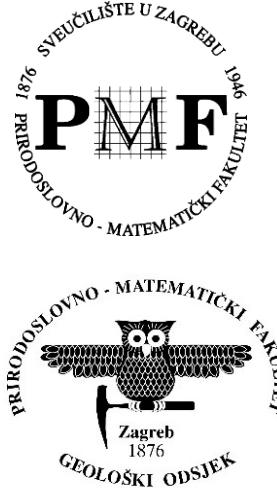


Mineralogija

Sveučilišni prijediplomski studij Znanosti o okolišu – 1. godina (253566)

Doc. dr. sc. Petra Schneider

akad. god.: 2024./25.



Sadržaj

- Kristalne klase
- Kristalni sustavi
- Triklinski sustav
- Monoklinski sustav
- Rompski sustav
- Tetragonski sustav
- Heksagonski sustav
- Kubični sustav
- Komplementarne forme
- Pseudoforme
- Određivanje simetrije

Kristalne klase

Kristalni razredi

Točkine (točkaste) grupe

→ 32 moguće kombinacije elemenata simetrije

Podjela klasa na temelju stupnja simetrije:

- **holoedrija** – klasa maksimalnog stupnja simetrije u nekom sustavu, njena simetrija jednaka je simetriji osnog križa
- **hemiedrija, tetartoedrija, ogdoedrija** – klase kod kojih je stupanj simetrije sведен na $1/2$, $1/4$, $1/8$
- **hemimorfija** – zbog gubitka horizontalne ravnine simetrije razlikuju se gornja i donja polovica kristala

KRISTALNI SUSTAVI = skup kristalnih razreda koji pokazuju zajednička simetrijska obilježja, tj. sukladnosti sa simetrijom osnog križa

- 1) kubični
- 2) tetragonski
- 3) heksagonski (+/- trigonski)
- 4) rompski
- 5) monoklinski
- 6) triklinski

Prilog 1

! Prisutnost nekih elemenata simetrije nužno zahtijeva prisutnost dodatnih elemenata simetrije:

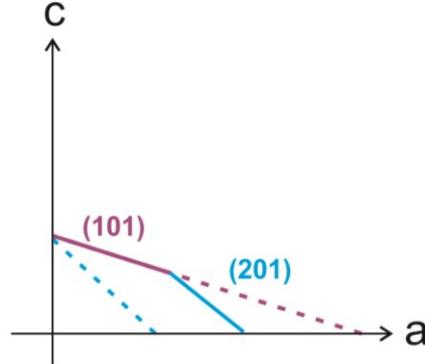
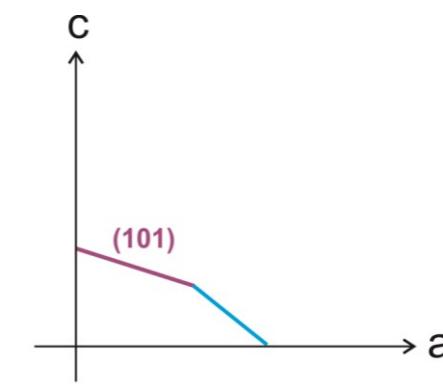
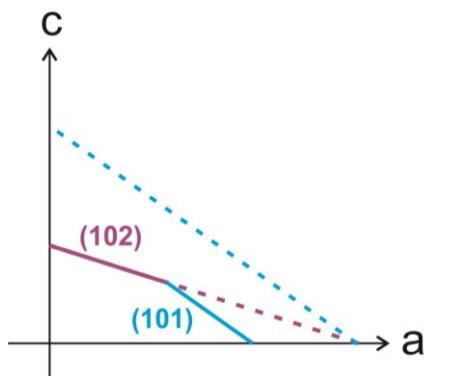
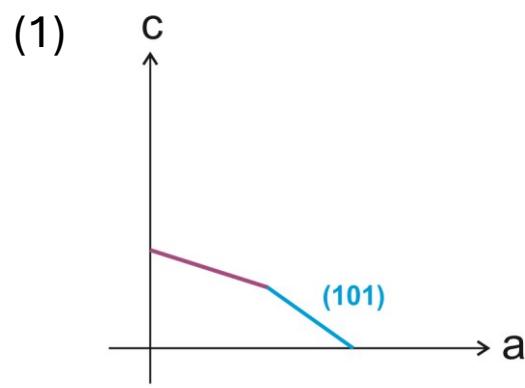
- parna os simetrije i na nju okomita ravnina simetrije zahtijevaju prisustvo centra simetrije

