Riješeni primjeri i zadaci s glavnim testom za FO

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Primjer 1

Ispitajmo valjanost formule $(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x))$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \to \exists x B(x) \quad \top$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \quad \bot \qquad (..a..)$$

$$A(a) \quad \bot$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @a$$

$$A(a) \to B(a) \quad \bot$$

$$A(a) \quad \top$$

$$B(a) \quad \bot$$

$$X$$

$$\exists x B(x) \quad \top \qquad (..b..)$$

$$B(b) \quad \top$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @b$$

$$A(b) \to B(b) \quad \bot$$

$$A(b) \quad \top$$

$$B(b) \quad \bot$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \to \exists x B(x) \quad \top$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \quad \bot \quad (..a..)$$

$$A(a) \quad \bot$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @$$

$$A(a) \to B(a) \quad \bot$$

$$A(a) \quad \top$$

$$B(a) \quad \bot$$

$$X$$

$$\exists x B(x) \quad \top \quad (..b..)$$

$$B(b) \quad \top$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @$$

$$A(b) \to B(b) \quad \bot$$

$$A(b) \quad \top$$

$$B(b) \quad \bot$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \to \exists x B(x) \quad \top$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \quad \bot \quad (..a..) \qquad\qquad \exists x B(x) \quad \top \quad (..b..)$$

$$A(a) \quad \bot$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @a$$

$$A(a) \to B(a) \quad \bot$$

$$A(a) \quad \top$$

$$B(a) \quad \bot$$

$$X$$

$$B(b) \quad \top$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @b$$

$$A(b) \to B(b) \quad \bot$$

$$A(b) \quad \top$$

$$B(b) \quad \bot$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \to \exists x B(x) \quad \top$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \quad \bot \qquad (..a..)$$

$$A(a) \quad \bot$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @a$$

$$A(a) \to B(a) \quad \bot$$

$$A(a) \quad \top$$

$$B(a) \quad \bot$$

$$X$$

$$\exists x B(x) \quad \top \qquad (..b..)$$

$$B(b) \quad \top$$

$$(*) \quad \exists x (A(x) \to B(x)) \quad \bot \quad @b$$

$$A(b) \to B(b) \quad \bot$$

$$A(b) \quad \top$$

$$B(b) \quad \bot$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad (\bot)$$

$$\forall x A(x) \to \exists x B(x) \quad (\top)$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \bot$$

$$\forall x A(x) \; (\bot) \quad (..a..) \qquad \exists x B(x) \; (\top) \quad (..b..)$$

$$A(a) \; (\bot) \qquad B(b) \; (\top)$$

$$(*) \quad \exists x (A(x) \to B(x)) \; \bot \; @a \qquad (*) \quad \exists x (A(x) \to B(x)) \; \bot \; @b$$

$$A(a) \to B(a) \; (\bot) \qquad A(b) \to B(b) \; (\bot)$$

$$A(a) \; (\top) \qquad A(b) \; (\top)$$

$$B(a) \; (\bot) \qquad B(b) \; (\bot)$$

$$X \qquad X$$

$(\forall x A(x) \to \exists x B(x)) \to \exists x(A(x) \to B(x))$ $\bot$

$\forall x A(x) \to \exists x B(x)$ $\top$

$(*)$ $\exists x(A(x) \to B(x))$ $\bot$

$\forall x A(x)$ $\bot$ $(..a..)$

$A(a)$ $\bot$

$(*)$ $\exists x(A(x) \to B(x))$ $\bot$ ⓐ

$A(a) \to B(a)$ $\bot$

$A(a)$ $\top$

$B(a)$ $\bot$

X

$\exists x B(x)$ $\top$ $(..b..)$

$B(b)$ $\top$

$(*)$ $\exists x(A(x) \to B(x))$ $\bot$ ⓑ

$A(b) \to B(b)$ $\bot$

$A(b)$ $\top$

$B(b)$ $\bot$

X

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓐ \qquad\qquad (*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓑ$$

$$A(a) \to B(a) \textcircled{\perp} \qquad\qquad A(b) \to B(b) \textcircled{\perp}$$

$$A(a) \textcircled{\top} \qquad\qquad A(b) \textcircled{\top}$$

$$B(a) \textcircled{\perp} \qquad\qquad B(b) \textcircled{\perp}$$

$$X \qquad\qquad X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓐ$$

$$A(a) \to B(a) \textcircled{\perp}$$

$$A(a) \ \textcircled{\top}$$

$$B(a) \ \perp$$

$$X$$

$$B(b) \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓑ$$

$$A(b) \to B(b) \textcircled{\perp}$$

$$A(b) \textcircled{\top}$$

$$B(b) \textcircled{\perp}$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓐ \qquad\qquad (*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓑ$$

$$A(a) \to B(a) \textcircled{\perp} \qquad\qquad A(b) \to B(b) \textcircled{\perp}$$

$$A(a) \textcircled{\top} \qquad\qquad A(b) \textcircled{\top}$$

$$B(a) \ \perp \qquad\qquad B(b) \textcircled{\perp}$$

$$X \qquad\qquad X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓐ$$

$$A(a) \to B(a) \textcircled{\perp}$$

$$A(a) \textcircled{\top}$$

$$B(a) \textcircled{\perp}$$

$$X$$

$$B(b) \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓑ$$

$$A(b) \to B(b) \textcircled{\perp}$$

$$A(b) \textcircled{\top}$$

$$B(b) \textcircled{\perp}$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓐ \qquad\qquad (*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓑ$$

$$A(a) \to B(a) \textcircled{\perp} \qquad\qquad A(b) \to B(b) \textcircled{\perp}$$

$$A(a) \textcircled{\top} \qquad\qquad A(b) \textcircled{\top}$$

$$B(a) \textcircled{\perp} \qquad\qquad B(b) \textcircled{\perp}$$

$$X \qquad\qquad X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓐ$$

$$A(a) \to B(a) \textcircled{\perp}$$

$$A(a) \textcircled{\top}$$

$$B(a) \textcircled{\perp}$$

$$X$$

$$B(b) \ \top$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓑ$$

$$A(b) \to B(b) \textcircled{\perp}$$

$$A(b) \textcircled{\top}$$

$$B(b) \textcircled{\perp}$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓐ$$

$$A(a) \to B(a) \textcircled{\perp}$$

$$A(a) \textcircled{\top}$$

$$B(a) \textcircled{\perp}$$

$$X$$

$$B(b) \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \quad ⓑ$$

$$A(b) \to B(b) \textcircled{\perp}$$

$$A(b) \textcircled{\top}$$

$$B(b) \textcircled{\perp}$$

$$X$$

$(\forall x A(x) \to \exists x B(x)) \to \exists x(A(x) \to B(x))$ $\bot$

$\forall x A(x) \to \exists x B(x)$ $\top$

$(*)$ $\exists x(A(x) \to B(x))$ $\bot$

$\forall x A(x)$ $\bot$ $(..a..)$

$A(a)$ $\bot$

$(*)$ $\exists x(A(x) \to B(x))$ $\bot$ ⓐ

$A(a) \to B(a)$ $\bot$

$A(a)$ $\top$

$B(a)$ $\bot$

X

$\exists x B(x)$ $\top$ $(..b..)$

$B(b)$ $\top$

$(*)$ $\exists x(A(x) \to B(x))$ $\bot$ ⓑ

$A(b) \to B(b)$ $\bot$

$A(b)$ $\top$

$B(b)$ $\bot$

X

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \perp \quad \textcircled{a}$$

$$A(a) \to B(a) \textcircled{\perp}$$

$$A(a) \textcircled{\top}$$

$$B(a) \textcircled{\perp}$$

$$X$$

$$\exists x B(x) \quad \textcircled{\top} \quad (..b..)$$

$$B(b) \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \to B(x)) \perp \quad \textcircled{b}$$

$$A(b) \to B(b) \perp$$

$$A(b) \top$$

$$B(b) \perp$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..) \qquad\qquad \exists x B(x) \ \textcircled{\top} \quad (..b..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \ \perp \ \textcircled{a} \qquad\qquad (*) \quad \exists x (A(x) \to B(x)) \ \perp \ \textcircled{b}$$

$$A(a) \to B(a) \textcircled{\perp} \qquad\qquad A(b) \to B(b) \textcircled{\perp}$$

$$A(a) \textcircled{\top} \qquad\qquad A(b) \ \top$$

$$B(a) \textcircled{\perp} \qquad\qquad B(b) \ \perp$$

$$X \qquad\qquad X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \perp \quad \textcircled{a}$$

$$A(a) \to B(a) \textcircled{\perp}$$

$$A(a) \textcircled{\top}$$

$$B(a) \textcircled{\perp}$$

$$X$$

$$\exists x B(x) \quad \textcircled{\top} \quad (..b..)$$

$$B(b) \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \to B(x)) \perp \quad \textcircled{b}$$

$$A(b) \to B(b) \textcircled{\perp}$$

$$A(b) \textcircled{\top}$$

$$B(b) \perp$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \perp$$

$$\forall x A(x) \textcircled{\perp} \quad (..a..)$$

$$A(a) \textcircled{\perp}$$

$$(*) \quad \exists x (A(x) \to B(x)) \perp \quad \textcircled{a}$$

$$A(a) \to B(a) \textcircled{\perp}$$

$$A(a) \textcircled{\top}$$

$$B(a) \textcircled{\perp}$$

$$X$$

$$\exists x B(x) \quad \textcircled{\top} \quad (..b..)$$

$$B(b) \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \to B(x)) \perp \quad \textcircled{b}$$

$$A(b) \to B(b) \textcircled{\perp}$$

$$A(b) \textcircled{\top}$$

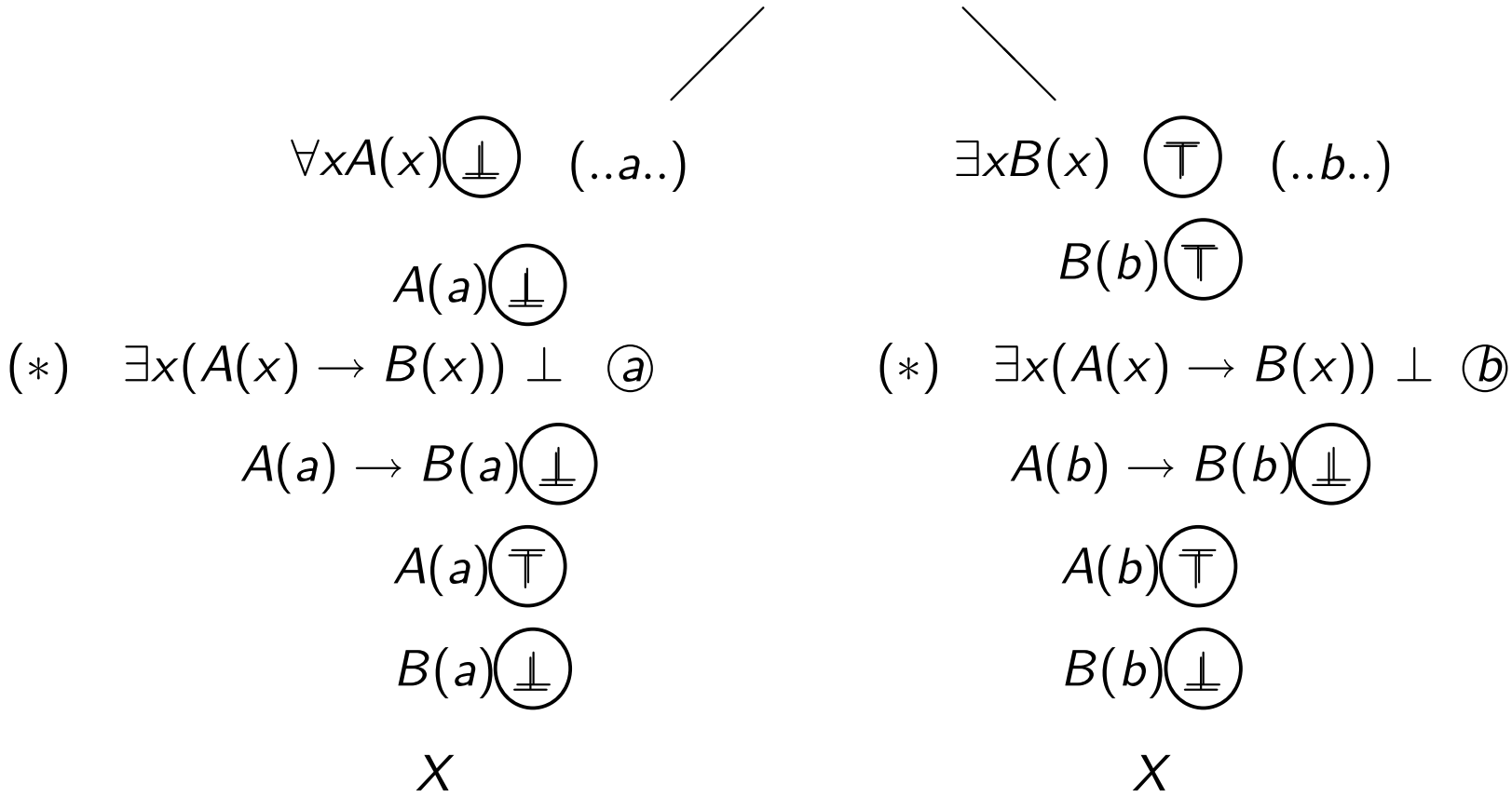
$$B(b) \textcircled{\perp}$$

$$X$$

$$(\forall x A(x) \to \exists x B(x)) \to \exists x (A(x) \to B(x)) \quad \textcircled{\bot}$$

$$\forall x A(x) \to \exists x B(x) \quad \textcircled{\top}$$

$$(*) \qquad \exists x (A(x) \to B(x)) \quad \bot$$



Sljedećim primjerom želimo istaknuti nužnost uvođenja novih elemenata u nosač prilikom analize formula oblika $\forall x G(x) \perp$ i $\exists x G(x) \top$.

Primjer 2

Pomoću glavnog testa ispitajmo vrijedi li

$$\forall x \exists y P(x, y) \rightarrow \forall x Q(x) \models \forall x \forall y (P(x, y) \rightarrow Q(x)).$$

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$ $\quad$ $\forall x Q(x)$ $\top$ @ a b

$\exists y P(c,y)$ $\bot$ @ a b c $\quad$ $Q(a)$ $\top$

$P(c,a)$ $\bot$ $\quad$ X

$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$ $\quad$ $\forall x Q(x)$ $\top$ @ a b

$\exists y P(c,y)$ $\bot$ @ a b c $\quad$ $Q(a)$ $\top$

$P(c,a)$ $\bot$ $\quad$ X

$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$ $\quad$ $\forall x Q(x)$ $\top$ @ b

$\exists y P(c,y)$ $\bot$ @ a b c $\quad$ $Q(a)$ $\top$

$P(c,a)$ $\bot$ $\quad$ X

$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$

$\exists y P(c,y)$ $\bot$ @a @b @c

$P(c,a)$ $\bot$

$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x Q(x)$ $\top$ @a b

$Q(a)$ $\top$

X

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$ $\exists y P(c,y)$ $\bot$ @a @b @c $P(c,a)$ $\bot$ $P(c,b)$ $\bot$ $P(c,c)$ $\bot$

$\forall x Q(x)$ $\top$ @a @b $Q(a)$ $\top$ X

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$ $\quad$ $\forall x Q(x)$ $\top$ a b

$\exists y P(c,y)$ $\bot$ a b c $\quad$ $Q(a)$ $\top$

$P(c,a)$ $\bot$ $\quad$ X

$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\mathbb{T}$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$

$\exists y P(c,y)$ $\bot$ @a @b @c

$P(c,a)$ $\bot$

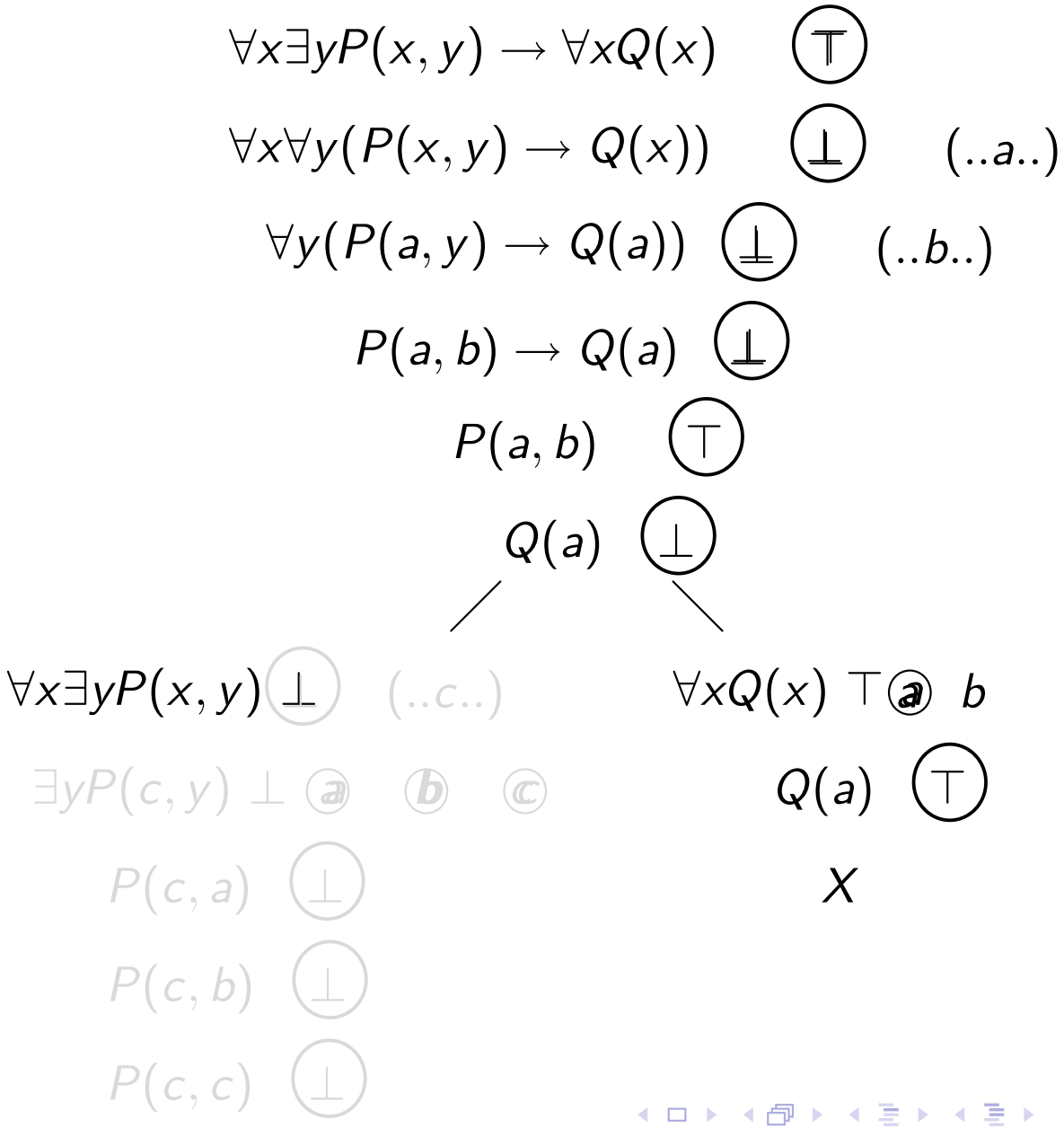
$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x Q(x)$ $\top$ @a b

