on the frequency for $C = 2.2$ µF and $R_D = \infty$ Ω, 470 Ω, 220 Ω (Fig. 10).
– Plot the impedance in dependence on the frequency for $C = 1$ µF, 2.2 µF, .47 µF and $R_D = \infty$ Ω (Fig. 11).
– Set up a series tuned circuit as seen in Fig. 2a with $R_D = 0$ Ω and $C = 2.2$ µF.
– Select "Gauge" > "Cobra3 Universal Writer" and select the parameters as seen in Fig. 12.
– Record current and voltage curves in dependence on time for different frequencies between 80 Hz and 360 Hz. For frequencies over 200 Hz it is necessary to switch the frequency range under "Configure FG module" to "High frequencies".

– Note which of the curves, current or voltage, was ahead of the other.
– Use "Analysis" > "Smooth…" with the options "left axis" and "add new" on both current and voltage curves. The curve that was clicked on before will be processed.
– Use "Measurement" > "Channel manager…" to select the "Current FG' "values as x-axis and the "Analog in 2' "-voltage values as y-axis (Fig. 13). The Lissajous-figure to be produced now is no function but a relation so select in the "Convert relation to function" window the option "Keep measurement in relation mode".
– Use the "Survey" tool to determine the maximal extension of the Lissajous-figure in x-direction ΔI_{max} (Fig. 14) and the extension of the figure on the y = 0 line ΔI_0 (Fig. 15).
– The ratio $\Delta I_0 / \Delta I_{max}$ equals the sine of the phase shift angle $\sin(\varphi)$ between current and voltage.
– Calculate φ and $\tan(\varphi)$ for the used frequencies and plot them over the frequency using "Measurement" > "Enter data manually…" (Fig. 16).
– You may use "Measurement" > "Function generator…" to compare calculated theoretical values with the measured values. Fig. 17 shows the equation for coil with $L = 0.3$ mH and d.c. resistance $R_L = 150$ Ω in series with a 2.2 µF capacitor with no additional damping resistor.

Fig. 10: Impedance in dependence on frequency for different damping resistors in a parallel tuned circuit
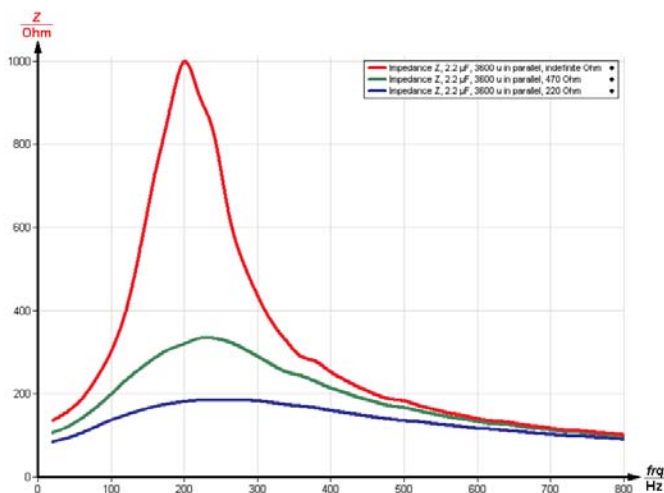