$$2, 4, 6 \perp m \rightarrow \mathbf{C}$$

- presjecištem dviju međusobno okomitih ravnina simetrije ide digira

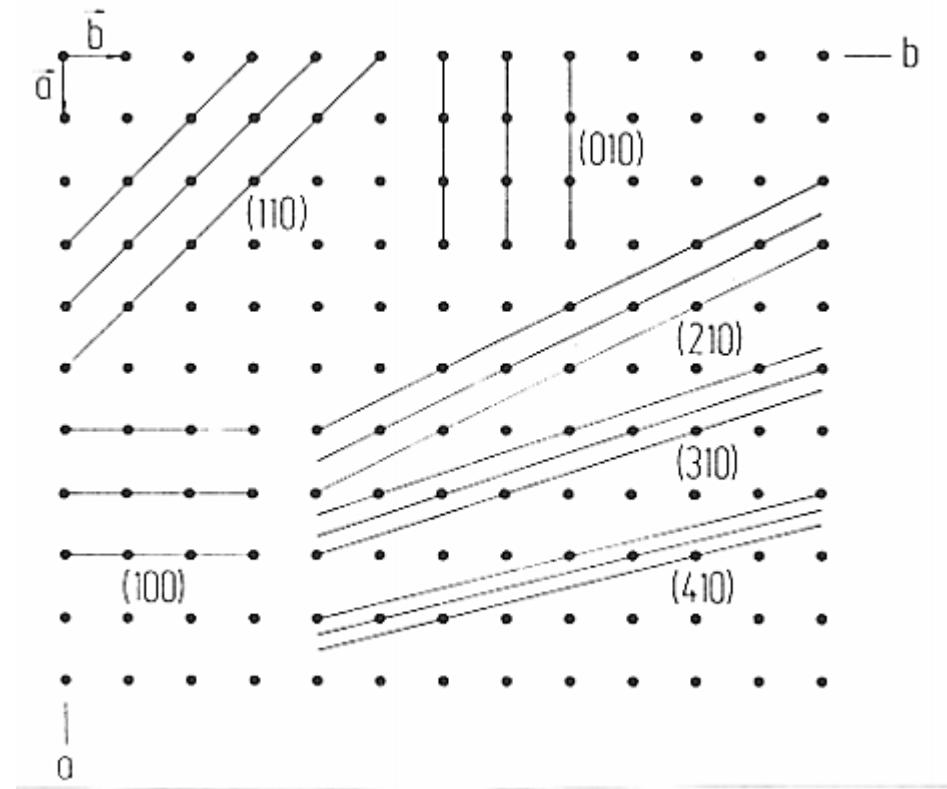
$$m \perp m \rightarrow \mathbf{2}$$

Indeksiranje ploha



! Zakon o stalnosti kutova:

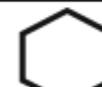
U različitim uzorcima istog minerala, kutovi između odgovarajućih ploha su isti pri konstantnoj temperaturi i tlaku.



Označavanje mrežnih ravnina

→ jedan indeks označava čitav set paralelnih mrežnih ravnina

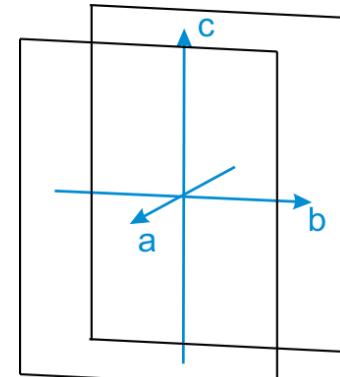
Stereografska projekcija - crtanje

Element simetrije	Oznaka	Grafički simbol
Centar simetrije	C, $\bar{1}$	○
Ravnina simetrije	P, m	---
Digira	$L^2, 2$	
Trigira	$L^3, 2$	
Tetragira	$L^4, 2$	
Heksagira	$L^6, 2$	
Rotoinverzna tetragira	$L_{(4)}^2, 2$	
Zona	-	-----

