Matematika 2: Zadaci za vježbu

Zadatak 1. Izračunajte determinante i inverze matrica (ako postoje):

(a) $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}$,

(b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$,

(c) $A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & 2 \end{bmatrix}$,

(d) $A = \begin{bmatrix} 2 & 1 & 3 & 1 \\ -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$

Rješenje. (a) $\det A = 2,\ A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 2 & -1 \end{bmatrix}$

(b) $\det A = 2,\ A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\det A = 15,\ A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ -\frac{1}{15} & -\frac{1}{5} & \frac{7}{15} \end{bmatrix}$,

(d) $\det A = 1,\ A^{-1} = \begin{bmatrix} -1 & -3 & 4 & 0 \\ 1 & 4 & -5 & 1 \\ 1 & 2 & -3 & 0 \\ -1 & -4 & 6 & -1 \end{bmatrix}$

□

Zadatak 2. Gaussovom metodom eliminacije riješite sustave i zapišite rješenje matrično:

(a) $\begin{cases} x_2 - x_3 = 0, \\ x_1 + 2x_2 - x_3 = 1. \end{cases}$

(b) $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0, \\ 7x_1 + 14x_2 + 20x_3 + 27x_4 = 0, \\ 5x_1 + 10x_2 + 16x_3 + 19x_4 = -2, \\ 3x_1 + 5x_2 + 6x_3 + 13x_4 = 5. \end{cases}$

(c) $\begin{cases} 2x_1 + x_2 + 4x_3 + x_4 = 1, \\ 3x_1 + 2x_2 - x_3 - 6x_4 = 0, \\ 7x_1 + 4x_2 + 6x_3 - 5x_4 = 0, \\ 3x_1 + 8x_3 + 7x_4 = 0. \end{cases}$

(d) $\begin{cases} x_3 - x_4 = 1, \\ x_1 + 2x_2 + 3x_3 = 0, \\ -x_1 + x_3 + 2x_4 = -1, \\ 2x_1 + 2x_2 + 3x_3 - 3x_4 = 0. \end{cases}$

(e) $\begin{cases} 7x_1 - 5x_2 - 2x_3 - 4x_4 = 8, \\ -3x_1 + 2x_2 + x_3 + 2x_4 = -3, \\ 2x_1 - x_2 - x_3 - 2x_4 = 1, \\ -x_1 + x_3 + 2x_4 = 1, \\ - x_2 + x_3 + 2x_4 = 3. \end{cases}$

Rješenje. (a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ za $t \in \mathbb{R}$.

(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$.

(c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -16 \\ 25 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ za $t \in \mathbb{R}$.

(d) nema rješenja.

(e) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$ za $t, s \in \mathbb{R}$.

□

Zadatak 3. Izračunajte rang matrice:

(a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 3 & 4 & 5 & 6 \end{bmatrix}$,

(b) $A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{bmatrix}$,

(c) $A = \begin{bmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{bmatrix}$,

(d) $A = \begin{bmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 86 \\ 16 & -428 & 1 & 1284 & 52 \end{bmatrix}$,

(e) $A = \begin{bmatrix} 2 & 2 & -1 & 6 & 4 \\ 4 & 4 & 1 & 10 & 13 \\ 6 & 6 & 0 & 20 & 19 \end{bmatrix}$,

(f) $A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 1 & 5 \\ 3 & 4 & 9 \\ 1 & -1 & -2 \\ 2 & 3 & 8 \end{bmatrix}$.

Rješenje. (a) $r(A) = 3$,

(b) $r(A) = 3$,

(c) $r(A) = 3$,

(d) $r(A) = 2$,

(e) $r(A) = 3$,

(f) $r(A) = 3$.

□

Zadatak 4. Izračunajte spektar matrice te za svaku svojstvenu vrijednost nađite neki svojstveni vektor.

(a) $A = \begin{bmatrix} 3 & 2 & -5 \\ 2 & 6 & -10 \\ 1 & 2 & -3 \end{bmatrix}$,

(b) $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix}$,

(c) $A = \begin{bmatrix} 2 & -5 & -3 \\ -1 & -2 & -3 \\ 3 & 15 & 12 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,

(e) $A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$

Rješenje. (a) $k_A(\lambda) = -(\lambda - 2)^3$, $\sigma(A) = \{2\}$ sa svojstvenim vektorom (npr.) $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$.

(b) $k_A(\lambda) = -\lambda(\lambda + 2)(\lambda + 3)$, $\sigma(A) = \{-3, -2, 0\}$ sa svojstvenim vektorima (npr.) $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ redom.

(c) $k_A(\lambda) = -(\lambda - 3)^2(\lambda - 6)$, $\sigma(A) = \{3, 6\}$ sa svojstvenim vektorima (npr.) $\begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$ redom.

(d) $k_A(\lambda) = (\lambda-3)(\lambda-1)(\lambda+1)(\lambda+3)$, $\sigma(A) = \{-3, -1, 1, 3\}$ sa svojstvenim vektorima (npr.) $\begin{bmatrix} -1 \\ 3 \\ -3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ redom.

(e) $k_A(\lambda) = (\lambda - 4)^2(\lambda - 1)(\lambda - 2)$, $\sigma(A) = \{1, 2, 4\}$ sa svojstvenim vektorima (npr.) $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ redom.

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