$Q(a)$ $\top$

X


$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$
$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$
$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$
$P(a,b) \to Q(a)$ $\bot$
$P(a,b)$ $\top$
$Q(a)$ $\bot$
$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$
$\exists y P(c,y)$ $\bot$ @ a @ b @ c
$P(c,a)$ $\bot$
$P(c,b)$ $\bot$
$P(c,c)$ $\bot$
$\forall x Q(x)$ $\top$ @ a b
$Q(a)$ $\top$
X

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\mathbb{T}$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$

$\exists y P(c,y)$ $\bot$ ⓐ ⓑ ⓒ

$P(c,a)$ $\bot$

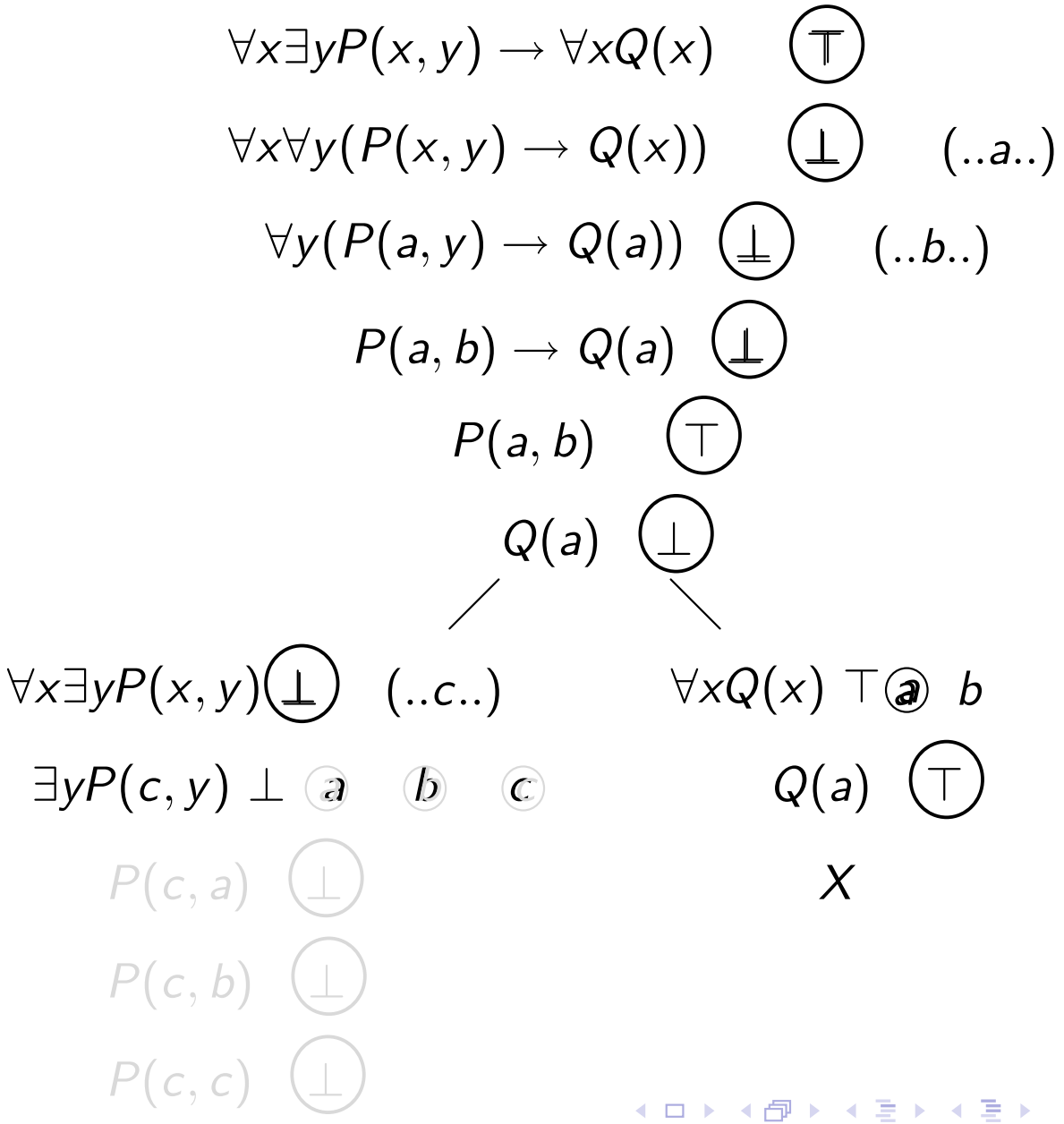
$P(c,b)$ $\bot$

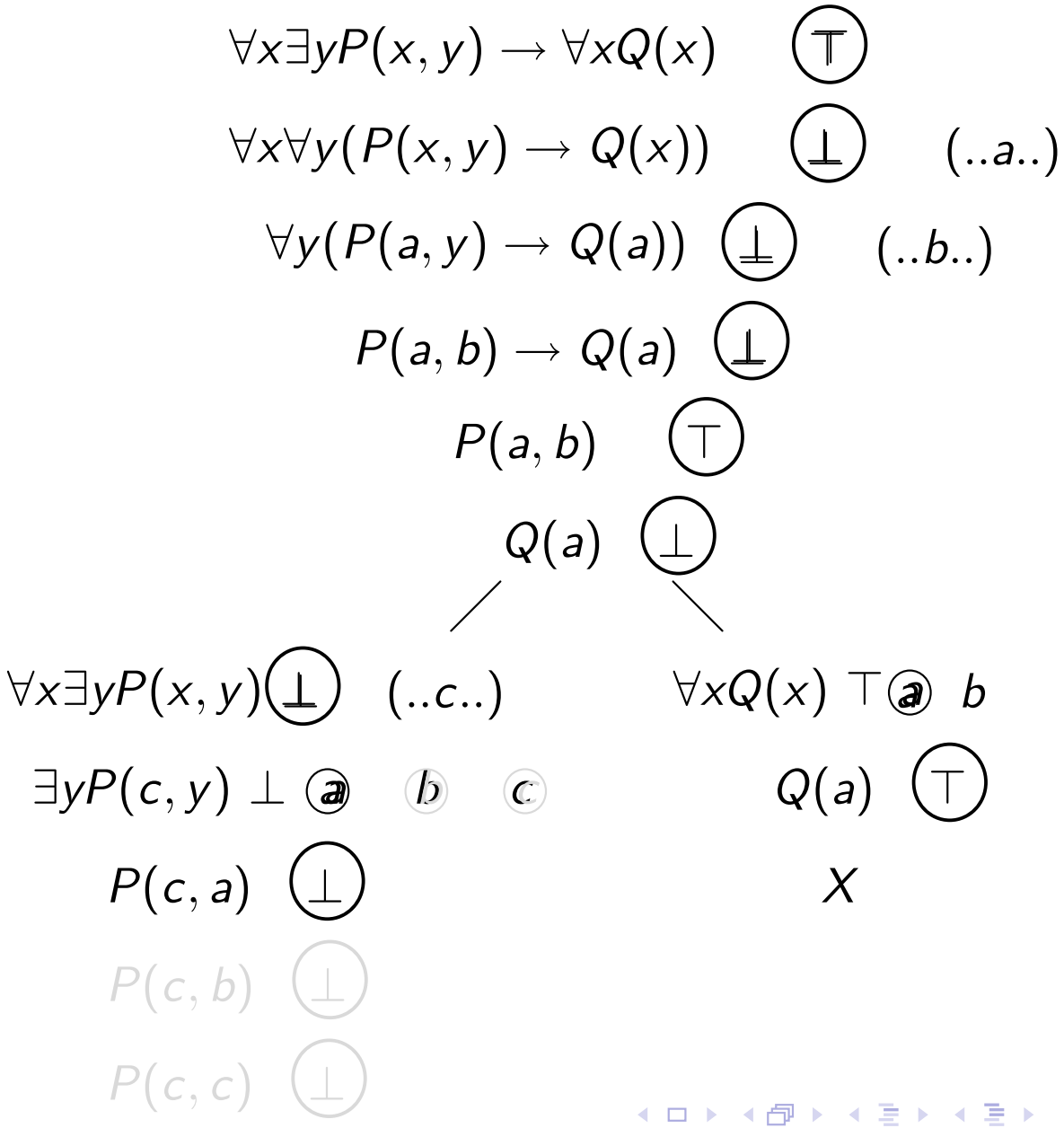
$P(c,c)$ $\bot$

$\forall x Q(x)$ $\top$ ⓐ b

$Q(a)$ $\top$

X


$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$
$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$
$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$
$P(a,b) \to Q(a)$ $\bot$
$P(a,b)$ $\top$
$Q(a)$ $\bot$
$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$
$\exists y P(c,y)$ $\bot$ a b c
$P(c,a)$ $\bot$
$P(c,b)$ $\bot$
$P(c,c)$ $\bot$
$\forall x Q(x)$ $\top$ a b
$Q(a)$ $\top$
X


$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$
$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$
$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$
$P(a,b) \to Q(a)$ $\bot$
$P(a,b)$ $\top$
$Q(a)$ $\bot$
$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$
$\exists y P(c,y)$ $\bot$ @ a b c
$P(c,a)$ $\bot$
$P(c,b)$ $\bot$
$P(c,c)$ $\bot$
$\forall x Q(x)$ $\top$ @ a b
$Q(a)$ $\top$
X

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\mathbb{T}$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$

$\exists y P(c,y)$ $\bot$ @ a @ b c

$P(c,a)$ $\bot$

$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x Q(x)$ $\top$ @ a b

$Q(a)$ $\top$

X

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$

$\forall x \exists y P(x,y)$ $\bot$ $(..c..)$

$\exists y P(c,y)$ $\bot$ @a @b @c

$P(c,a)$ $\bot$

$P(c,b)$ $\bot$

$P(c,c)$ $\bot$

$\forall x Q(x)$ $\top$ @a b

$Q(a)$ $\top$

X

Pošto lijeva grana nije završila oznakom za kontradikciju zaključujemo da dana tvrdnja nije istinita.

S te lijeve grane možemo pročitati strukturu za koju početna tvrdnja nije istinita.

Nosač strukture je $|\mathfrak{M}| = \{a, b, c\}$, te je $P^{\mathfrak{M}} = \{(a, b)\}$ i $Q^{\mathfrak{M}} = \emptyset$.

Uočimo još da na desnoj grani prvi redak oblika $\forall x Q(x)\ \top$ nismo analizirali u odnosu na element b. To nije nužno jer smo već naišli na kontradikciju.

Pogledajmo sada što se događa kada u rješavanju prethodnog zadatka koristimo "stare" elemente.

$\forall x \exists y P(x,y) \to \forall x Q(x)$ $\top$

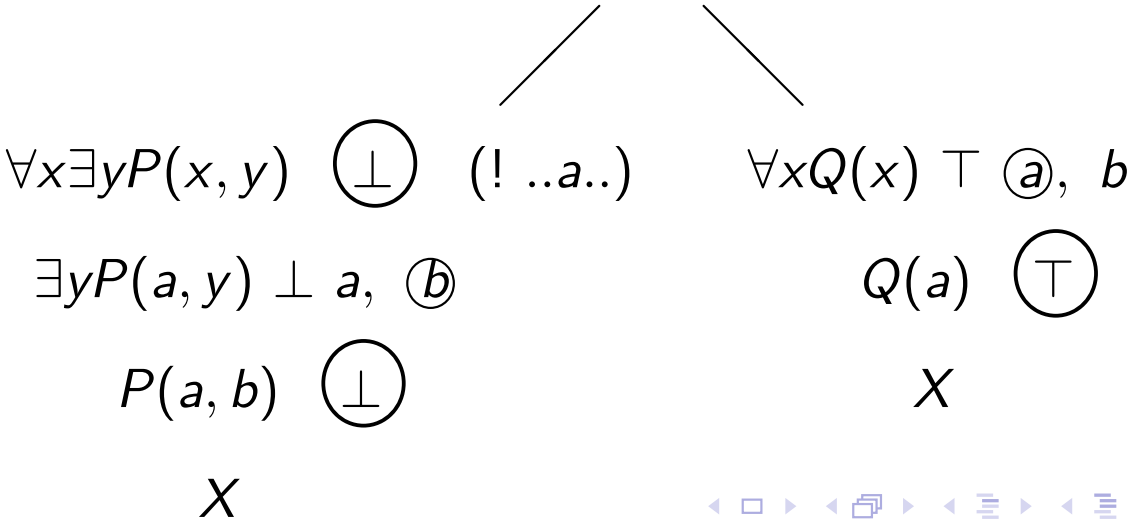
$\forall x \forall y (P(x,y) \to Q(x))$ $\bot$ $(..a..)$

$\forall y (P(a,y) \to Q(a))$ $\bot$ $(..b..)$

$P(a,b) \to Q(a)$ $\bot$

$P(a,b)$ $\top$

$Q(a)$ $\bot$



U sedmom retku smo sa (! ..*a*..) označili da ne uvodimo novi element već koristimo stari.

Dani test je na svim granama završio kontradikcijom, pa bi brzopleto (i krivo) mogli zaključiti da je dana formula valjana.

Iz prethodnog testa znamo da formula nije valjana.

Korištenjem "starog" elementa a mi smo posljednjim testom zapravo dokazali da ne postoji strukura s točno dva elementa koja nije model za F.

U sljedećem primjeru želimo istaknuti kako se glavni test koristi za ispitivanje je li neka formula F **oboriva**.

Početni redak u testu je oblika $F\bot$. To znači da pokušavamo odrediti strukturu koja nije model za formulu F.

Primjer 3

Ispitajmo pomoću glavnog testa je li sljedeća formula oboriva:

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y))) \rightarrow \exists x(Q(x) \vee R(x,x)).$$

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y))) \rightarrow \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y)) \quad \top$$

$$(*) \quad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \quad \top \quad (..a..)$$

$$\forall y R(a,y) \quad \top \; @a$$

$$R(a,a) \quad \top$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \quad \bot \; @a$$

$$Q(a) \vee R(a,a) \quad \bot$$

$$Q(a) \quad \bot$$

$$R(a,a) \quad \bot$$

$$X$$

$$\forall y(\exists x P(y,x) \rightarrow \forall x R(x,y)) \quad \top \quad (..b..) \; @b$$

$$(**) \quad \exists x P(b,x) \rightarrow \forall x R(x,b) \quad \top$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \quad \bot \; @b$$

$$Q(b) \vee R(b,b) \quad \bot$$

$$Q(b) \quad \bot$$

$$R(b,b) \quad \bot$$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \quad \bigcirc\!\!\!\!\perp$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \quad \bigcirc\!\!\!\!\top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \perp$$

$\exists x \forall y R(x,y) \quad \bigcirc\!\!\!\!\top \quad (..a..)$

$\forall y R(a,y) \ \top \ @a$

$R(a,a) \quad \bigcirc\!\!\!\!\top$

$(*) \ \exists x(Q(x) \vee R(x,x)) \ \perp \ @a$

$Q(a) \vee R(a,a) \quad \bigcirc\!\!\!\!\perp$

$Q(a) \quad \bigcirc\!\!\!\!\perp$

$R(a,a) \quad \bigcirc\!\!\!\!\perp$

X

$\forall y(\exists x P(y,x) \to \forall x R(x,y)) \ \top \qquad (..b..) \ \textcircled{b}$

$(**) \qquad \exists x P(b,x) \to \forall x R(x,b) \ \top$

$(*) \ \exists x(Q(x) \vee R(x,x)) \ \perp \ \textcircled{b}$

$Q(b) \vee R(b,b) \quad \bigcirc\!\!\!\!\perp$

$Q(b) \quad \bigcirc\!\!\!\!\perp$

$R(b,b) \quad \bigcirc\!\!\!\!\perp$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \; \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \; \top \quad (..a..) \qquad \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top \quad (..b..) \; b$$

$$\forall y R(a,y) \; \top \; a \qquad (**) \quad \exists x P(b,x) \to \forall x R(x,b) \; \top$$

$$R(a,a) \; \top \qquad (*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; b$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; a \qquad Q(b) \vee R(b,b) \; \bot$$

$$Q(a) \vee R(a,a) \; \bot \qquad Q(b) \; \bot$$

$$Q(a) \; \bot \qquad R(b,b) \; \bot$$

$$R(a,a) \; \bot$$

$$X$$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \; \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \; \top \quad (..a..) \qquad \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top \quad (..b..)$$

$$\forall y R(a,y) \; \top \; @$$

$$(**) \qquad \exists x P(b,x) \to \forall x R(x,b) \; \top$$

$$R(a,a) \; \top$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; @$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; @$$

$$Q(b) \vee R(b,b) \; \bot$$

$$Q(a) \vee R(a,a) \; \bot$$

$$Q(b) \; \bot$$

$$Q(a) \; \bot$$

$$R(b,b) \; \bot$$

$$R(a,a) \; \bot$$

X

Nastavak na sljedećem slajdu

$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y))) \rightarrow \exists x(Q(x) \vee R(x,x))\ \textcircled{\bot}$

$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y))\ \textcircled{\top}$

$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$

$\exists x \forall y R(x,y)\ \textcircled{\top} \quad (..a..)$

$\forall y R(a,y)\ \top\ @a$

$R(a,a)\ \textcircled{\top}$

$(*)\ \exists x(Q(x) \vee R(x,x))\ \bot\ @a$

$Q(a) \vee R(a,a)\ \textcircled{\bot}$

$Q(a)\ \textcircled{\bot}$

$R(a,a)\ \textcircled{\bot}$

X

$\forall y(\exists x P(y,x) \rightarrow \forall x R(x,y))\ \top \qquad (..b..)\ @b$