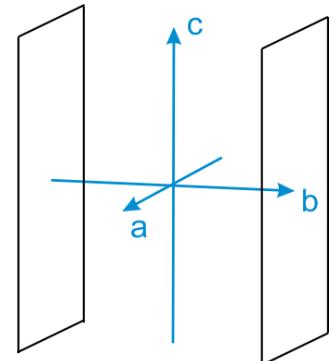
Forme

PINAKOID = otvorena forma od 2 paralelne plohe:

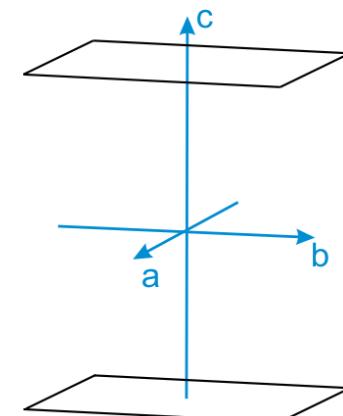
- **prednji (prvi)** – plohe sijeku samo os a , $\{100\}$
- **bočni (drugi)** – plohe sijeku samo os b , $\{010\}$
- **bazni (treći)** – plohe sijeku samo os c , $\{001\}$
- **I. položaj (a-pinakoid)** – plohe ne sijeku os a (paralelan s a);
siječe osi b i c , $\{0kl\}$
- **II. položaj (b-pinakoid)** – plohe ne sijeku os b (paralelan s b);
siječe osi a i c , $\{h0l\}$
- **III. položaj (c-pinakoid)** – plohe ne sijeku os c (paralelan s c);
siječe osi a i b , $\{hk0\}$
- **IV. (opći) položaj** – plohe sijeku sve 3 osi, $\{hkl\}$



prednji pinakoid



bočni pinakoid



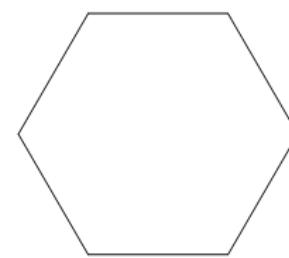
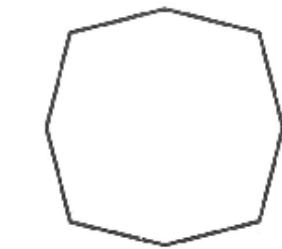
bazni pinakoid

Forme

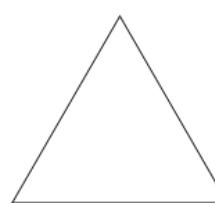
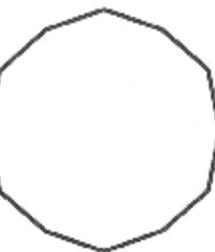
PRIZMA = otvorena forma od 3 ili više (4, 6, 8, 12) paralelnih plohe koje se sjeku u međusobno paralelnim
bridovima, tj. paralelne su s nekim pravcem
→ naziv prema **kristalnom sustavu** (presjeku) i **položaju**



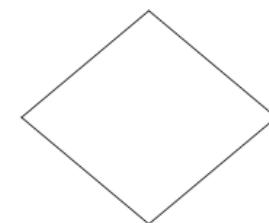
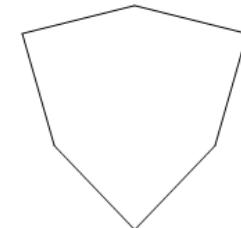
tetragonska ditetragonska



heksagonska diheksagonska



trigonska ditrigonska



rompska monoklinska

Forme

PRIZMA = otvorena forma od 3 ili više (4, 6, 8, 12) paralelnih plohe koje se sijeku u međusobno paralelnim bridovima, tj. paralelne su s nekim pravcem
→ naziv prema **kristalnom sustavu** (presjeku) i **položaju**