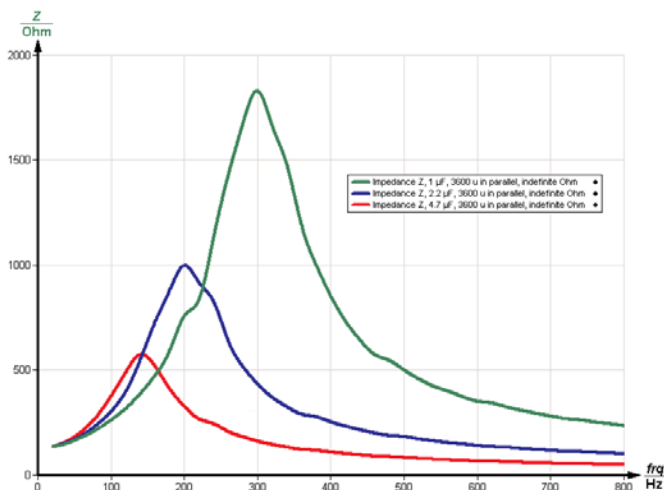Fig. 11: Impedance in dependence on frequency for different capacitors in a parallel tuned circuit.
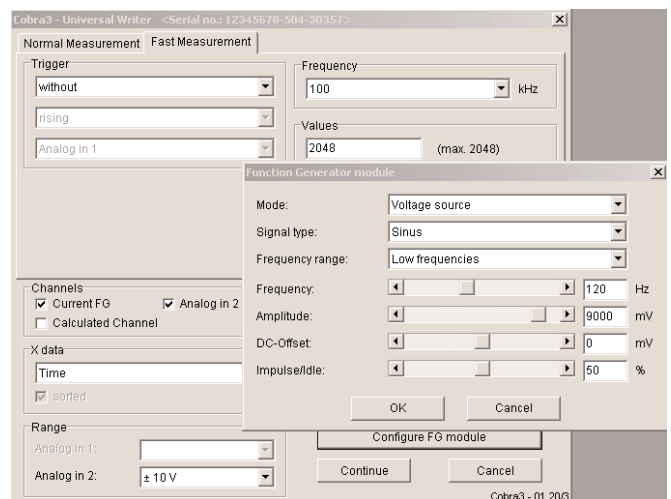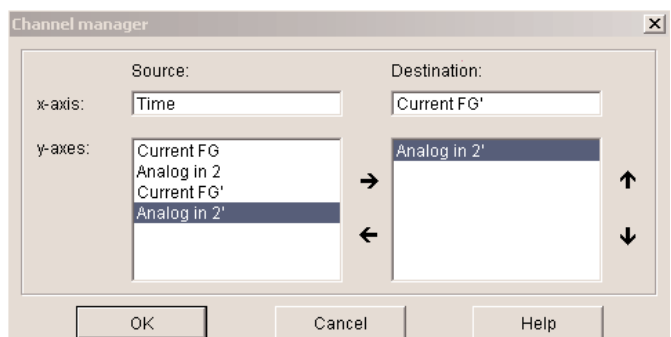

Fig. 12: Universal Writer settings.

Fig. 13: Channel manager.

Theory and evaluation
– Series tuned circuit
A coil with inductance L and ohmic resistance R_L , a capacitance C and an ohmic resistance R_D are connected in series to an alternating voltage source

$$U(t) = \hat{U} \cdot e^{i\omega t}$$

with the angular frequency $\omega = 2\,\pi\,f$. The ohmic resistances add up to a total ohmic resistance $R = R_L + R_D$. Inductance L and ohmic resistance R_L of the coil are in series because all the current going through the coil is affected by the ohmic resistance of the long coil wire. Though Lenz's rule states $U_L = -L \cdot dI/dt$, here the polarity of the voltage on the coil has to be included as positive, because if a voltage is switched on on an ideal coil, the induced voltage on the coil is such, that the positive pole of the coil is there, where it is connected to the positive pole of the voltage source.

Fig. 14: Determining ΔI_{max} on the Lissajous-figure displaying the phase shift between current and voltagetuned circuit.

Fig. 15: Determining ΔI_0 on the Lissajous-figure displaying the phase shift between current and voltage.
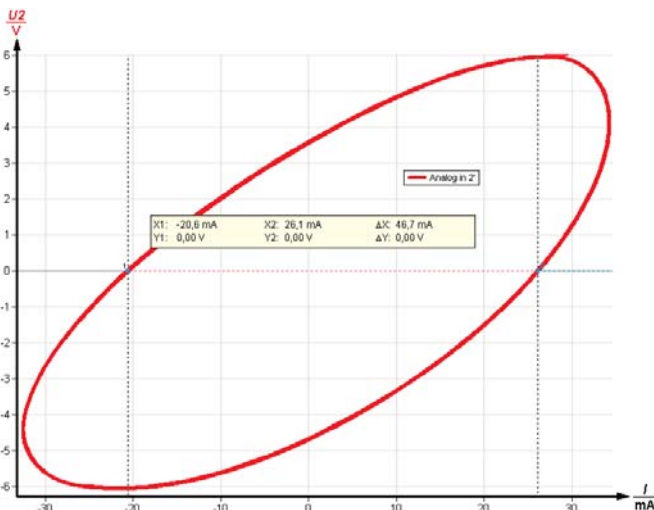
So Kirchhoff's voltage law becomes