$(**) \qquad \exists x P(b,x) \rightarrow \forall x R(x,b)\ \top$

$(*)\ \exists x(Q(x) \vee R(x,x))\ \bot\ @b$

$Q(b) \vee R(b,b)\ \textcircled{\bot}$

$Q(b)\ \textcircled{\bot}$

$R(b,b)\ \textcircled{\bot}$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \; \bigcirc\!\!\!\!\perp$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \bigcirc\!\!\!\!\top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \perp$$

$$\exists x \forall y R(x,y) \; \bigcirc\!\!\!\!\top \quad (..a..)$$

$$\forall y R(a,y) \; \top \; @$$

$$R(a,a) \; \bigcirc\!\!\!\!\top$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \perp \; @$$

$$Q(a) \vee R(a,a) \; \bigcirc\!\!\!\!\perp$$

$$Q(a) \; \bigcirc\!\!\!\!\perp$$

$$R(a,a) \; \bigcirc\!\!\!\!\perp$$

$$X$$

$$\forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top \qquad (..b..) \; ⓑ$$

$$(**) \qquad \exists x P(b,x) \to \forall x R(x,b) \; \top$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \perp \; ⓑ$$

$$Q(b) \vee R(b,b) \; \bigcirc\!\!\!\!\perp$$

$$Q(b) \; \bigcirc\!\!\!\!\perp$$

$$R(b,b) \; \bigcirc\!\!\!\!\perp$$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \; \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$\exists x \forall y R(x,y) \; \top \quad (..a..)$

$\forall y R(a,y) \; \top \; @$

$R(a,a) \; \top$

$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; @$

$Q(a) \vee R(a,a) \; \bot$

$Q(a) \; \bot$

$R(a,a) \; \bot$

X

$\forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top \qquad (..b..) \; ⓑ$

$(**) \qquad \exists x P(b,x) \to \forall x R(x,b) \; \top$

$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; ⓑ$

$Q(b) \vee R(b,b) \; \bot$

$Q(b) \; \bot$

$R(b,b) \; \bot$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \; \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \; \top \quad (..a..)$$

$$\forall y R(a,y) \; \top \; @$$

$$R(a,a) \; \top$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; @$$

$$Q(a) \vee R(a,a) \; \bot$$

$$Q(a) \; \bot$$

$$R(a,a) \; \bot$$

$$X$$

$$\forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top \qquad (..b..) \; ⓑ$$

$$(**) \qquad \exists x P(b,x) \to \forall x R(x,b) \; \top$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; ⓑ$$

$$Q(b) \vee R(b,b) \; \bot$$

$$Q(b) \; \bot$$

$$R(b,b) \; \bot$$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y))) \rightarrow \exists x(Q(x) \vee R(x,x)) \; \textcircled{\bot}$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y)) \; \textcircled{\top}$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \textcircled{\top} \quad (..a..) \qquad \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y)) \; \top \qquad (..b..) \; \textcircled{b}$$

$$\forall y R(a,y) \; \top \; \textcircled{a} \qquad (**) \qquad \exists x P(b,x) \rightarrow \forall x R(x,b) \; \top$$

$$R(a,a) \; \textcircled{\top} \qquad (*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; \textcircled{b}$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; \textcircled{a} \qquad Q(b) \vee R(b,b) \textcircled{\bot}$$

$$Q(a) \vee R(a,a) \textcircled{\bot} \qquad Q(b) \; \textcircled{\bot}$$

$$Q(a) \; \textcircled{\bot} \qquad R(b,b) \; \textcircled{\bot}$$

$$R(a,a) \; \textcircled{\bot}$$

$$X$$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \; \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \; \top \quad (..a..) \qquad \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top \quad (..b..) \; ⓑ$$

$$\forall y R(a,y) \; \top \; ⓐ \qquad (**) \quad \exists x P(b,x) \to \forall x R(x,b) \; \top$$

$$R(a,a) \; \top \qquad (*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; ⓑ$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; ⓐ \qquad Q(b) \vee R(b,b) \; \bot$$

$$Q(a) \vee R(a,a) \; \bot \qquad Q(b) \; \bot$$

$$Q(a) \; \bot \qquad R(b,b) \; \bot$$

$$R(a,a) \; \bot$$

$$X$$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y))) \rightarrow \exists x(Q(x) \vee R(x,x)) \; \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y)) \; \top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$$\exists x \forall y R(x,y) \; \top \quad (..a..) \qquad \forall y(\exists x P(y,x) \rightarrow \forall x R(x,y)) \; \top \quad (..b..)\ ⓑ$$

$$\forall y R(a,y) \; \top \; ⓐ \qquad (**) \quad \exists x P(b,x) \rightarrow \forall x R(x,b) \; \top$$

$$R(a,a) \; \top \qquad (*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; ⓑ$$

$$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; ⓐ \qquad Q(b) \vee R(b,b) \; \bot$$

$$Q(a) \vee R(a,a) \; \bot \qquad Q(b) \; \bot$$

$$Q(a) \; \bot \qquad R(b,b) \; \bot$$

$$R(a,a) \; \bot$$

$$X$$

Nastavak na sljedećem slajdu

$$(\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y))) \to \exists x(Q(x) \vee R(x,x)) \; \bot$$

$$\exists x \forall y R(x,y) \vee \forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top$$

$$(*) \qquad \exists x(Q(x) \vee R(x,x)) \quad \bot$$

$\exists x \forall y R(x,y) \; \top \quad (..a..)$

$\forall y R(a,y) \; \top \; @a$

$R(a,a) \; \top$

$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; @a$

$Q(a) \vee R(a,a) \; \bot$

$Q(a) \; \bot$

$R(a,a) \; \bot$

X

$\forall y(\exists x P(y,x) \to \forall x R(x,y)) \; \top \qquad (..b..) \; @b$

$(**) \qquad \exists x P(b,x) \to \forall x R(x,b) \; \top$

$(*) \; \exists x(Q(x) \vee R(x,x)) \; \bot \; @b$

$Q(b) \vee R(b,b) \; \bot$

$Q(b) \; \bot$

$R(b,b) \; \bot$

Nastavak na sljedećem slajdu

$(**)$ $\quad \exists x P(b,x) \to \forall x R(x,b)\ \top$

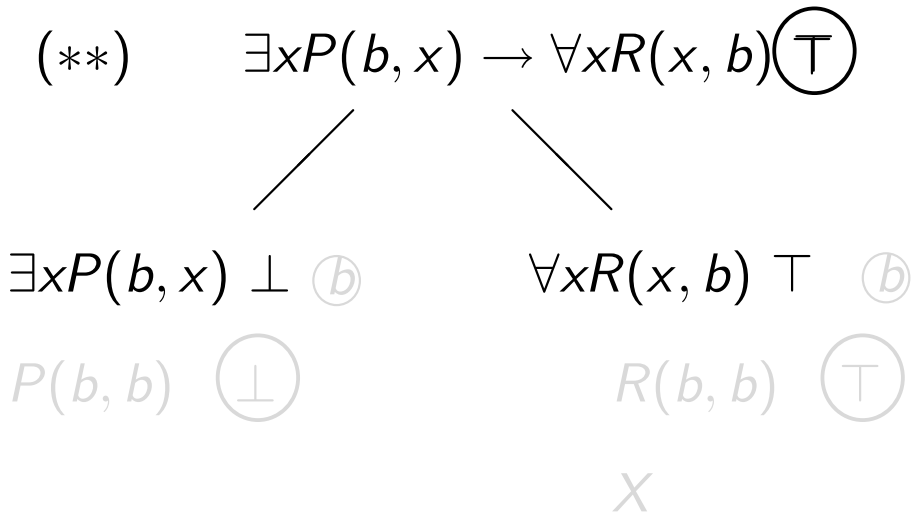
$\exists x P(b,x)\ \bot$ (b)

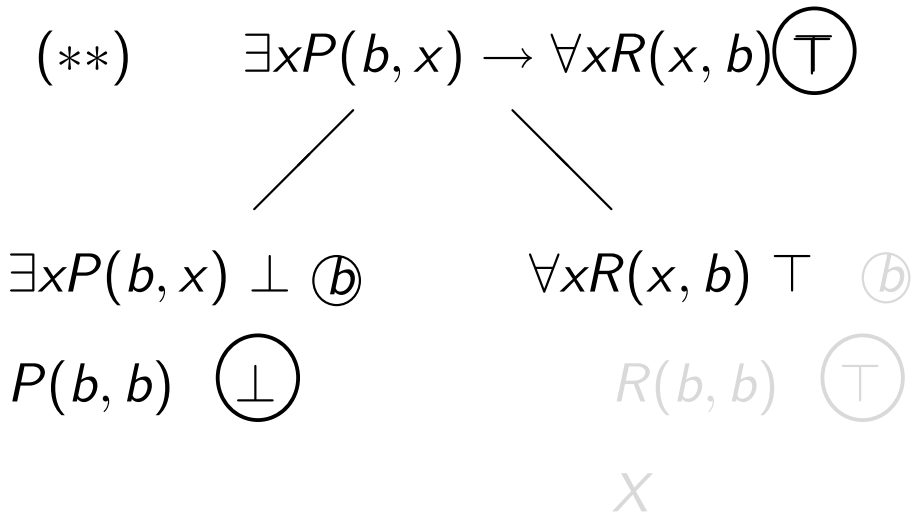
$P(b,b)\ \bot$

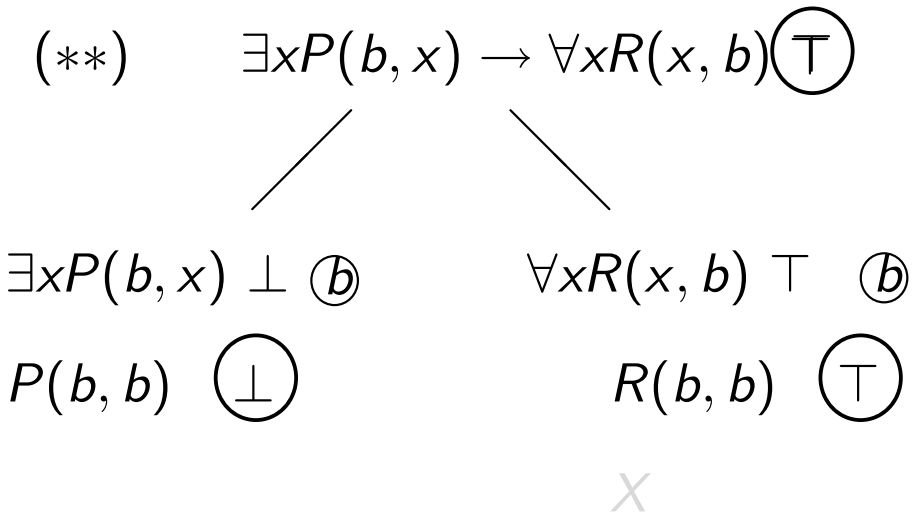
$\forall x R(x,b)\ \top$ (b)

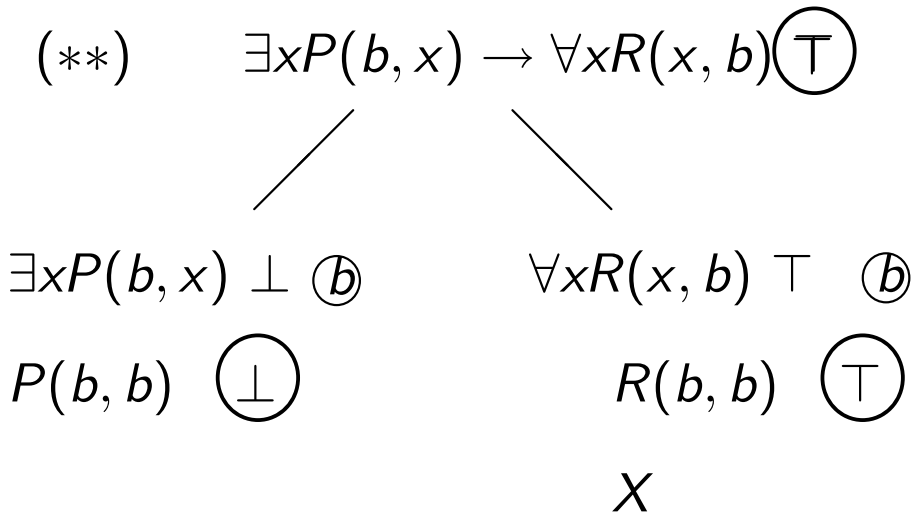
$R(b,b)\ \top$

X


(**) ∃xP(b, x) → ∀xR(x, b) ⊤
∃xP(b, x) ⊥ b
P(b, b) ⊥
∀xR(x, b) ⊤ b
R(b, b) ⊤
X


(**)
$\exists x P(b,x) \to \forall x R(x,b)$ ⊤
$\exists x P(b,x)$ ⊥ b
$P(b,b)$ ⊥
$\forall x R(x,b)$ ⊤ b
$R(b,b)$ ⊤
X


(**) ∃xP(b, x) → ∀xR(x, b) ⊤
∃xP(b, x) ⊥ b
P(b, b) ⊥
∀xR(x, b) ⊤ b
R(b, b) ⊤
X


(**) ∃xP(b, x) → ∀xR(x, b) ⊤
∃xP(b, x) ⊥ b
P(b, b) ⊥
∀xR(x, b) ⊤ b
R(b, b) ⊤
X

Zadana formula je oboriva.

Jedna struktura $\mathfrak{M}$ koja to dokazuje zadana je sa: $|\mathfrak{M}| = \{b\}$, te $Q^{\mathfrak{M}} = R^{\mathfrak{M}} = P^{\mathfrak{M}} = \emptyset$.

Sljedeći primjer pokazuje da nekad test ne mora završiti, ali mi ipak možemo odrediti jednu traženu (beskonačnu) strukturu.

Primjer 4

Ispitajmo je li formula $\forall x \exists y A(x, y) \rightarrow \exists y \forall x A(x, y)$ *valjana.*

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y) \; \bot$

$\forall x \exists y A(x,y) \; \top \quad (..a_1..) \quad a_1 \; a_2 \; a_3 \; a_4 \; a_5$

$\exists y \forall x A(x,y) \; \bot \quad a_1 \; a_2 \; a_3 \; a_4 \; a_5$

$\exists y A(a_1,y) \; \top \quad (..a_2..)$

$\forall x A(x,a_1) \; \bot \quad (..a_3..)$

$A(a_1,a_2) \; \top$

$A(a_3,a_1) \; \bot$

$\exists y A(a_2,y) \; \top \quad (..a_4..)$

$\forall x A(x,a_2) \; \bot \quad (..a_5..)$

$A(a_2,a_4) \; \top$

$A(a_5,a_2) \; \bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

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$A(a_1,a_2)$ $\top$

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$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\forall x \exists y A(x,y) \to \exists y \forall x A(x,y)$ $\bot$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ a_1 a_2 a_3 a_4 a_5

$\exists y \forall x A(x,y)$ $\bot$ a_1 a_2 a_3 a_4 a_5

$\exists y A(a_1,y)$ $\top$ $(..a_2..)$

$\forall x A(x,a_1)$ $\bot$ $(..a_3..)$

$A(a_1,a_2)$ $\top$

$A(a_3,a_1)$ $\bot$

$\exists y A(a_2,y)$ $\top$ $(..a_4..)$

$\forall x A(x,a_2)$ $\bot$ $(..a_5..)$

$A(a_2,a_4)$ $\top$

$A(a_5,a_2)$ $\bot$

$\vdots$

Neka je $|\mathfrak{M}| = \{a_n : n \in \mathbb{N} \setminus \{0\}\}$, te $A^{\mathfrak{M}} = \{(a_n, a_{2n}) : n \in \mathbb{N}\}$

Nije teško vidjeti da vrijedi $\mathfrak{M} \not\models \forall x \exists y A(x, y) \rightarrow \exists y \forall x A(x, y)$.

To znači da dana formula nije valjana.

Primjer 5

Odredimo preneksnu normalnu formu sljedeće formule, a zatim ispitajmo njenu valjanost pomoću glavnog testa

$$(\exists z F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w))$$

Rješenje. Određujemo prvo preneksnu normalnu formu dane formule:

$$(\exists z F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ \forall x \forall y (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z \forall x \forall y (F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\forall z \exists x \exists y \exists w [(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))]$$

Na posljednju formulu primjenjujemo glavni test.

Rješenje. Određujemo prvo preneksnu normalnu formu dane formule:

$$(\exists z F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ \forall x \forall y (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z \forall x \forall y (F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\forall z \exists x \exists y \exists w [(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))]$$

Na posljednju formulu primjenjujemo glavni test.

Rješenje. Određujemo prvo preneksnu normalnu formu dane formule:

$$(\exists z F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w(F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z(F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w(F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z(F(z) \ \wedge \ \forall x \forall y(F(y) \rightarrow G(x))) \rightarrow \exists w(F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z \forall x \forall y(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow \exists w(F(w) \wedge G(w)) \Leftrightarrow$$

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))]$$

Na posljednju formulu primjenjujemo glavni test.

Rješenje. Određujemo prvo preneksnu normalnu formu dane formule:

$$(\exists z F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ \forall x \forall y (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z \forall x \forall y (F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\forall z \exists x \exists y \exists w [(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))]$$

Na posljednju formulu primjenjujemo glavni test.

Rješenje. Određujemo prvo preneksnu normalnu formu dane formule:

$$(\exists z F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ (\exists y F(y) \rightarrow \forall x G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z (F(z) \ \wedge \ \forall x \forall y (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\exists z \forall x \forall y (F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow \exists w (F(w) \wedge G(w)) \Leftrightarrow$$

$$\forall z \exists x \exists y \exists w [(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))]$$

Na posljednju formulu primjenjujemo glavni test.

