

Elementarne funkcije

$$e^{z_1} \cdot e^{z_2} = e^{z_1+z_2}$$

$$e^{z+2\pi i} = e^z$$

$$(e^z)' = e^z$$

$$\sin^2 z + \cos^2 z = 1$$

$$\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \cos z_1 \sin z_2$$

$$\sin(z + k\pi) = (-1)^k \sin z$$

$$\sin z = 0 \Leftrightarrow z = k\pi$$

$$\sin' z = \cos z$$

$$e^{\pm iz} = \cos z \pm i \sin z$$

$$\cos(z_1 \pm z_2) = \cos z_1 \cos z_2 \mp \sin z_1 \sin z_2$$

$$\cos(z + k\pi) = (-1)^k \cos z$$

$$\cos z = 0 \Leftrightarrow z = \frac{1}{2}(2k+1)\pi$$

$$\cos' z = -\sin z$$

$$\sin iz = i \operatorname{sh} z$$

$$\cos iz = \operatorname{ch} z$$

$$\operatorname{tg} iz = i \operatorname{th} z$$

$$\operatorname{sh} iz = i \sin z$$

$$\operatorname{ch} iz = \cos z$$

$$\operatorname{th} iz = i \operatorname{tg} z$$

$$\operatorname{ch}^2 z - \operatorname{sh}^2 z = 1$$

$$\operatorname{sh}' z = \operatorname{ch} z$$

$$\operatorname{ch}' z = \operatorname{sh} z$$

$$\operatorname{sh}(z_1 + z_2) = \operatorname{sh} z_1 \operatorname{ch} z_2 + \operatorname{ch} z_1 \operatorname{sh} z_2$$

$$\operatorname{ch}(z_1 + z_2) = \operatorname{ch} z_1 \operatorname{ch} z_2 + \operatorname{sh} z_1 \operatorname{sh} z_2$$

$$\sin z = \sin x \operatorname{ch} y + i \cos x \operatorname{sh} y$$

$$\cos z = \cos x \operatorname{ch} y - i \sin x \operatorname{sh} y$$

$$|\operatorname{sh} y| \leq |\sin z| \leq \operatorname{ch} y$$

$$|\operatorname{sh} y| \leq |\cos z| \leq \operatorname{ch} y$$

$$z^a = e^{a \operatorname{Ln} z}, \quad z \in \mathbb{C} \setminus \{0\}, \quad a \in \mathbb{C}.$$

Taylorovi redovi elementarnih funkcija oko 0

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \dots, \quad z \in \mathbb{C}$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} = z - \frac{z^3}{6} + \frac{z^5}{120} - \dots, \quad z \in \mathbb{C}$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} = 1 - \frac{z^2}{2} + \frac{z^4}{24} - \dots, \quad z \in \mathbb{C}$$

$$\operatorname{sh} z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} = z + \frac{z^3}{6} + \frac{z^5}{120} + \dots, \quad z \in \mathbb{C}$$

$$\operatorname{ch} z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} = 1 + \frac{z^2}{2} + \frac{z^4}{24} + \dots, \quad z \in \mathbb{C}$$

$$\ln(1+z) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{z^n}{n} = z - \frac{z^2}{2} + \frac{z^3}{3} - \dots, \quad |z| < 1$$

$$(1+z)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} z^n = 1 + \alpha z + \frac{\alpha(\alpha-1)}{2} z^2 + \dots, \quad |z| < 1$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n = 1 + z + z^2 + z^3 + \dots, \quad |z| < 1$$

$$\frac{1}{1+z} = \sum_{n=0}^{\infty} (-1)^n z^n = 1 - z + z^2 - z^3 + \dots, \quad |z| < 1$$

Reziduum

Ako je a pol n -tog reda za funkciju f , tada je

$$\operatorname{res}(f, a) = \frac{1}{(n-1)!} \lim_{z \rightarrow a} \frac{d^{n-1}}{dz^{n-1}} [(z-a)^n f(z)].$$

Reziduum funkcije f u beskonačnosti može se računati kao

$$\operatorname{res}(f, \infty) = -\operatorname{res}\left(\frac{1}{z^2} f\left(\frac{1}{z}\right), 0\right),$$

a ako je f analitička (holomorfna) u beskonačnosti i kao

$$\operatorname{res}(f, \infty) = \lim_{z \rightarrow \infty} z(f(\infty) - f(z)) \text{ ili}$$

$$\operatorname{res}(f, \infty) = -f_1'(0), \text{ gdje je } f_1(z) := f\left(\frac{1}{z}\right).$$