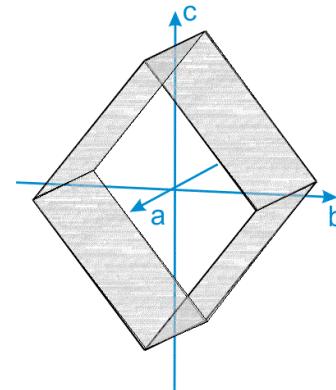
! U **rompskom** sustavu i **monoklinskom** sustavu:

I. položaj – plohe paralelne s osi *a* (siječe osi *b* i *c*)
 $\{0kl\}$ pr. $\{011\}$

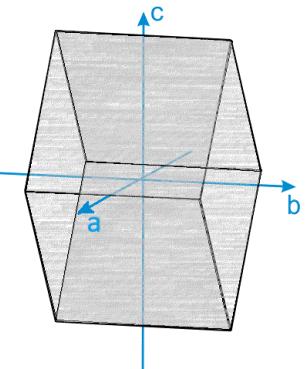
II. položaj – plohe paralelne s osi *b* (siječe osi *a* i *c*)
 $\{h0l\}$ pr. $\{101\}$

III. položaj – plohe paralelne s osi *c* (siječe osi *a* i *b*)
 $\{hk0\}$ pr. $\{110\}$

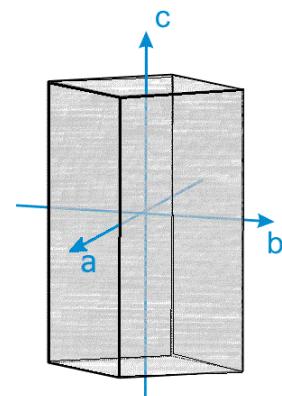
IV. položaj – plohe sijeku sve tri osi
 $\{hkl\}$ pr. $\{111\}$