$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))]$ $\bot$ $(..a..)$

$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))]$ $\bot$ @a

$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))]$ $\bot$ @a

$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))]$ $\bot$ @a

$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a))$ $\perp\!\!\!\perp$

$F(a) \ \wedge \ (F(a) \rightarrow G(a))$ $\top\!\!\!\top$

$F(a) \wedge G(a)$ $\bot$

$F(a)$ $\top\!\!\!\top$

$F(a) \rightarrow G(a)$ $\top$

Nastavak na sljedećem slajdu

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \ \textcircled{\perp} \quad (..a..)$$

$$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a)) \ \textcircled{\perp}$$

$$F(a) \ \wedge \ (F(a) \rightarrow G(a)) \ \textcircled{\top}$$

$$F(a) \wedge G(a) \quad \perp$$

$$F(a) \ \textcircled{\top}$$

$$F(a) \rightarrow G(a) \quad \top$$

Nastavak na sljedećem slajdu

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \textcircled{\perp} \quad (..a..)$$

$$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a)) \quad \textcircled{\perp}$$

$$F(a) \ \wedge \ (F(a) \rightarrow G(a)) \quad \textcircled{\top}$$

$$F(a) \wedge G(a) \quad \perp$$

$$F(a) \quad \textcircled{\top}$$

$$F(a) \rightarrow G(a) \quad \top$$

Nastavak na sljedećem slajdu

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \textcircled{\perp} \quad (..a..)$$

$$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a)) \quad \textcircled{\perp}$$

$$F(a) \ \wedge \ (F(a) \rightarrow G(a)) \quad \textcircled{\top}$$

$$F(a) \wedge G(a) \quad \perp$$

$$F(a) \quad \textcircled{\top}$$

$$F(a) \rightarrow G(a) \quad \top$$

Nastavak na sljedećem slajdu

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \mathbb{1} \quad (..a..)$$

$$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \bot\,@$$

$$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \bot\,@$$

$$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \bot\,@$$

$$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a)) \quad \mathbb{1}$$

$$F(a) \ \wedge \ (F(a) \rightarrow G(a)) \quad \mathbb{T}$$

$$F(a) \wedge G(a) \quad \bot$$

$$F(a) \quad \mathbb{T}$$

$$F(a) \rightarrow G(a) \quad \top$$

Nastavak na sljedećem slajdu

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \mathbb{1} \quad (..a..)$$

$$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \bot\,@$$

$$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \bot\,@$$

$$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \bot\,@$$

$$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a)) \quad \mathbb{1}$$

$$F(a) \ \wedge \ (F(a) \rightarrow G(a)) \quad \top$$

$$F(a) \wedge G(a) \quad \bot$$

$$F(a) \quad \top$$

$$F(a) \rightarrow G(a) \quad \top$$

Nastavak na sljedećem slajdu

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \textcircled{\perp} \quad (..a..)$$

$$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a)) \quad \textcircled{\perp}$$

$$F(a) \ \wedge \ (F(a) \rightarrow G(a)) \quad \textcircled{\top}$$

$$F(a) \wedge G(a) \quad \perp$$

$$F(a) \quad \top$$

$$F(a) \rightarrow G(a) \quad \top$$

Nastavak na sljedećem slajdu

$$\forall z \exists x \exists y \exists w[(F(z) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \textcircled{\perp} \quad (..a..)$$

$$\exists x \exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(x))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists y \exists w[(F(a) \ \wedge \ (F(y) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$\exists w[(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(w) \wedge G(w))] \quad \perp \textcircled{a}$$

$$(F(a) \ \wedge \ (F(a) \rightarrow G(a))) \rightarrow (F(a) \wedge G(a)) \quad \textcircled{\perp}$$

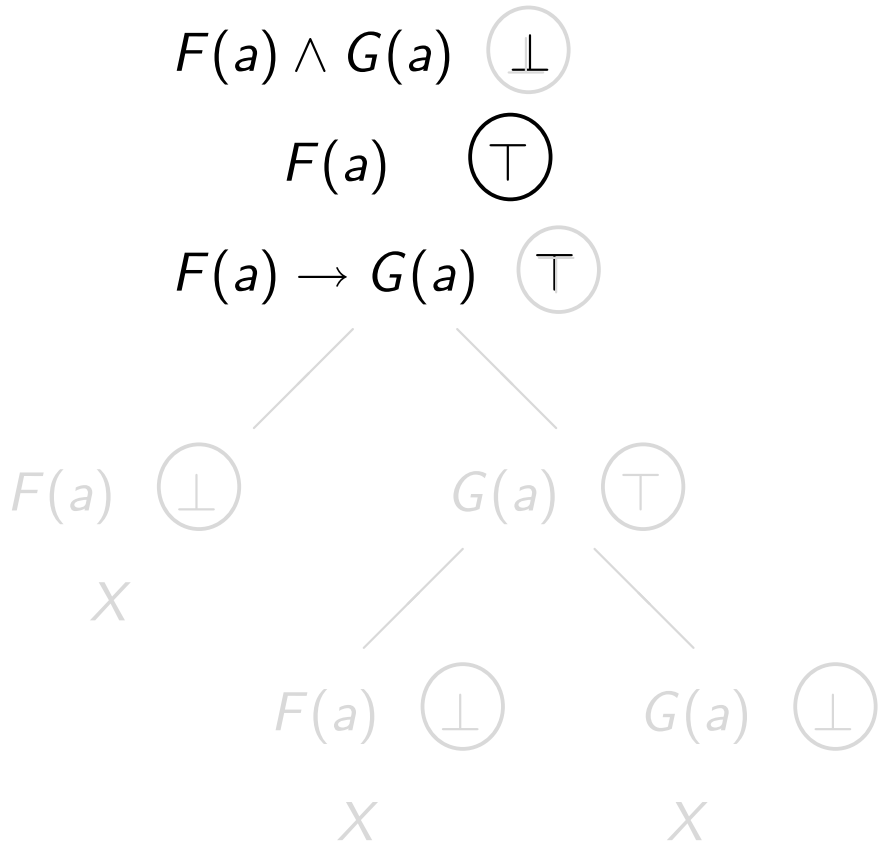
$$F(a) \ \wedge \ (F(a) \rightarrow G(a)) \quad \textcircled{\top}$$

$$F(a) \wedge G(a) \quad \perp$$

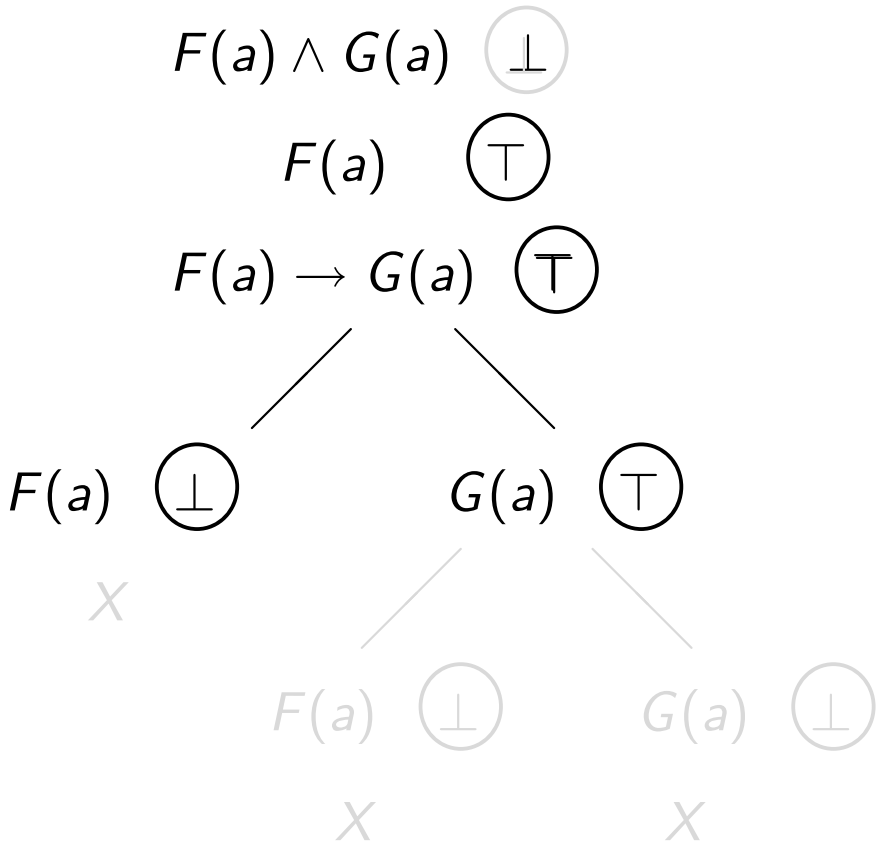
$$F(a) \quad \textcircled{\top}$$

$$F(a) \rightarrow G(a) \quad \top$$

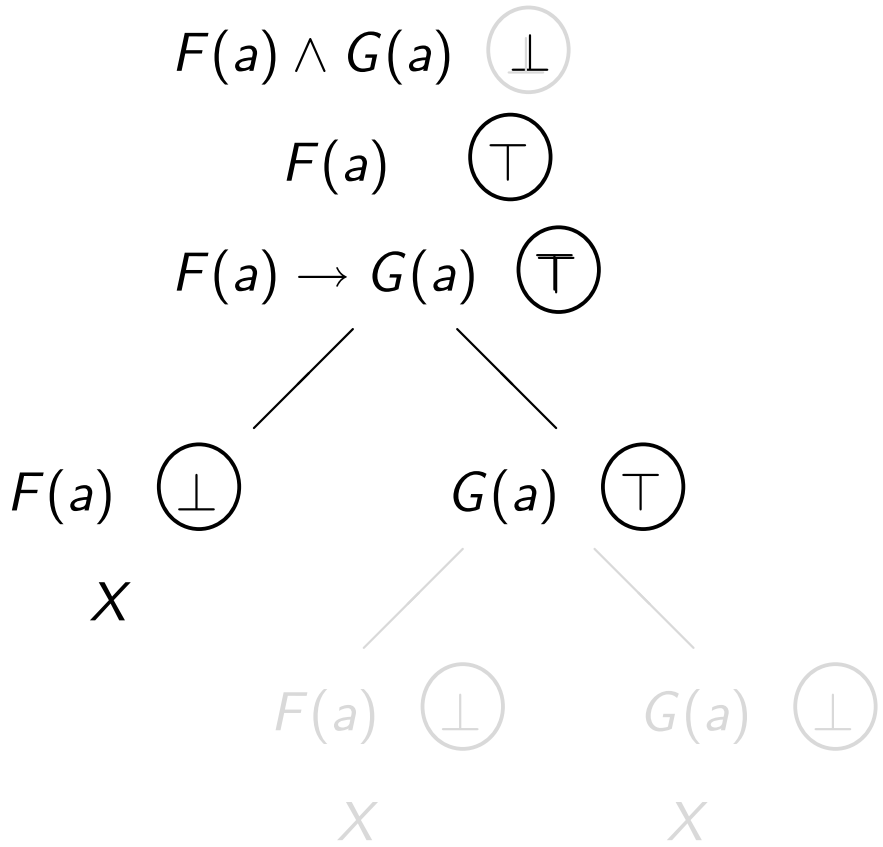
Nastavak na sljedećem slajdu



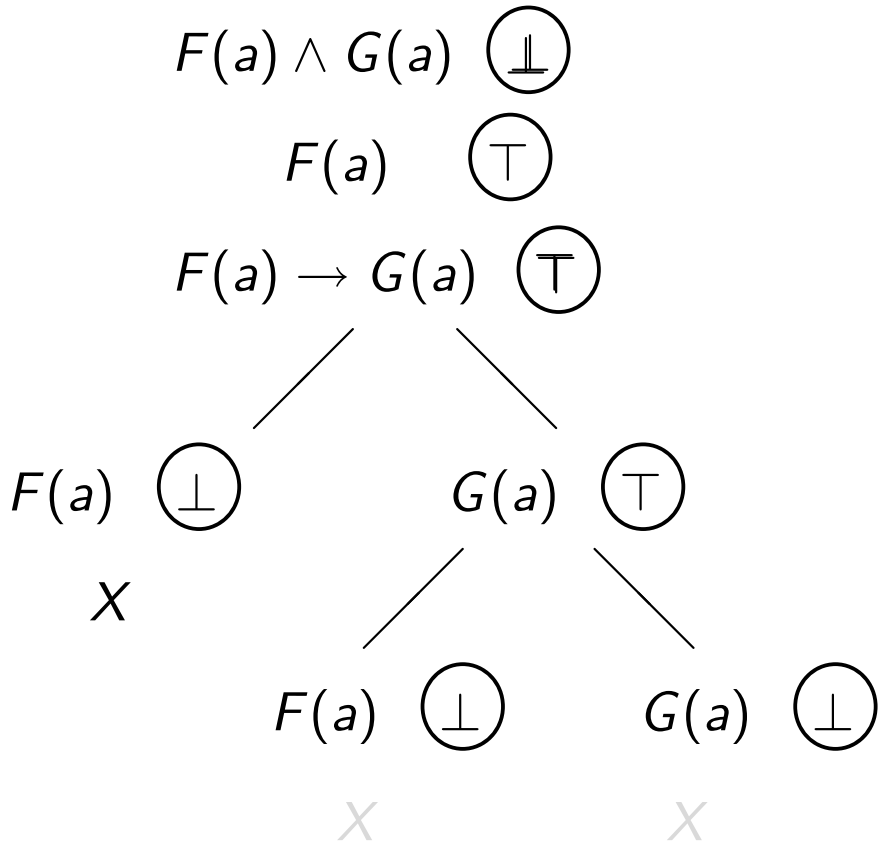
Sve grane su završile kontradikcijom pa zaključujemo da je dana formula valjana.



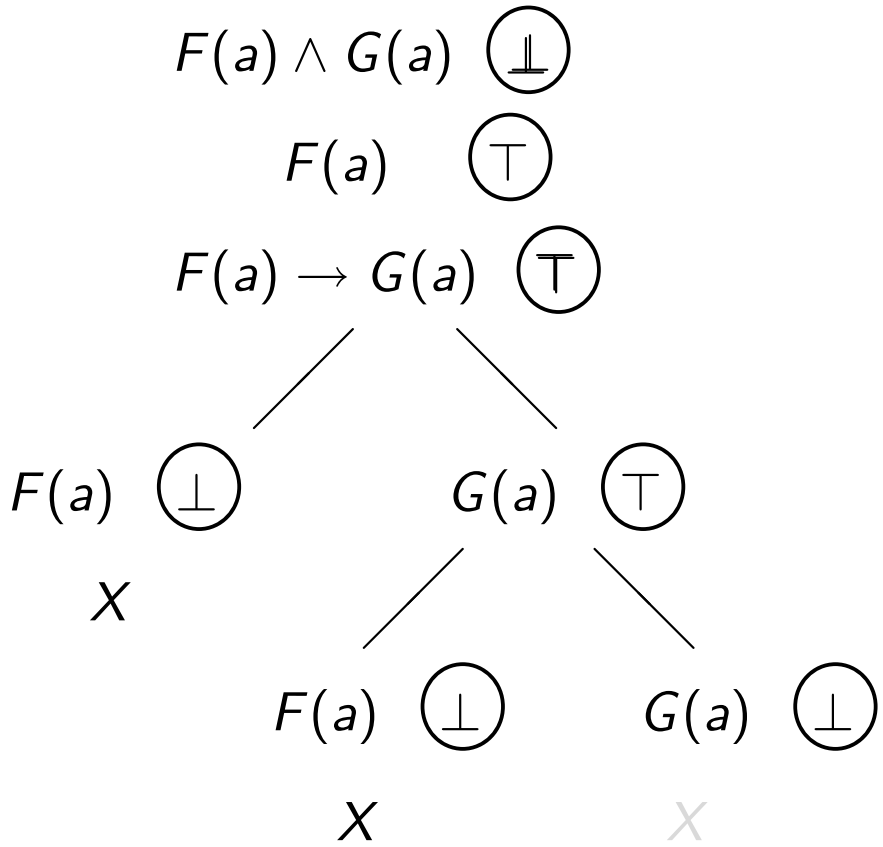
Sve grane su završile kontradikcijom pa zaključujemo da je dana formula valjana.



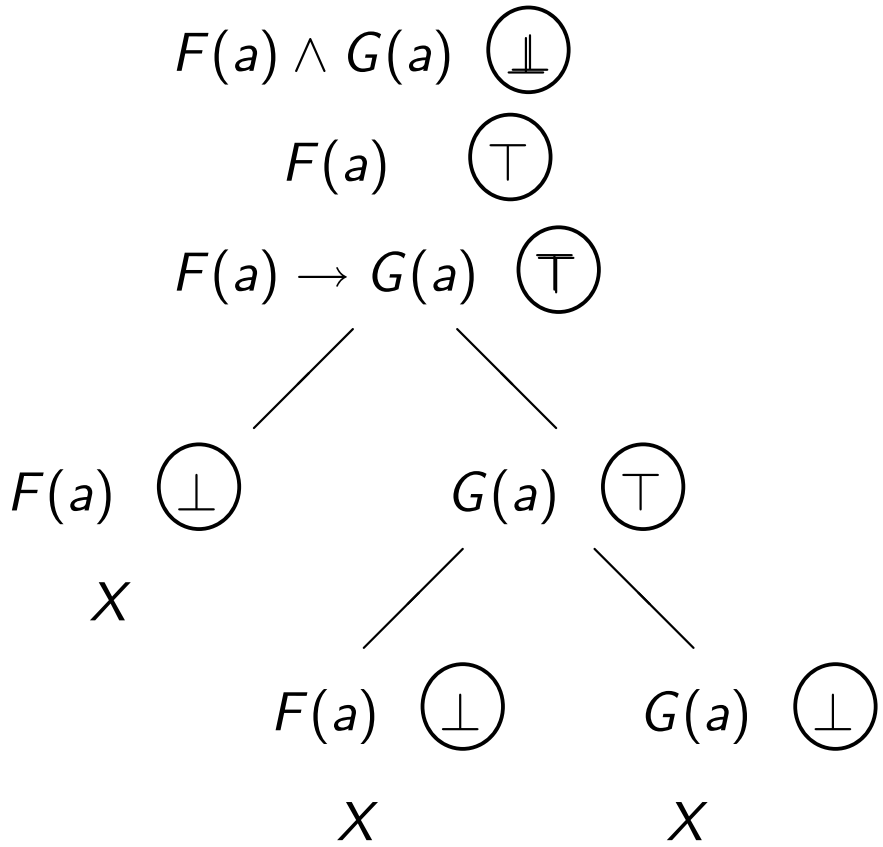
Sve grane su završile kontradikcijom pa zaključujemo da je dana formula valjana.



Sve grane su završile kontradikcijom pa zaključujemo da je dana formula valjana.



Sve grane su završile kontradikcijom pa zaključujemo da je dana formula valjana.



Sve grane su završile kontradikcijom pa zaključujemo da je dana formula valjana.

