Title: Generalised upper Banach density, VC dimension, and Euclidean point configurations

Abstract: We study two related problems of finding large copies of Euclidean configurations in large sets. We begin by recalling the basic conjecture of Székely, Bourgain's approach to proving the conjecture, as well as briefly recalling some of the results inspired by Bourgain's proof. The problem is concerned with finding a condition (the titular generalisation of upper Banach density) on two measurable sets, which ensures that all sufficiently large distances between them are attained, thus obtaining a result analogous to the classical Szekely's conjecture. Interestingly enough, there is a related problem about determining the VC dimension of certain families of (parts of) curves. The first part of the talk will focus on motivating tese problems and explaining the basic definititions needed for their statement. The second part of the talk will be dedicated to the harmonic analysis toolkit needed to prove these results.