$$U(t) = I(t) \cdot R + L\,\dot{I}(t) + \frac{1}{C}\,Q(t) \qquad (1)$$

with the current $I(t)$ and the charge on the capacitor $Q(t)$. With

$$I(t) = \dot{Q}(t)\,,$$

differentiating (1) yields

$$\dot{U} = \frac{1}{C}\,I + R\dot{I} + L\ddot{I} \qquad (2).$$

With $\dot{U} = i\omega\hat{U} \cdot e^{i\omega t}$ and the approach

$$I = e^{-i\varphi}\hat{I}\,e^{i\omega t}, \dot{I} = i\omega e^{-i\varphi}\,\hat{I}\,e^{i\omega t}, \ddot{I} = -\omega^2 e^{-i\varphi}\hat{I}\,e^{i\omega t}$$

and the impedance $Z = \dfrac{\hat{U}}{\hat{I}}$ equation(2) becomes

$$i\omega Z = e^{-i\varphi}\left(\frac{1}{C} + i\omega R - \omega^2 L\right) \qquad (3).$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

So the impedance of the circuit becomes infinite for low frequencies – the capacitor blocks all d.c. current. For low frequencies the capacitor dominates the behavior of the circuit. The impedance has a minimum for

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{thus} \quad \omega_0 = \frac{1}{\sqrt{LC}} \ ,$$

where only the pure ohmic resistance comes to effect. For high frequencies the high coil's impedance prevails. The series tuned circuit is a bandpass filter that has a low impedance only for the frequencies around it's resonance frequency ω_0.

Fig. 16: Phase shift angle and tangent of phase shift angle between current and voltage on a series tuned circuit.
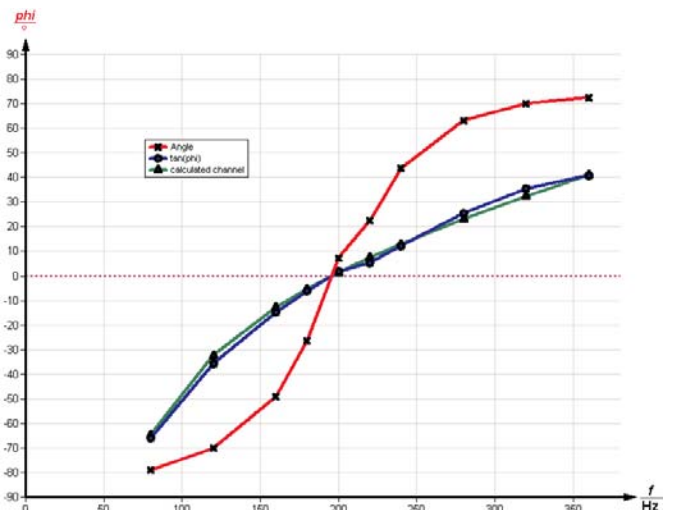
Using $e^{-i\varphi} = \cos\varphi - i\sin\varphi$ and splitting (3) into real and imaginary part yields for the imaginary part (since Z is real):

$$i\left(\omega L - \frac{1}{\omega C}\right)\cos\varphi - iR\sin\varphi = 0,$$

$$\tan\varphi = \frac{\omega L - \dfrac{1}{\omega C}}{R} \tag{4}$$

This term is negative for low frequencies – i.e. the exponent of the current function $e^{i(\omega t - \varphi)}$ has a higher value than the one for the voltage function $e^{i\omega t}$. This means the current is ahead of the voltage for low frequencies and behind the voltage for high frequencies.