Primjer 6

Ispitajmo pomoću glavnog testa vrijedi li

$$F(a) \to G(b),\ \forall x(\neg F(x)) \models \neg G(b).$$

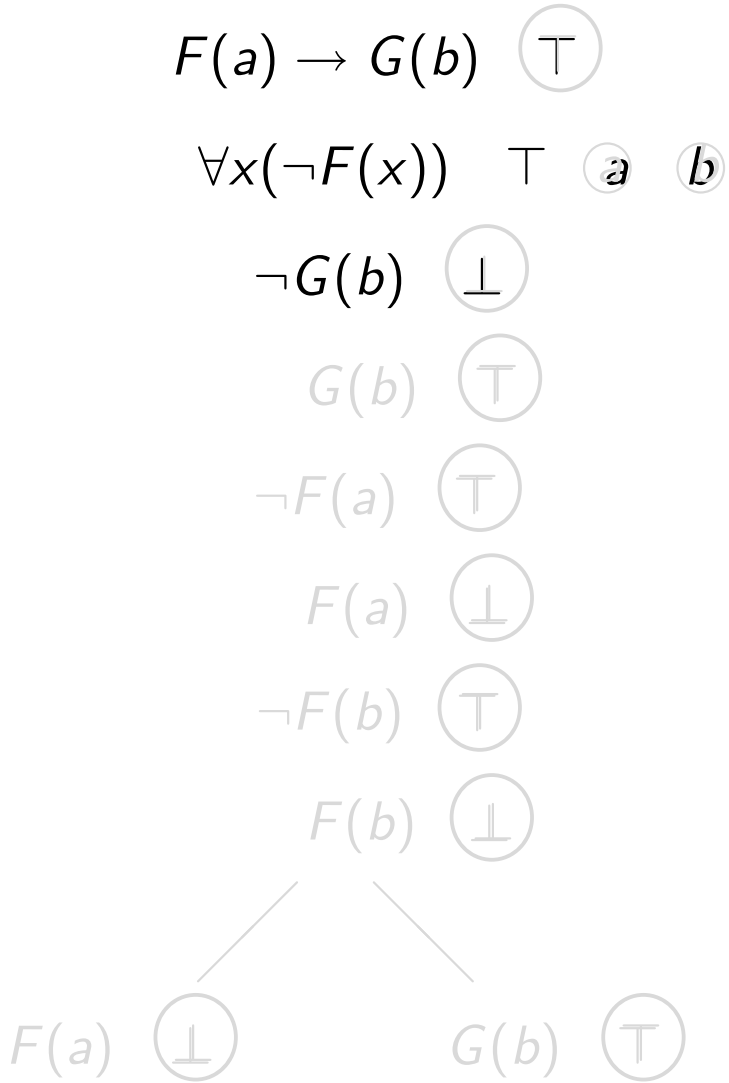
Rješenje: Pošto dana formula sadrži konstantske simbole a i b moramo prvo reći što raditi s njima prilikom glavnog testa.

Po definiciji strukture za svaki konstantski simbol mora postojati element u nosaču.

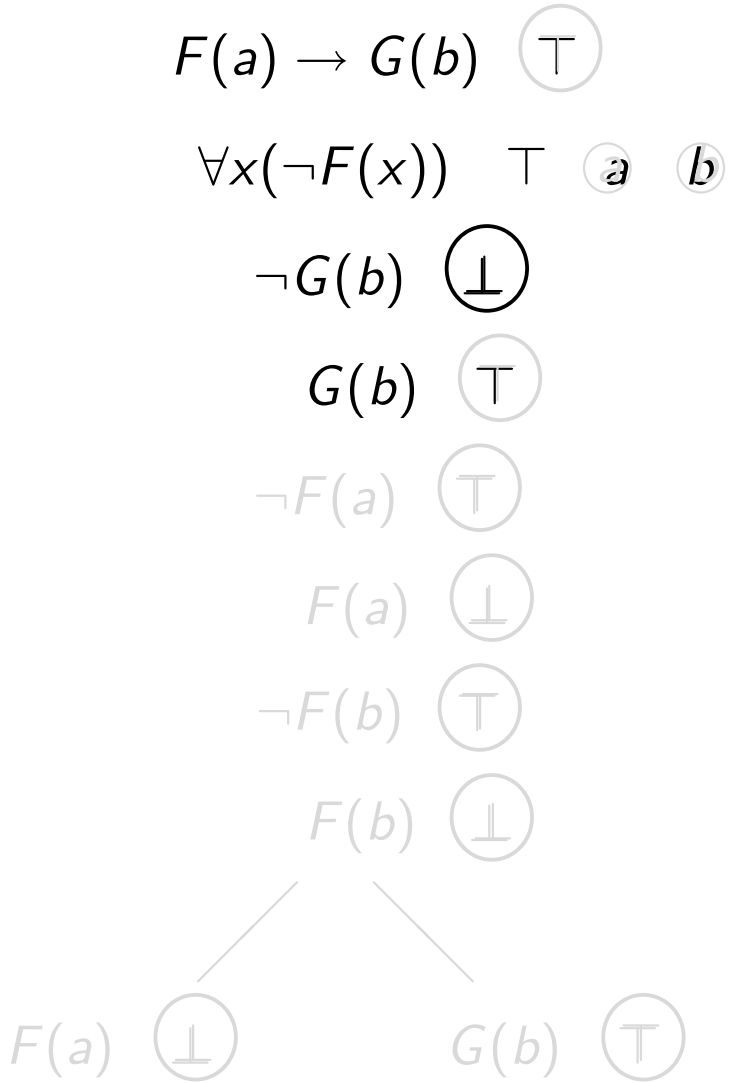
To znači da prije početka testiranja smatramo da nosač strukture sadrži barem dva elementa.

Interpretacije konstatskih simbola, kao i obično u ovoj točki, označavamo istim znakovima.

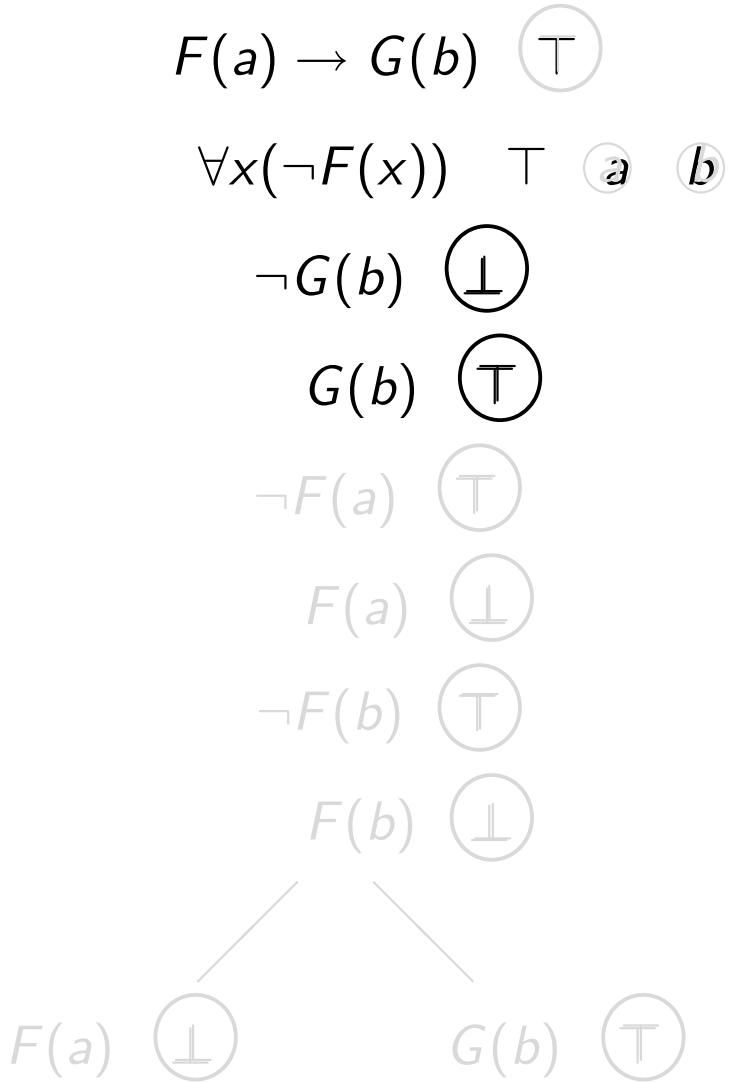
Glavni test zapisujemo u obliku stabla ovako:



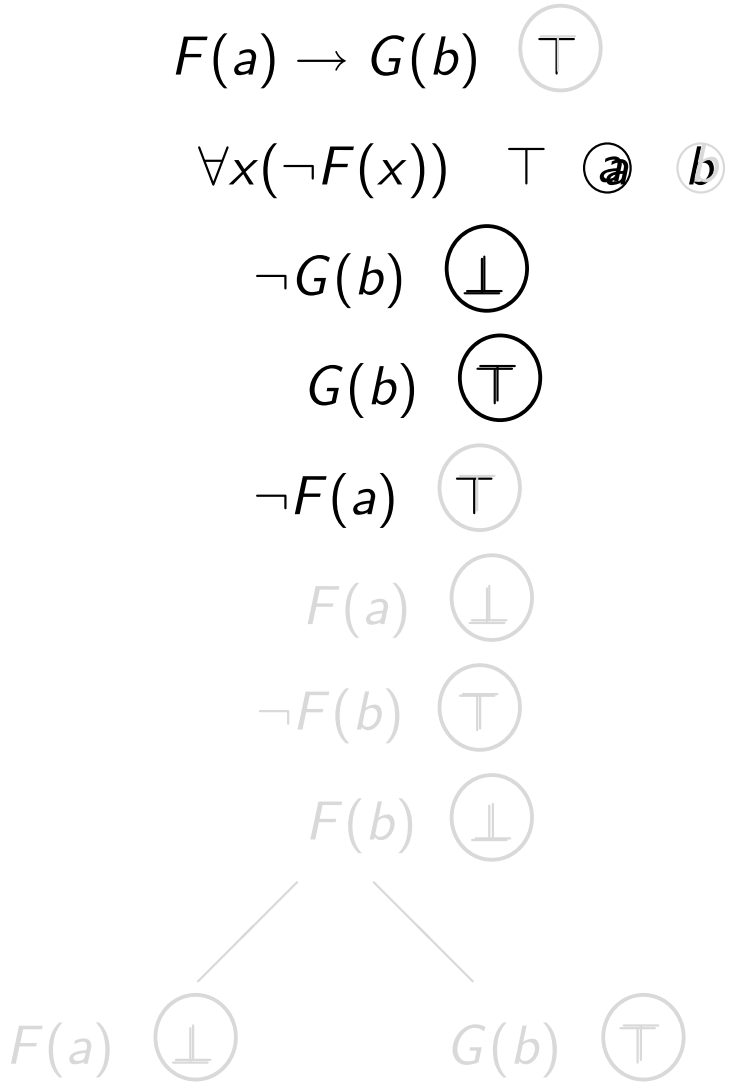
Glavni test zapisujemo u obliku stabla ovako:



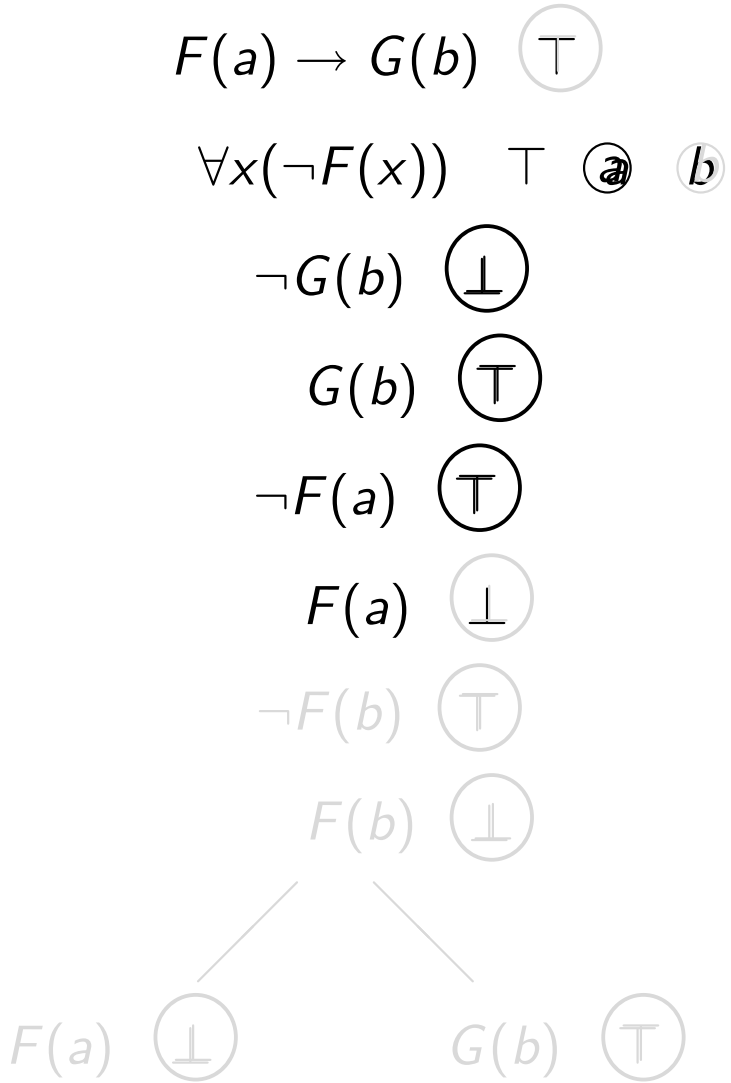
Glavni test zapisujemo u obliku stabla ovako:



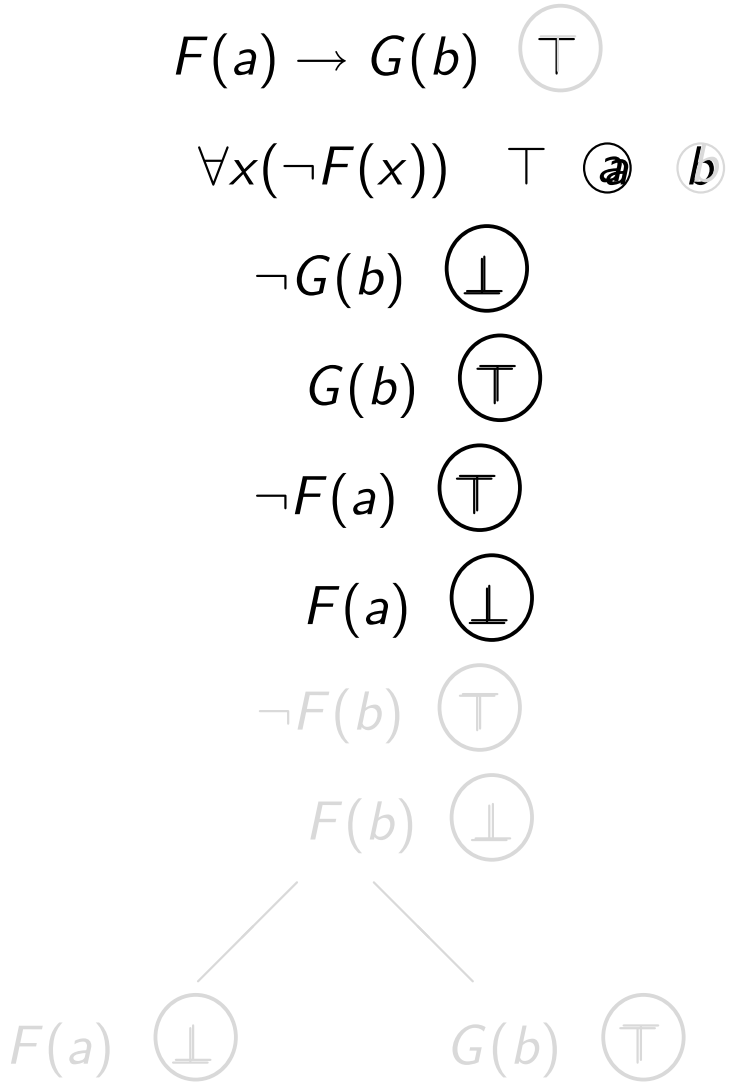
Glavni test zapisujemo u obliku stabla ovako:



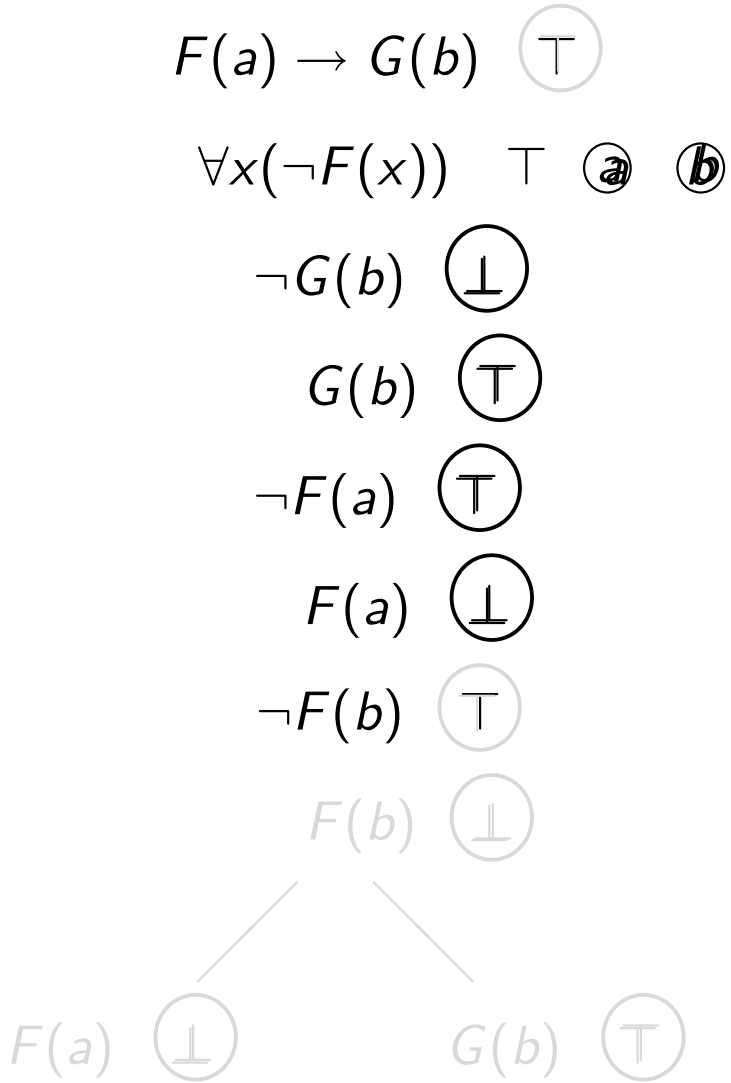
Glavni test zapisujemo u obliku stabla ovako:



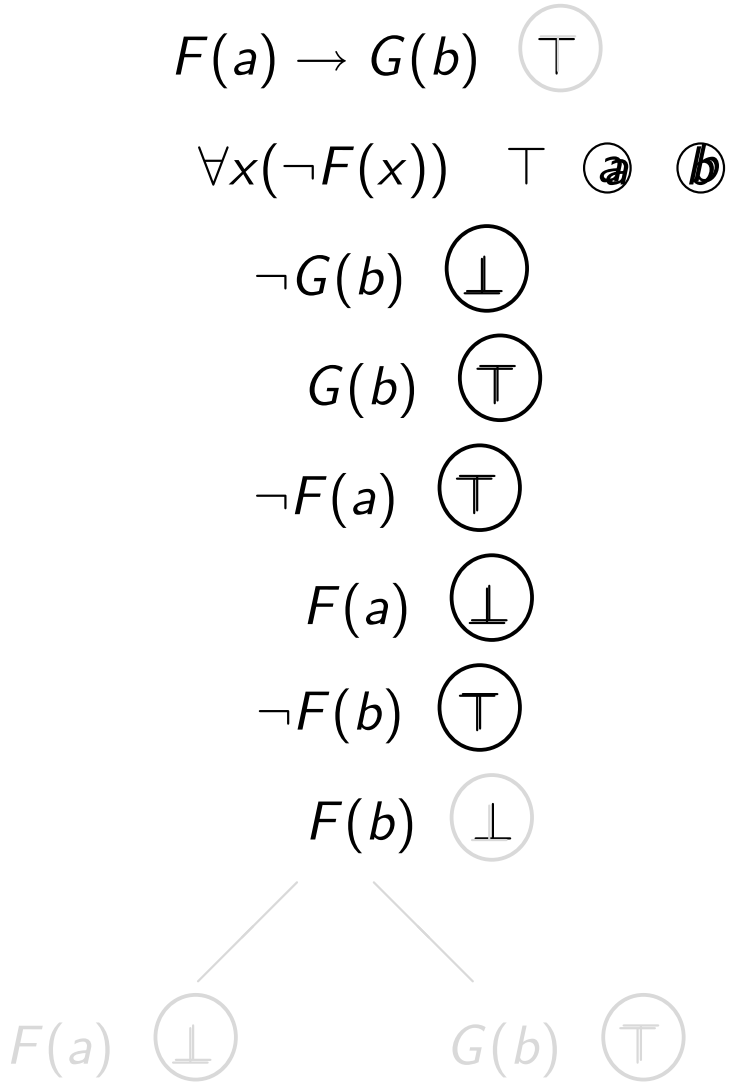
Glavni test zapisujemo u obliku stabla ovako:



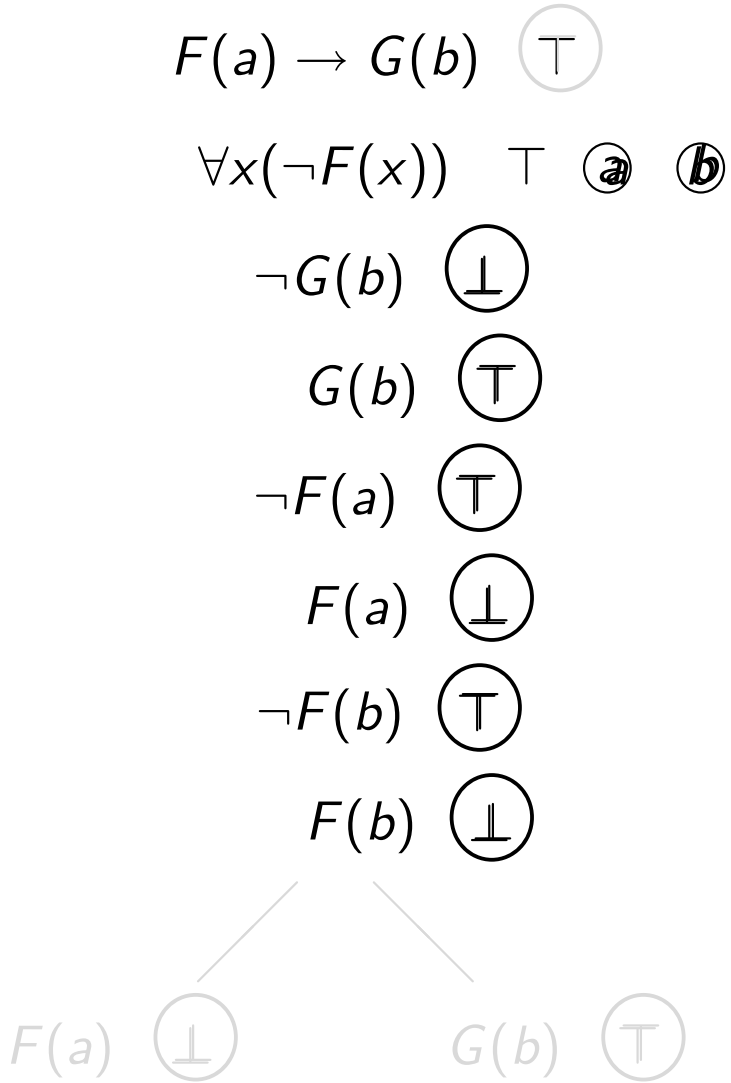
Glavni test zapisujemo u obliku stabla ovako:



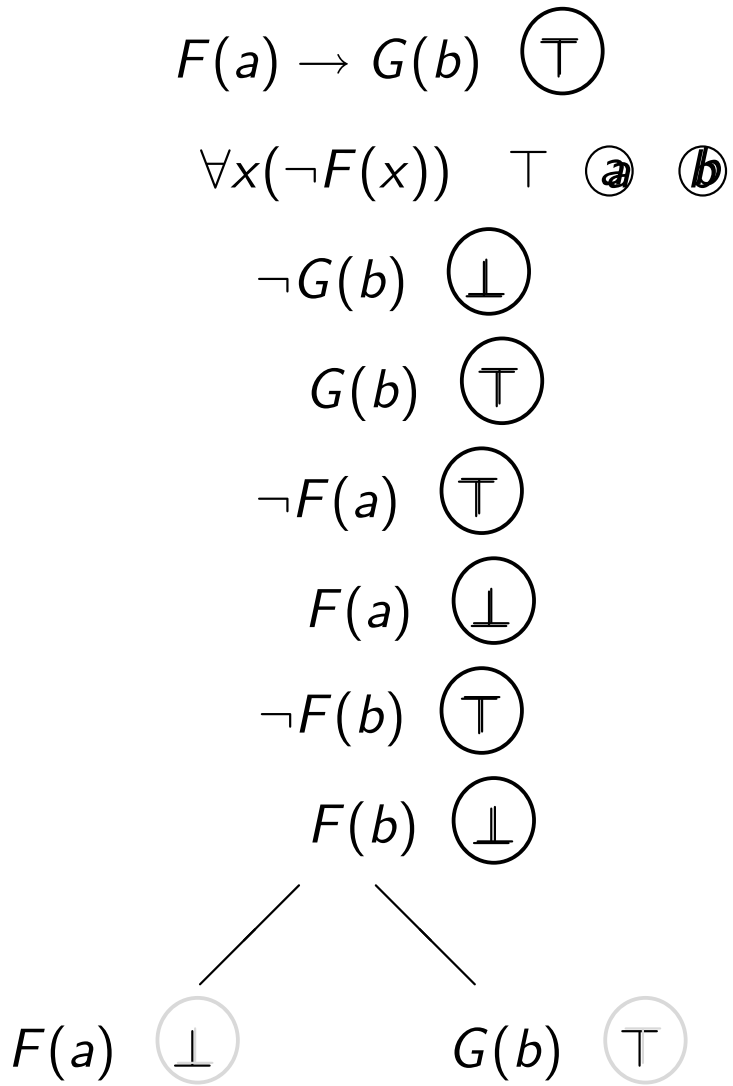
Glavni test zapisujemo u obliku stabla ovako:



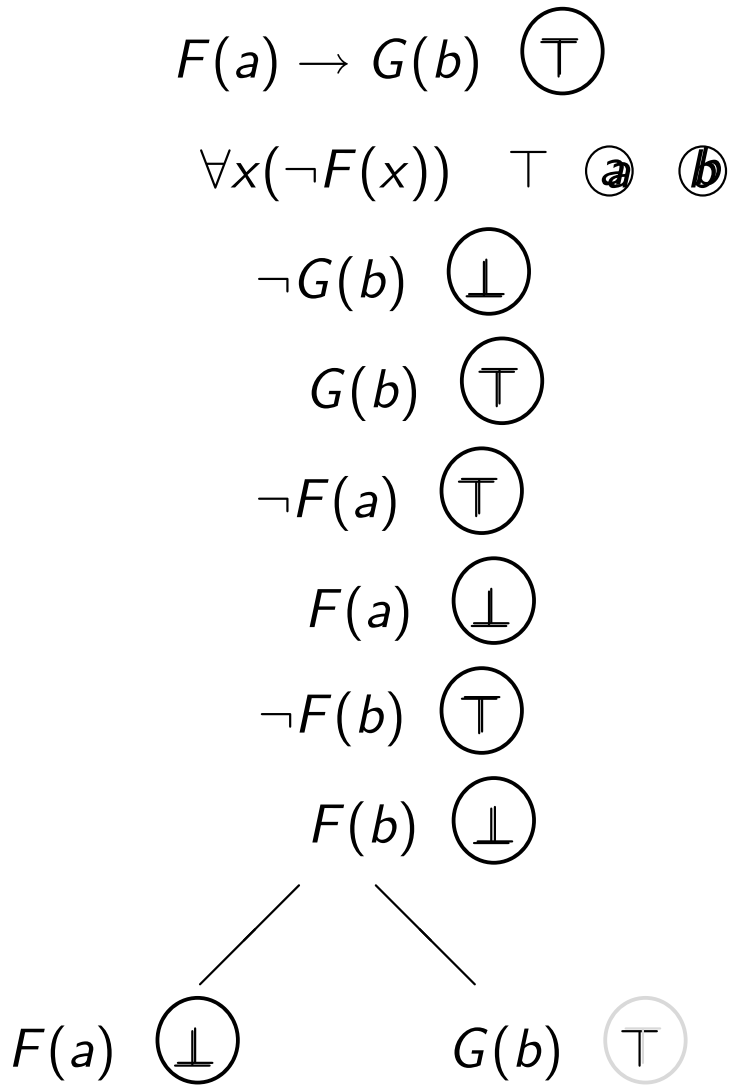
Glavni test zapisujemo u obliku stabla ovako:



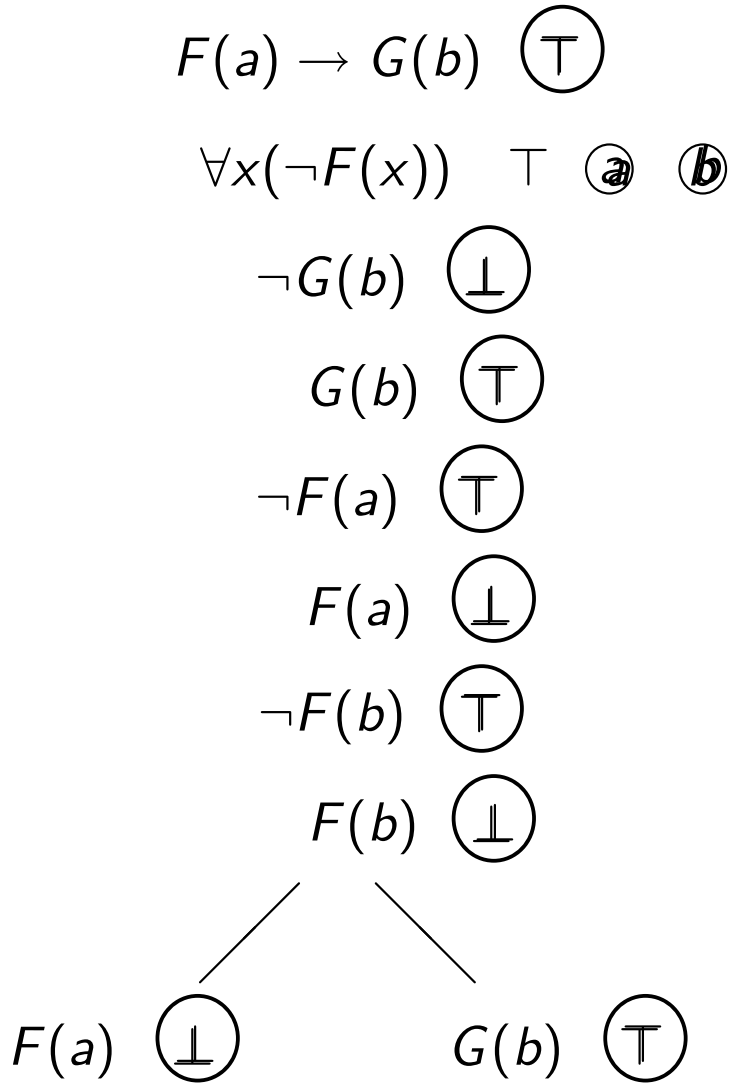
Glavni test zapisujemo u obliku stabla ovako:



Glavni test zapisujemo u obliku stabla ovako:



Glavni test zapisujemo u obliku stabla ovako:



Pošto nisu sve grane završile kontradikcijom (nije niti jedna!) zaključujemo da dana tvrdnja nije istinita, tj. formula $\neg G(b)$ logički ne slijedi iz skupa formula $\{F(a) \rightarrow G(b),\ \forall x(\neg F(x))\}$.

Primjer 7

Pomoću glavnog testa odredimo barem dvije strukture koje dokazuju

$$\forall x(A(x) \to B(x)) \not\models \forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))).$$

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \quad @ \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**) \quad \exists x(B(a) \wedge C(x,x)) \quad \bot \quad @ \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

X $\qquad$ Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; @ \; b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**) \quad \exists x(B(a) \wedge C(x,x)) \quad \bot \; @ \; b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

X

Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; @ \; b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**) \quad \exists x(B(a) \wedge C(x,x)) \quad \bot \; @ \; b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

X $\qquad$ Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; @\; a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**) \quad \exists x(B(a) \wedge C(x,x)) \quad \bot \; @\; a \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

X $\qquad$ Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**)$ $\exists x(B(a) \wedge C(x,x)) \quad \bot \; a \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

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X $\qquad$ Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**)$ $\exists x(B(a) \wedge C(x,x)) \quad \bot \; a \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot \qquad\qquad B(a) \quad \top$

X Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \; \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \; \bot$

$\exists x(A(a) \wedge C(a,x)) \; \top \quad (..b..)$

$(**)$ $\exists x(B(a) \wedge C(x,x)) \quad \bot \; a \quad b$

$A(a) \wedge C(a,b) \; \top$

$A(a) \; \top$

$C(a,b) \; \top$

Iz $(*)$ $\quad A(a) \to B(a) \; \top$

$A(a) \; \bot \qquad\qquad B(a) \; \top$

X Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \quad a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**)$ $\quad \exists x(B(a) \wedge C(x,x)) \quad \bot \quad a \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

X $\qquad$ Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \; \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \; \bot$

$\exists x(A(a) \wedge C(a,x)) \; \top \quad (..b..)$

$(**)$ $\exists x(B(a) \wedge C(x,x)) \quad \bot \; a \quad b$

$A(a) \wedge C(a,b) \; \top$

$A(a) \; \top$

$C(a,b) \; \top$

Iz $(*)$ $\quad A(a) \to B(a) \; \top$

$A(a) \; \bot$ $\qquad$ $B(a) \; \top$

X

Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \; a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \; \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \; \bot$

$\exists x(A(a) \wedge C(a,x)) \; \top \quad (..b..)$

$(**)$ $\quad \exists x(B(a) \wedge C(x,x)) \quad \bot \; a \quad b$

$A(a) \wedge C(a,b) \; \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \; \top$

$A(a) \; \bot$ $\qquad$ $B(a) \; \top$

X $\qquad$ Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \ @_a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**)$ $\exists x(B(a) \wedge C(x,x)) \quad \bot \ @_a \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

X $\qquad$ Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \ @_a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**)$ $\exists x(B(a) \wedge C(x,x)) \quad \bot \ @_a \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot \qquad\qquad B(a) \quad \top$

X

Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \ @_a \quad b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x))) \quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x)) \quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**)$ $\exists x(B(a) \wedge C(x,x)) \quad \bot \ @_a \quad b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

$C(a,b) \quad \top$

Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

X

Nastavak na sljedećem slajdu

$(*)$ $\quad \forall x(A(x) \to B(x)) \quad \top \ @ \ b$

$\forall y(\exists x(A(y) \wedge C(y,x)) \to \exists x(B(y) \wedge C(x,x)))$ $\quad \bot \quad (..a..)$