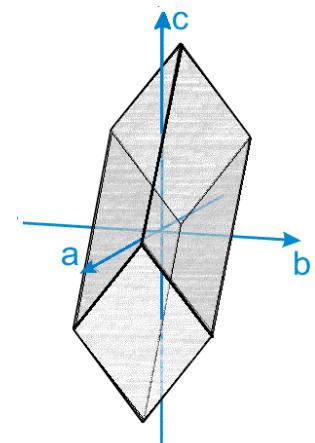
I. položaj



II. položaj

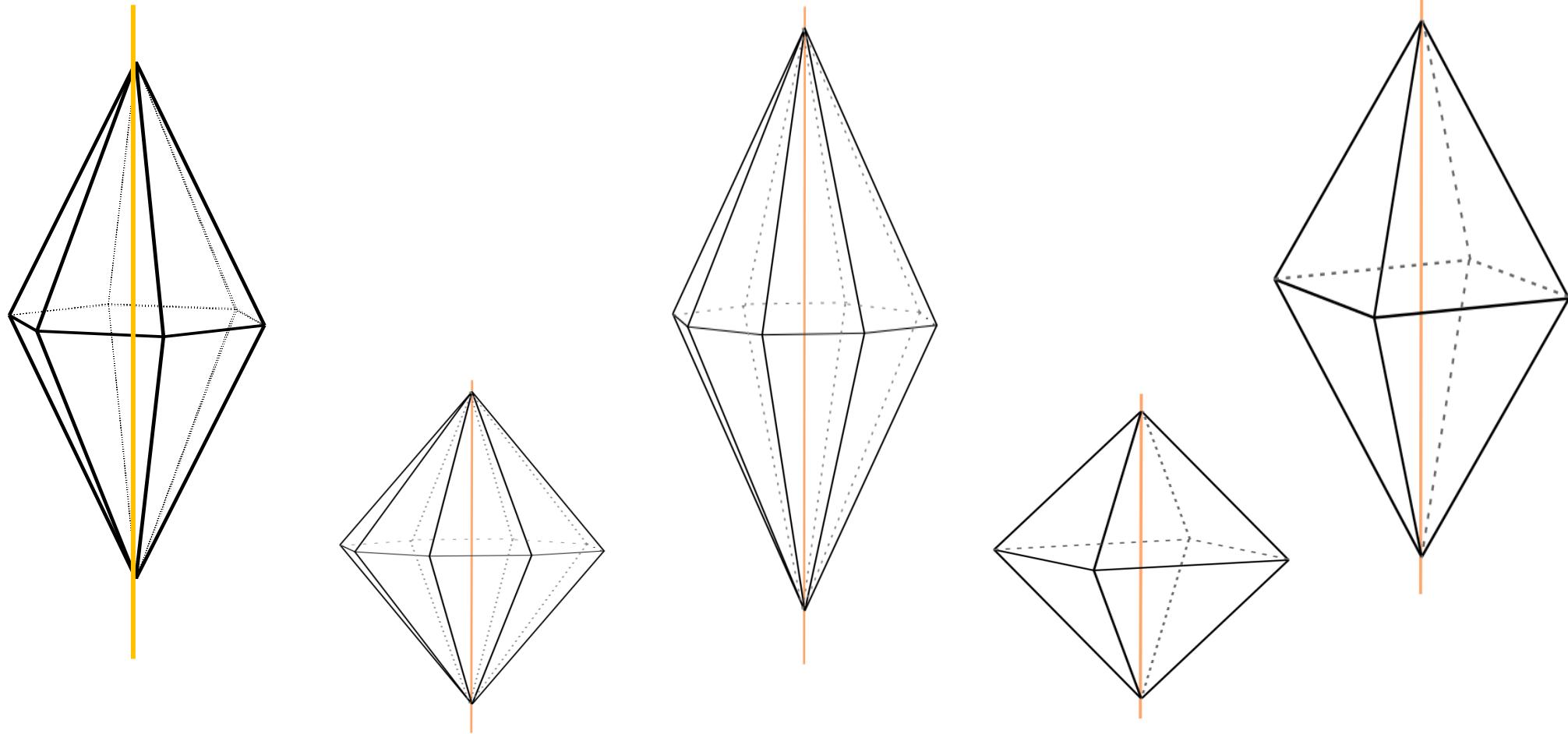


III. položaj



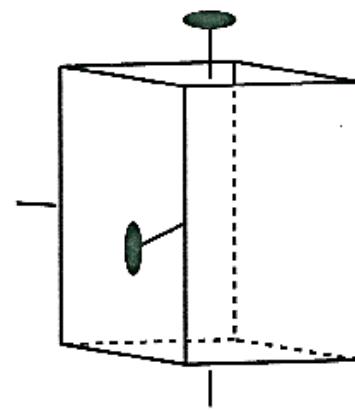
IV. položaj

DIPIRAMIDA (BIPIRAMIDA) = zatvorena forma od 6 ili više (8, 12, 16 ili 24) ploha koje su pod jednakim kutom nagnute u odnosu na neki pravac (=os c)
→ naziv prema **kristalnom sustavu** (presjeku)