For the series tuned circuit the quality factor Q_S is defined as

$$Q_S = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The quality factor determines the bandwidth of the circuit $\Delta\omega = \omega_2 - \omega_1$ with

$$Q_S = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} \text{ and } Z(\omega_{1,2}) = \sqrt{2}\cdot R.$$

and . The resonance frequency is the geometric mean value of the frequencies ω_2 and ω_1:

$$\omega_0 = \sqrt{\omega_1\cdot\omega_2} = 1/\sqrt{LC}$$

In the measurement data of Fig. 8 and Fig. 9 the quality factor can be determined with the "Survey" tool of "measure". The measured resistance value includes the ohmic resistance of the coil and is about that amount greater than the nominal R_D. A digital multimeter measured the coil's d.c. resistance to 132 Ω.

– Parallel tuned circuit
The correct calculation for the circuit of Fig. 2b is more complicated since there is a relevant ohmic resistance both parallel and in series to the inductance. The calculation is left out here but can be carried out in the same manner as above using now both Kirchhoff's current and Kirchhoff's voltage rule.

The resonance frequency is for low R_L again

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

Here the impedance is maximal for the resonance frequency. Below it the coil acts as a shortcut and at zero frequency the impedance curve starts at the d.c. resistance of the coil, if no damping resistor is connected ($R_D = \infty$ Ω). For high frequencies the capacitor acts as a shortcut and the impedance goes to zero for $f \to \infty$.

For low R_L: The quality factor Q_P for the parallel tuned circuit is defined as

$$Q_P = R_D\sqrt{\frac{C}{L}}$$

again with $\Delta\omega = \omega_2 - \omega_1$ and $Q_P = \dfrac{\omega_0}{\Delta\omega} = \dfrac{f_0}{\Delta f}$, but here

$$Z(\omega_{1,2}) = \frac{1}{\sqrt{2}}Z(\omega_0),\ \omega_0 = \sqrt{\omega_1\cdot\omega_2} = 1/\sqrt{LC}.$$

Note
The 100 Ω resistor is in the circuit to minimize possible interferences between the parallel tuned circuit and the function generator module output.

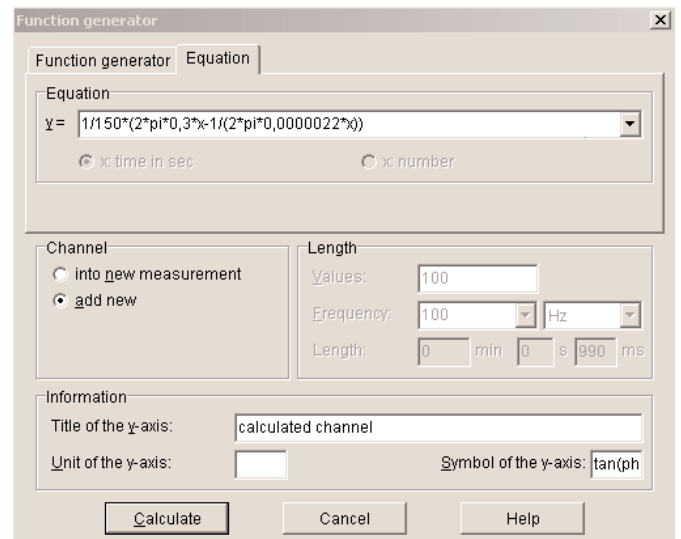Fig. 17: "Equation" chart of the Function generator.

Table 1: Series tuned circuit with coil with nominal 0.3 mH and $R_L = 150$ Ω

C / µF (nom.)	R_D / Ω (nom.)	R / Ω (meas.)	Δf / Hz (meas.)	f_0 / Hz (meas.)	f_0 / Hz (theor.)	Q (meas.)
1	0	133	67	290	291	4.3
2.2	0	134	68	195	196	2.9
4.7	0	133	70	140	134	2.0
2.2	220	351	186	200	196	1.1
2.2	470	598	315	206	196	0.7