$\exists x(A(a) \wedge C(a,x)) \to \exists x(B(a) \wedge C(x,x))$ $\quad \bot$

$\exists x(A(a) \wedge C(a,x)) \quad \top \quad (..b..)$

$(**)$ $\quad \exists x(B(a) \wedge C(x,x)) \quad \bot \ @ \ b$

$A(a) \wedge C(a,b) \quad \top$

$A(a) \quad \top$

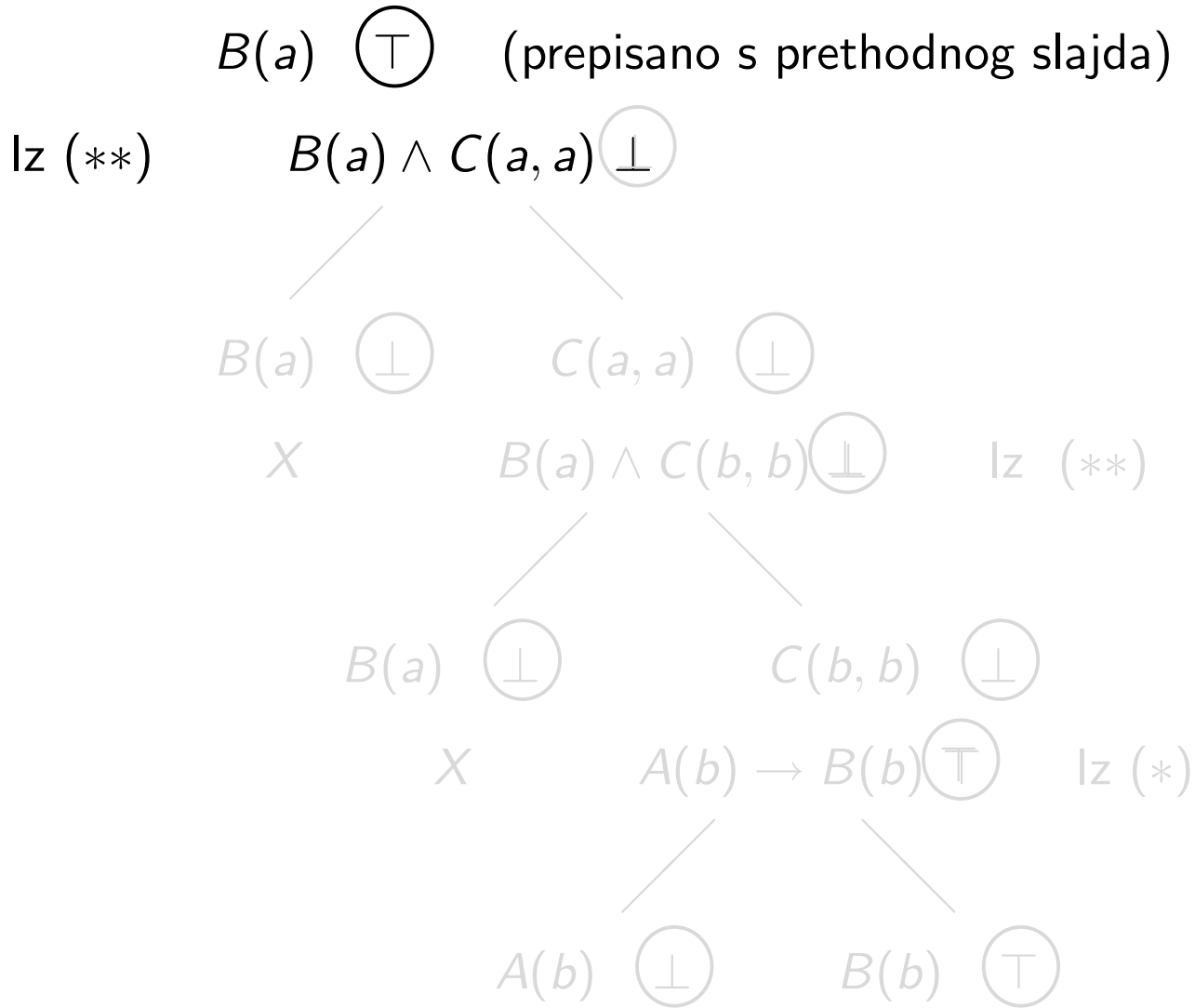
$C(a,b) \quad \top$

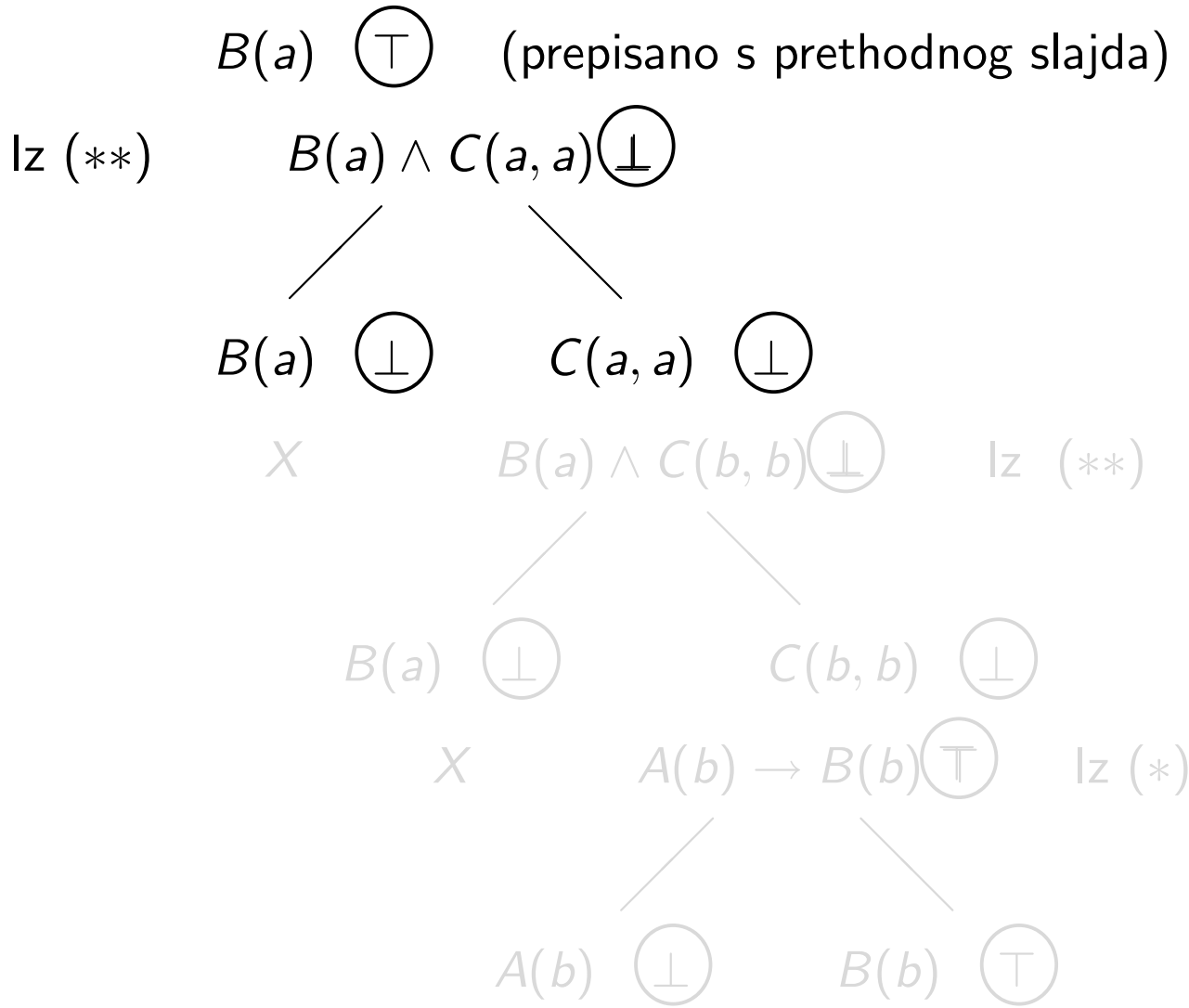
Iz $(*)$ $\quad A(a) \to B(a) \quad \top$

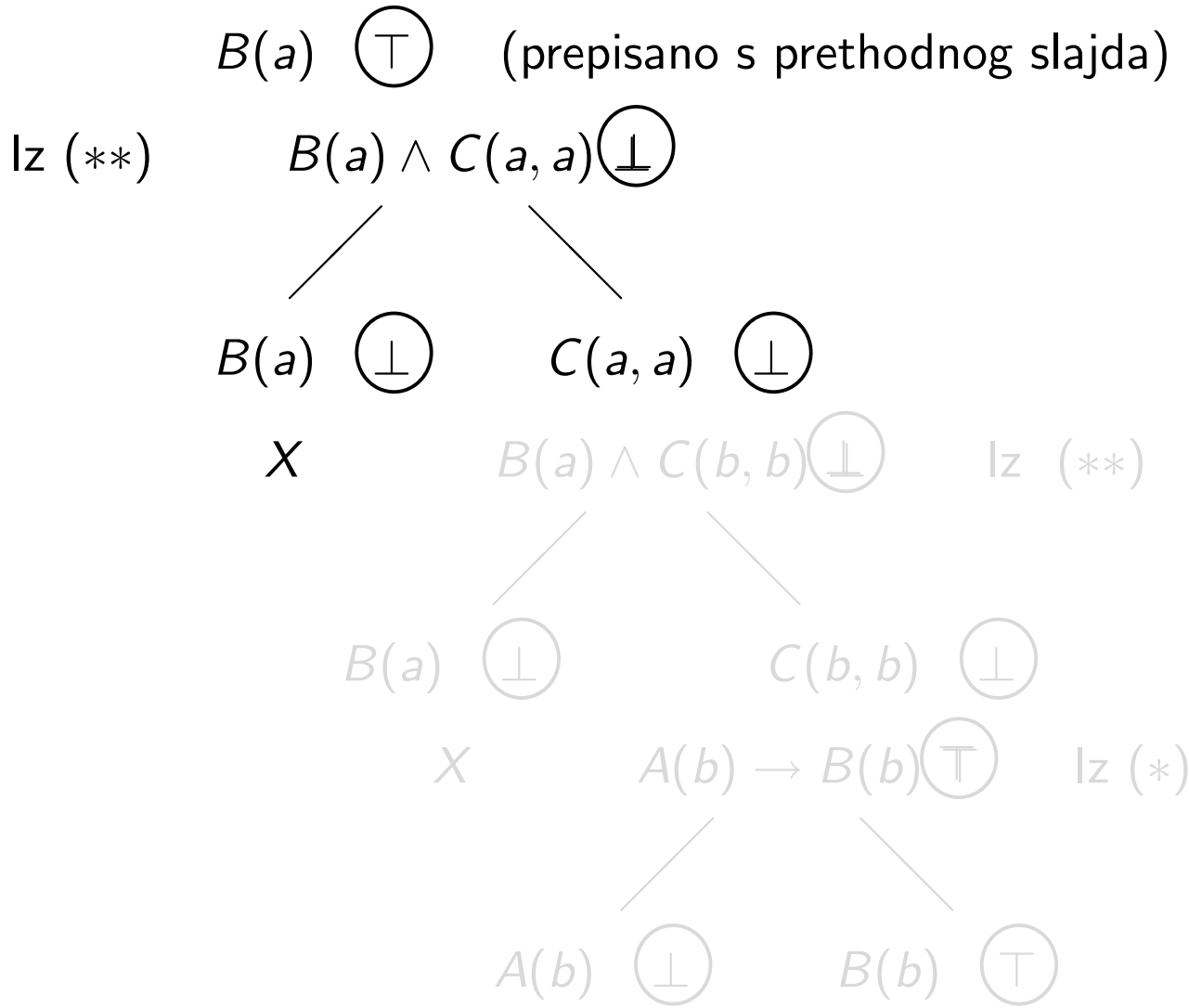
$A(a) \quad \bot$ $\qquad$ $B(a) \quad \top$

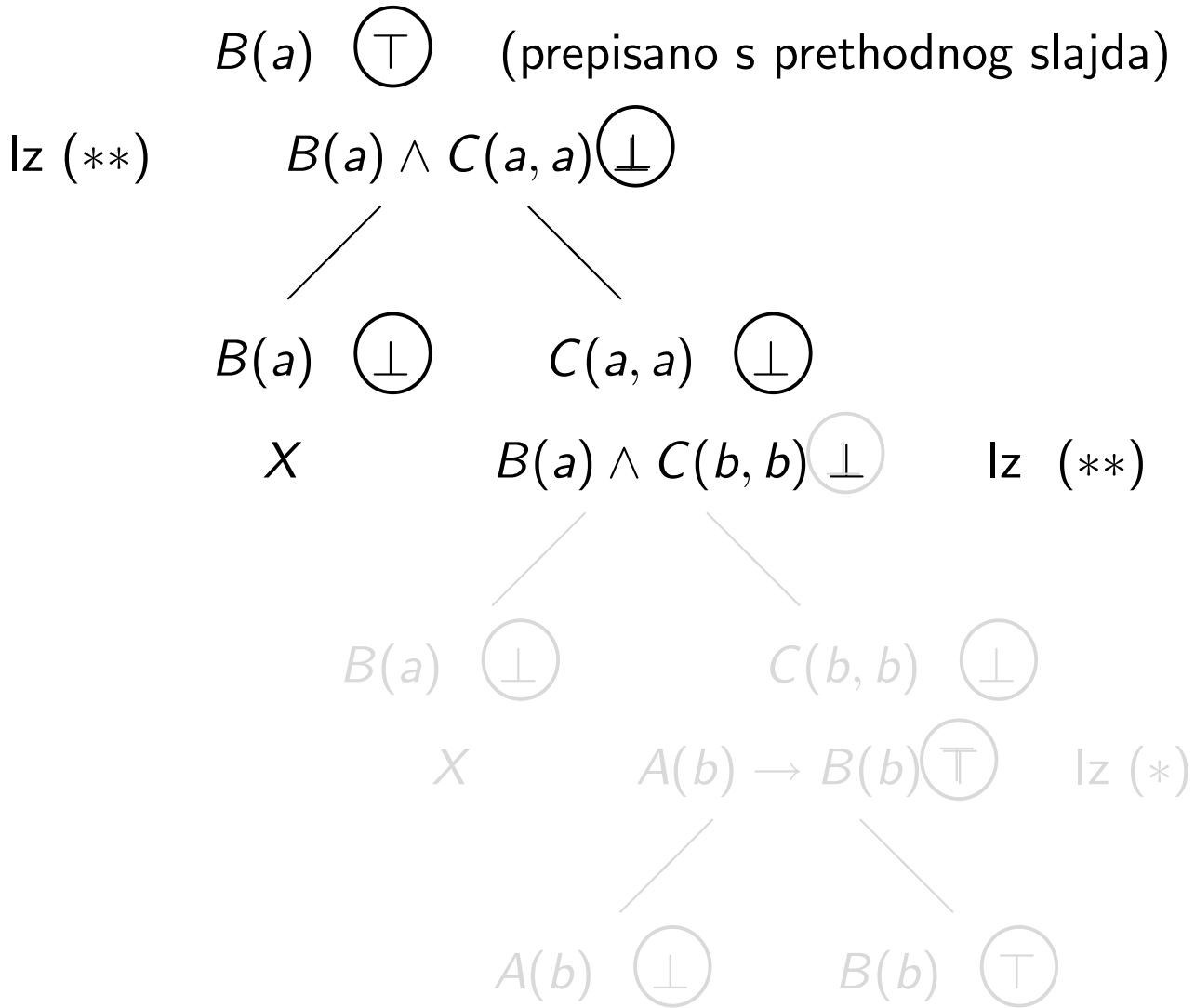
X $\qquad$ Nastavak na sljedećem slajdu

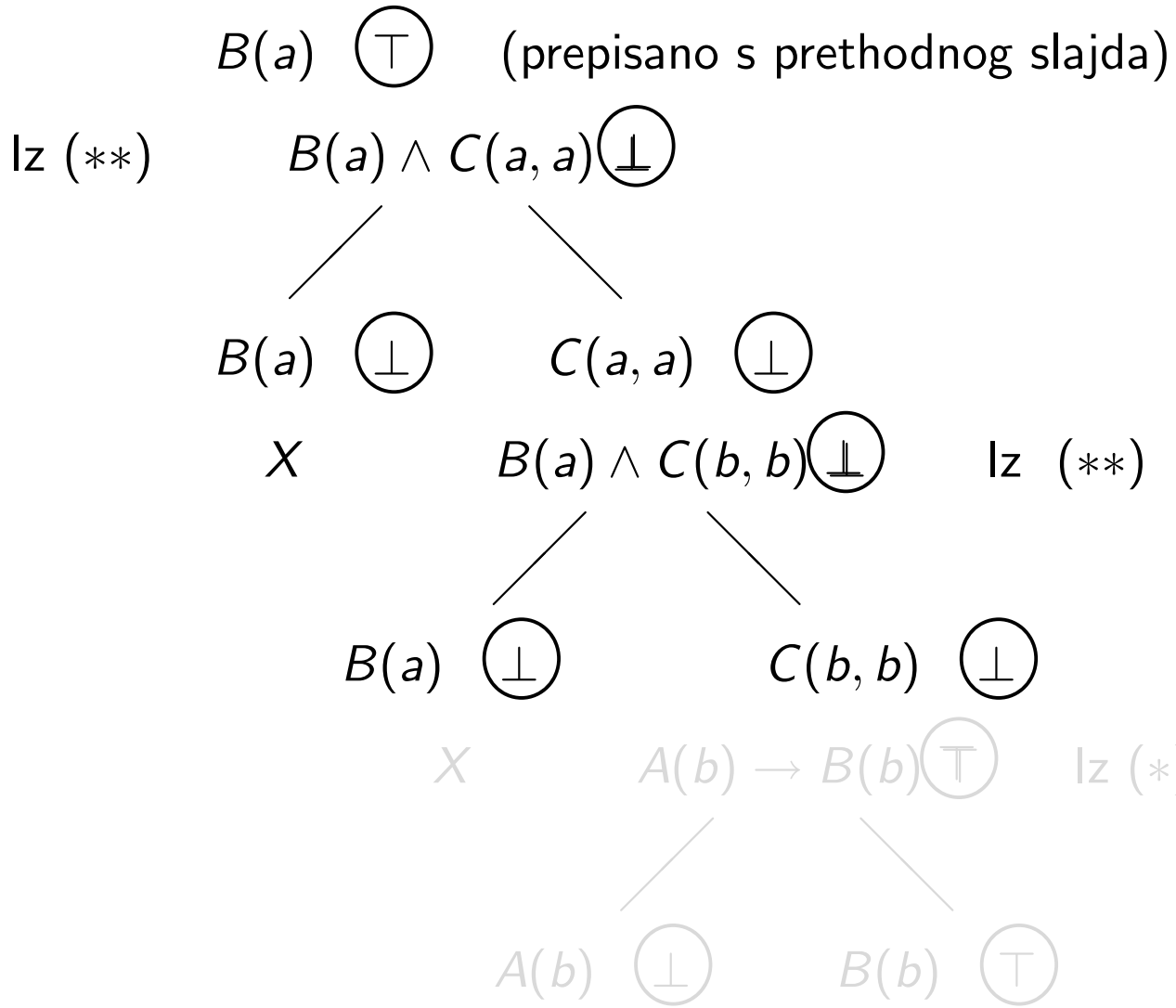
$B(a)$ $\top$ (prepisano s prethodnog slajda)

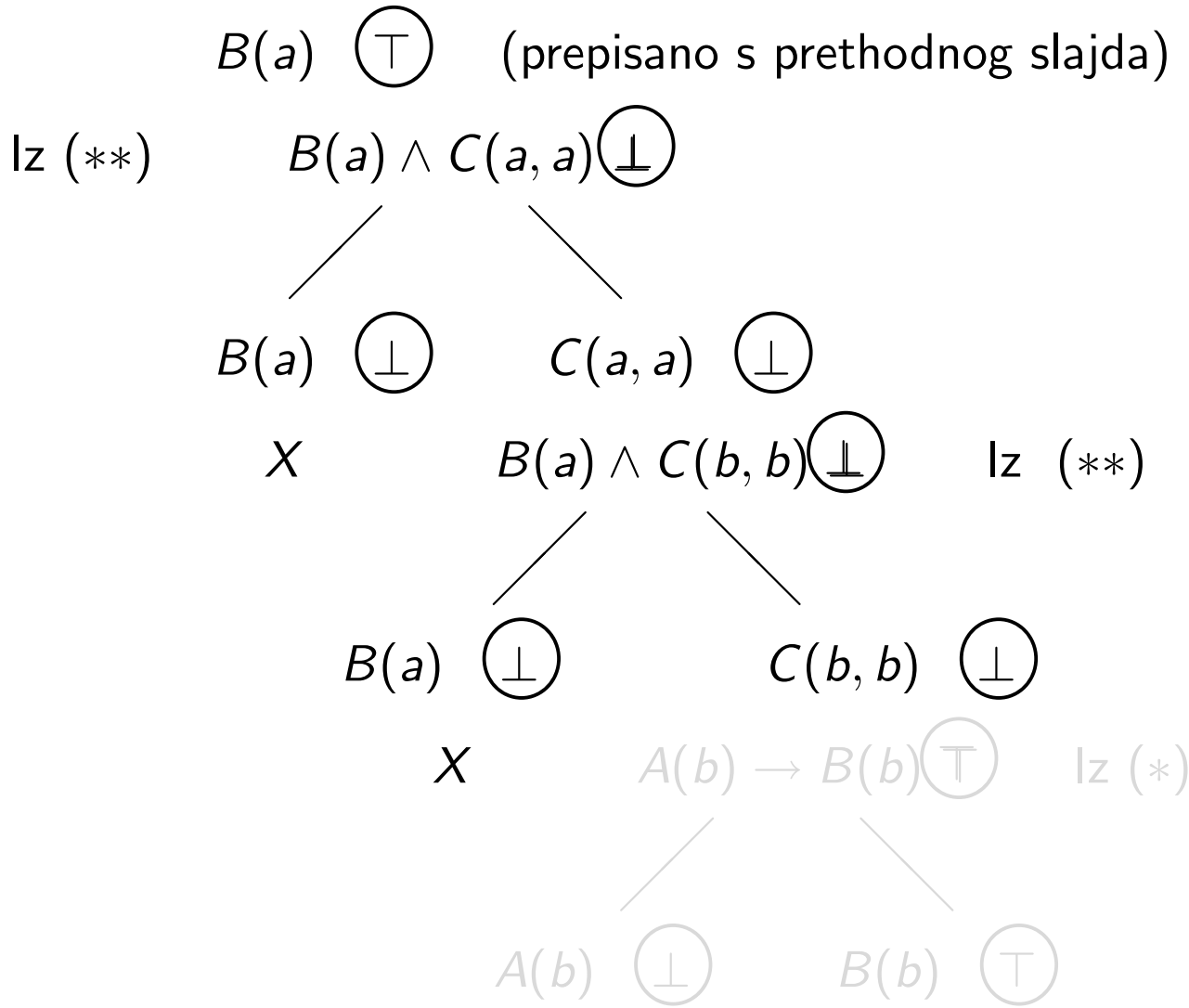

$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$ $B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
$C(b,b)$ $\bot$
X
$A(b) \to B(b)$ $\top$
Iz $(*)$
$A(b)$ $\bot$
$B(b)$ $\top$