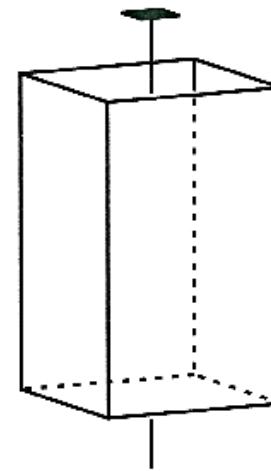


prizma

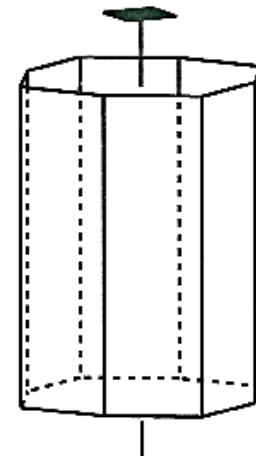
rompski



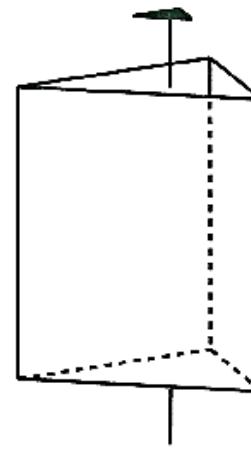
tetragonski



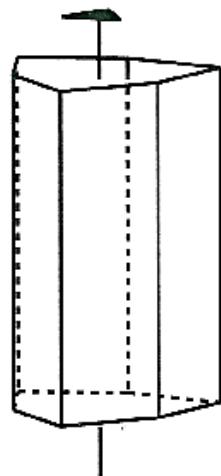
ditetragonski



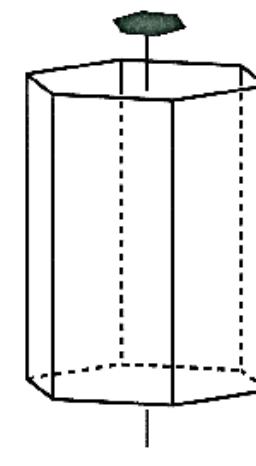
trigonski



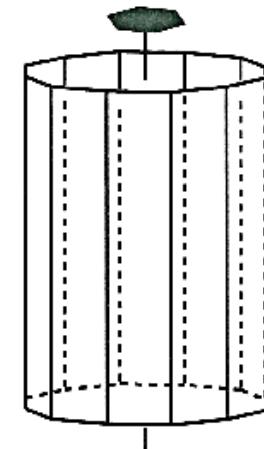
ditrigonski



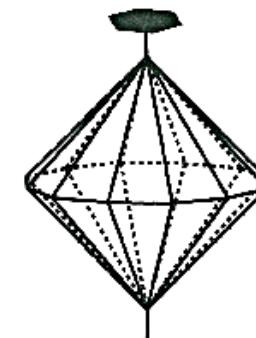
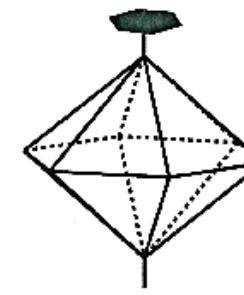
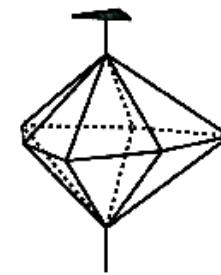
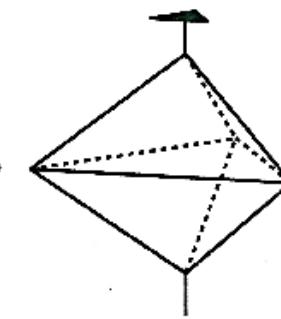
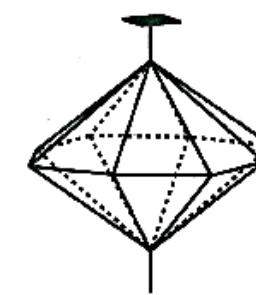
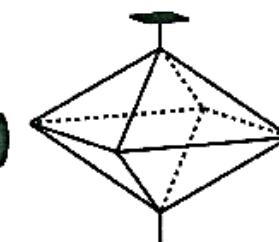
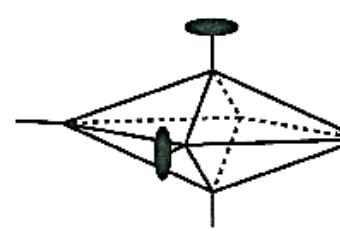
heksagonski



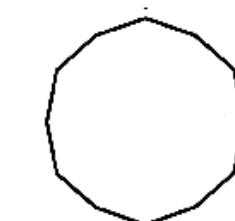
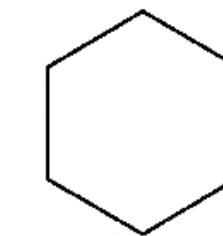
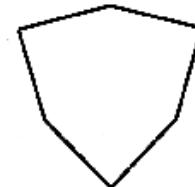
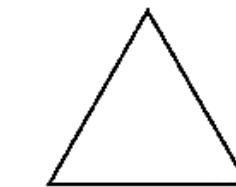
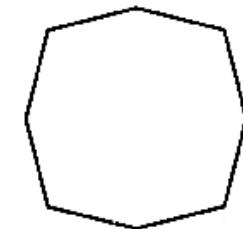
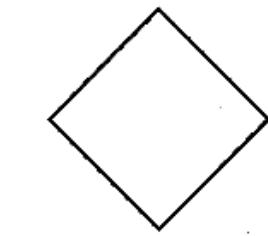
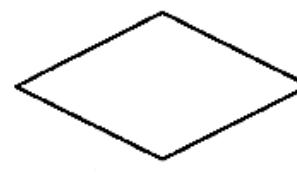
diheksagonski