$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$
$B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
$C(b,b)$ $\bot$
X
$A(b) \rightarrow B(b)$ $\top$
Iz $(*)$
$A(b)$ $\bot$
$B(b)$ $\top$


$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$
$B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
$C(b,b)$ $\bot$
X
$A(b) \to B(b)$ $\top$
Iz $(*)$
$A(b)$ $\bot$
$B(b)$ $\top$


$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$ $B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
$C(b,b)$ $\bot$
X
$A(b) \to B(b)$ $\top$
Iz $(*)$
$A(b)$ $\bot$
$B(b)$ $\top$


$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$ $B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
$C(b,b)$ $\bot$
X
$A(b) \to B(b)$ $\top$
Iz $(*)$
$A(b)$ $\bot$
$B(b)$ $\top$


$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$
$B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
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X
$A(b) \to B(b)$ $\top$
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$A(b)$ $\bot$
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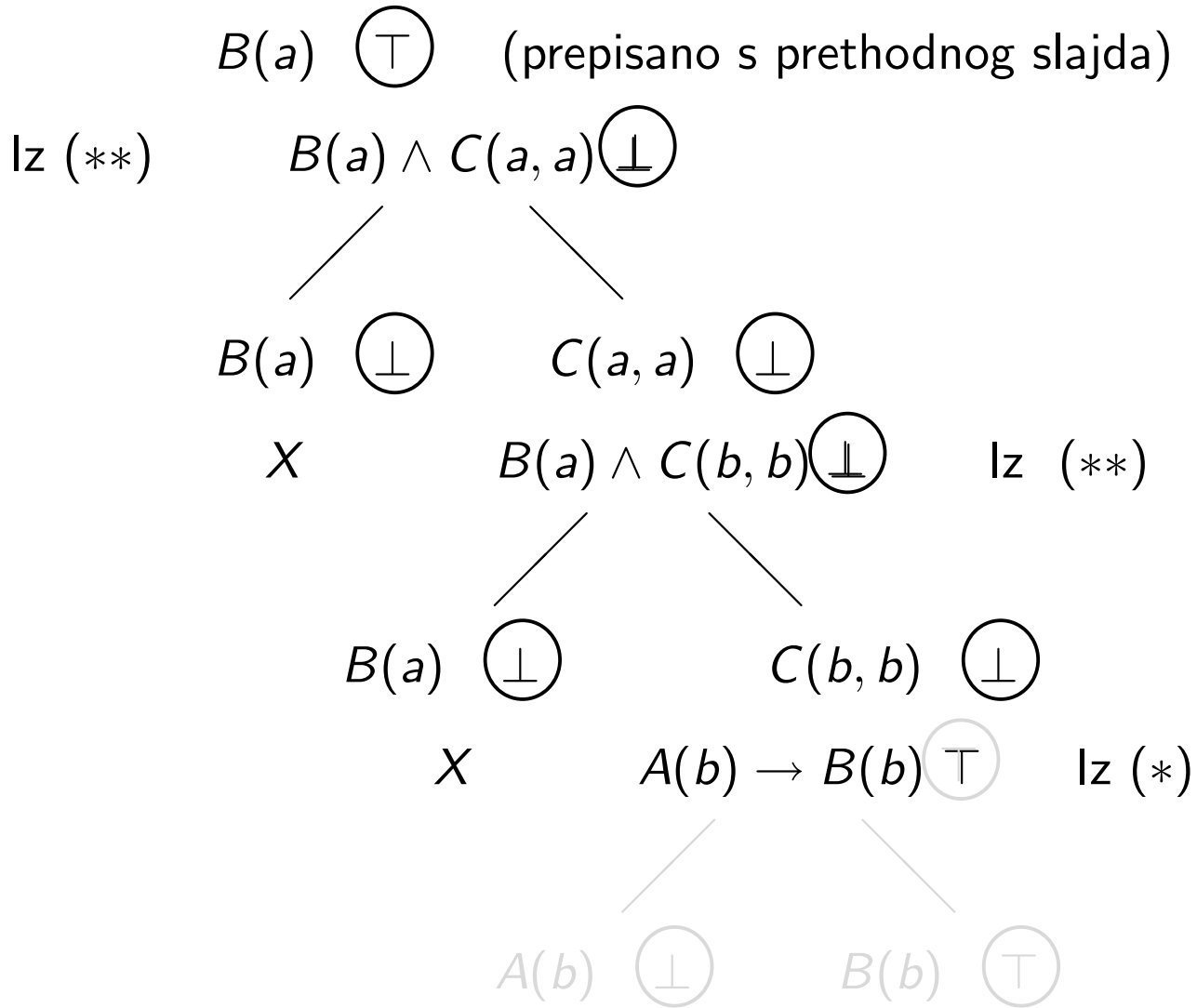
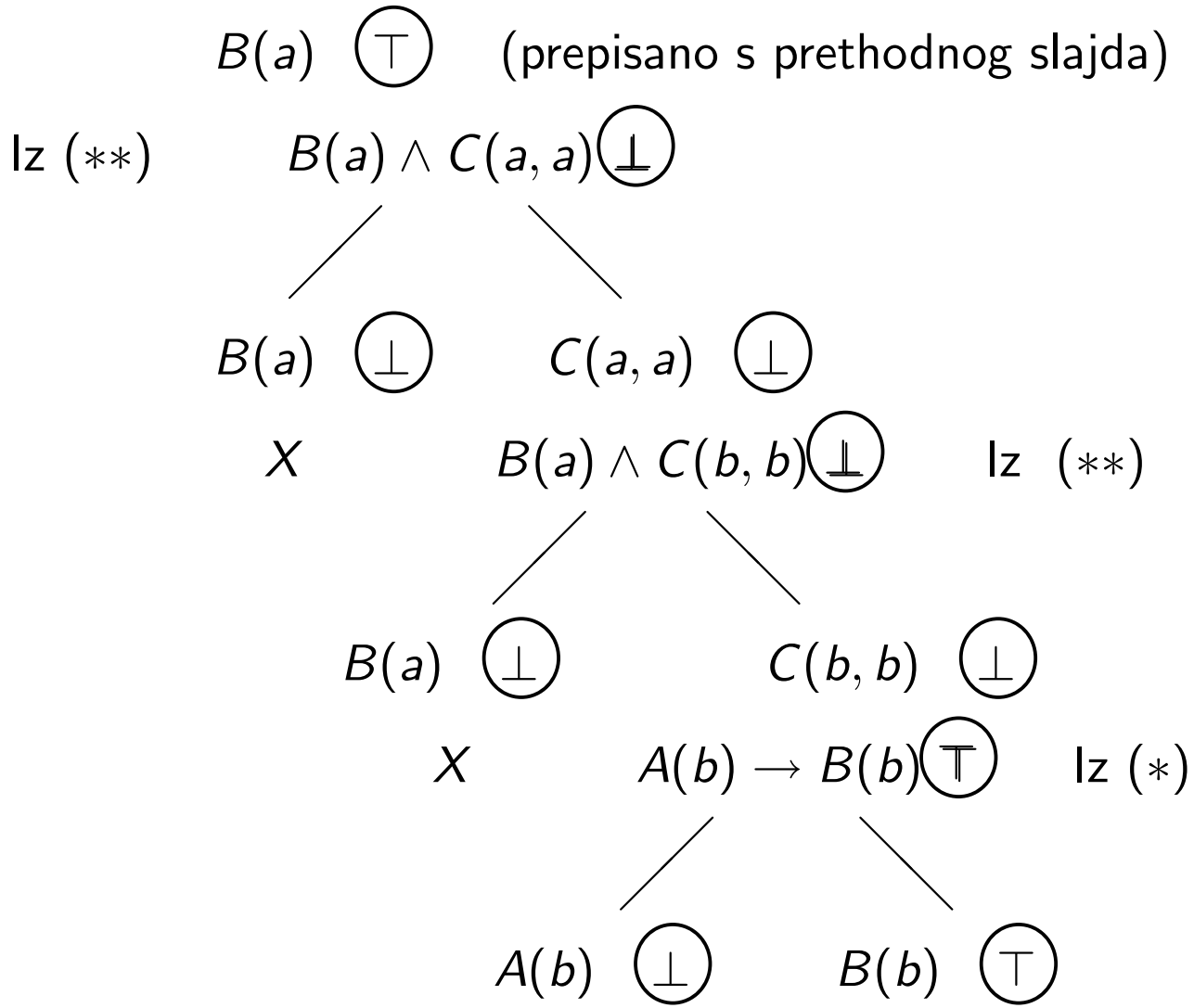

$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$
$B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
$C(b,b)$ $\bot$
X
$A(b) \to B(b)$ $\top$
Iz $(*)$
$A(b)$ $\bot$
$B(b)$ $\top$


$B(a)$ $\top$ (prepisano s prethodnog slajda)
Iz $(**)$ $B(a) \wedge C(a,a)$ $\bot$
$B(a)$ $\bot$
$C(a,a)$ $\bot$
X
$B(a) \wedge C(b,b)$ $\bot$
Iz $(**)$
$B(a)$ $\bot$
$C(b,b)$ $\bot$
X
$A(b) \to B(b)$ $\top$
Iz $(*)$
$A(b)$ $\bot$
$B(b)$ $\top$

Neka je $|\mathfrak{M}| = \{a, b\}$, te $A^{\mathfrak{M}} = \{a\}$, $B^{\mathfrak{M}} = \{a\}$, i $C^{\mathfrak{M}} = \{(a, b)\}$.

Zatim definiramo $|\mathfrak{N}| = \{a, b\}$, $A^{\mathfrak{N}} = \{a\}$, $B^{\mathfrak{N}} = \{a, b\}$ i $C^{\mathfrak{N}} = \{(a, b)\}$.

Tada su $\mathfrak{M}$ i $\mathfrak{N}$ tražene dvije strukture.

Primjer 8

Neka je F zatvorena formula, a R dvomjesni relacijski simbol.

Koristeći glavni test dokažite ili opovrgnite

$$\{\neg F \vee \exists x \forall y(\neg R(x,y)),\ \forall y \exists x R(y,x) \vee \exists x R(x,x)\} \models F \rightarrow \exists x \exists y R(x,y).$$

Rješenje:

$$\neg F \vee \exists x \forall y (\neg R(x,y)) \quad \textcircled{\top}$$

$$\forall y \exists x R(y,x) \vee \exists x R(x,x) \quad \textcircled{\top}$$

$$F \to \exists x \exists y R(x,y) \quad \textcircled{\bot}$$

$$F \quad \textcircled{\top}$$

$$(I) \qquad \exists x \exists y R(x,y) \quad \bot \quad @$$

$$\neg F \quad \textcircled{\top} \qquad\qquad \exists x \forall y (\neg R(x,y)) \quad \textcircled{\top} \quad (..a..)$$

$$F \quad \textcircled{\bot} \qquad\qquad (II) \quad \forall y (\neg R(a,y)) \quad \top \quad @$$

$$X \qquad\qquad \neg R(a,a) \quad \textcircled{\top}$$

$$R(a,a) \quad \textcircled{\bot}$$

$$(III) \qquad \exists y R(a,y) \quad \bot \quad @$$

(III) $\quad \exists y R(a,y)\ \bot\ \textcircled{a}$

$R(a,a)\ \ \textcircled{\bot}$

$\forall y \exists x R(y,x)\ \top \textcircled{a}\ , c$

$(I) \quad \exists x \exists y R(x,y)\ \bot\ c$

$(II) \quad \forall y(\neg R(a,y))\ \top\ c$

$(III) \quad \exists y R(a,y)\ \bot\ \textcircled{c}$

$\exists x R(a,x)\ \ \textcircled{\top}\ \ (..c..)$

$R(a,c)\ \ \textcircled{\top}$

$R(a,c)\ \ \textcircled{\bot}$

X

$\exists x R(x,x)\ \ \textcircled{\top}\ \ (..b..)$

$(I) \quad \exists x \exists y R(x,y)\ \bot\ \textcircled{b}$

$(II) \quad \forall y(\neg R(a,y))\ \top\ b$

$(III) \quad \exists y R(a,y)\ \bot\ b$

$R(b,b)\ \ \textcircled{\top}$

$\exists y R(b,y)\ \bot\ \textcircled{b}$

$R(b,b)\ \ \textcircled{\bot}$

X

Pošto su sve grane završile kontradikcijom zaključujemo da je početna tvrdnja istinita.

Zadaci

1. Pomoću glavnog testa ispitajte valjanost sljedećih formula:

   a) $(B \to (\forall x A(x) \wedge \forall x C(x))) \to (\neg B \vee \forall x(A(x) \wedge C(x)))$, pri čemu je B zatvorena formula;

   b) $\forall x \forall y P(x,y) \to (\exists y \exists x P(y,x) \ \vee \ \exists x \exists y P(x,y))$;

   c) $(\neg A \wedge (\exists x B(x) \vee \exists x C(x))) \ \leftrightarrow \ \neg(A \vee \forall x(\neg B(x) \wedge \neg C(x)))$;

   d) $\forall x \forall y (P(x,y) \wedge Q(x)) \to (\forall x \forall y P(x,y) \wedge \forall x Q(x))$.

Rješenje: Sve navedene formule su valjane.

2. Odredite prvo preneksnu normalnu formu formule

$$(\forall x F(x) \vee (\exists x F(x) \rightarrow \forall x G(x))) \rightarrow (\forall x F(x) \wedge \forall x G(x)),$$

a zatim ispitajte je li dobivena preneksna normalna forma oboriva.

3. Pomoću glavnog testa ispitajte vrijedi li:

a) $\{\forall x \forall y(F(x,y) \to \neg F(y,x))\} \models \neg \exists x F(x,x)$;

b) $\{\forall x(A(x) \to B(x))\} \models \forall y(\exists x(A(x) \wedge C(y,x)) \to \exists x(B(x) \wedge C(y,x)))$;

c) $\{\exists x \forall y B(x,y) \to A,\ \neg A \vee \exists x \exists y B(x,y)\} \models A \vee \neg \forall y \forall x B(y,x)$, gdje je formula A zatvorena;

d) $\forall x \forall y(R(x,y) \to \neg R(y,x)) \models \forall x \forall y(R(x,y) \to (R(y,x) \to R(x,x)))$;

e) $F \to \neg \forall y \exists x R(x,y),\ \forall x \exists y R(y,x) \vee \exists x R(x,x) \models \neg F \vee \forall x \forall y R(x,y)$;

f) $\exists x(R(x,x) \to \forall y R(x,y)) \models \forall x \forall y(\neg R(x,y) \to R(y,x))$. Ako tvrdnja ne vrijedi odredite barem jednu interpretaciju koja to dokazuje.

4. Pomoću glavnog testa ispitajte je li ispunjiva formula

$$(\forall x \exists y P(x, y) \wedge \forall x Q(x)) \wedge \neg \forall x \exists y (P(x, y) \wedge Q(x)).$$

5.

- Neka je A zatvorena formula, a B formula s točno jednom slobodnom varijablom.

  Koristeći glavni test dokažite ili opovrgnite

$$\{\exists x B(x) \to A,\ \neg A \vee \exists x B(x)\} \models A \vee \neg \exists x B(x).$$

Pitanje:

Prilikom ispitivanje valjanosti formule

$$\forall x \exists y A(x, y) \rightarrow \exists y \forall x A(x, y)$$

proveden je sljedeći glavni test:

$\forall x \exists y A(x,y) \rightarrow \exists y \forall x A(x,y)$ $\bigcirc\!\!\!\!\perp$

$\forall x \exists y A(x,y)$ $\top$ $(..a_1..)$ $\bigcirc\!\!\!\!a_1$, $\bigcirc\!\!\!\!a_2$

$\exists y \forall x A(x,y)$ $\perp$ $\bigcirc\!\!\!\!a_1$, $\bigcirc\!\!\!\!a_2$

$\exists y A(a_1,y)$ $\bigcirc\!\!\!\!\top$ $(..a_2..)$

$A(a_1,a_2)$ $\bigcirc\!\!\!\!\top$

$\forall x A(x,a_1)$ $\bigcirc\!\!\!\!\perp$ $(!..a_1..)$

$A(a_1,a_1)$ $\bigcirc\!\!\!\!\perp$

$\exists y A(a_2,y)$ $\bigcirc\!\!\!\!\top$ $(!..a_1..)$

$A(a_2,a_1)$ $\bigcirc\!\!\!\!\top$

$\forall x A(x,a_2)$ $\bigcirc\!\!\!\!\perp$ $(!..a_2..)$

$A(a_2,a_2)$ $\bigcirc\!\!\!\!\perp$

Uočite da je test završen, tj. provedena je analiza za sve formule i sve elemente.

Pošto test nije završen kontradikcijom možemo li zaključiti da dana formula nije valjana?

(Uočite da smo u primjeru 4 također ispitivali valjanost iste formule.

U gornjem testu smo s znakom ! označili da upotrebljavamo stari element, iako bismo po pravilu trebali uvoditi novi element.)