dipiramida



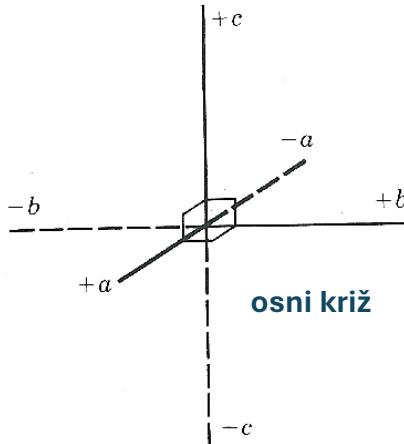
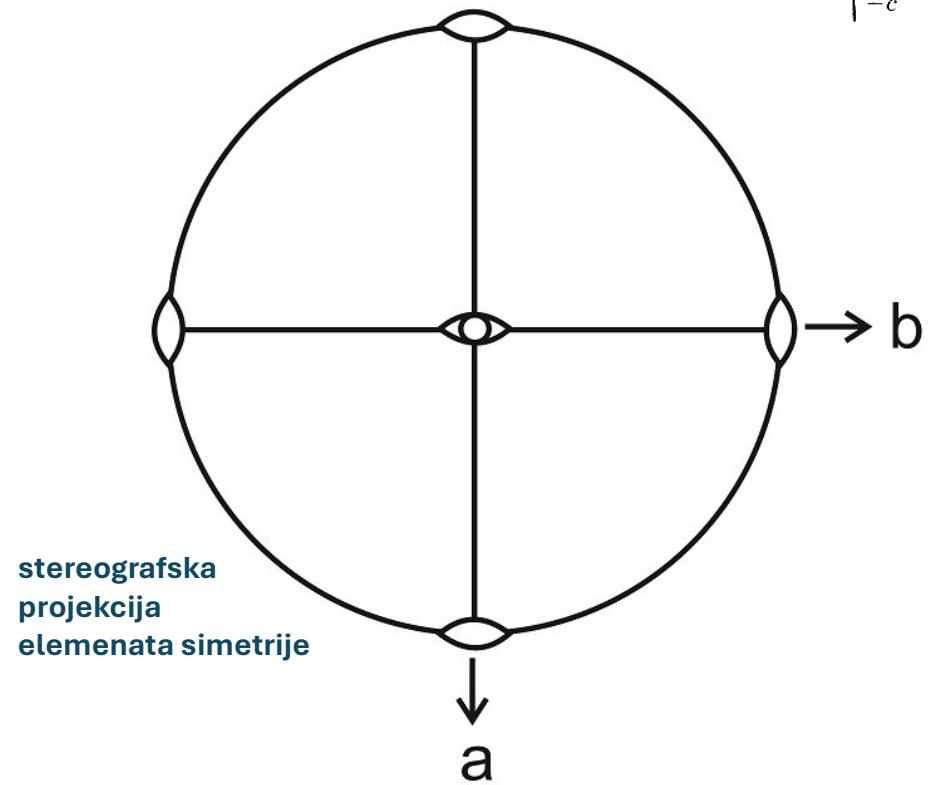
poprečni presjek



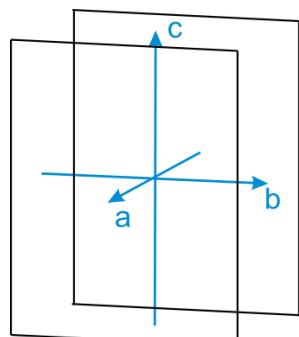
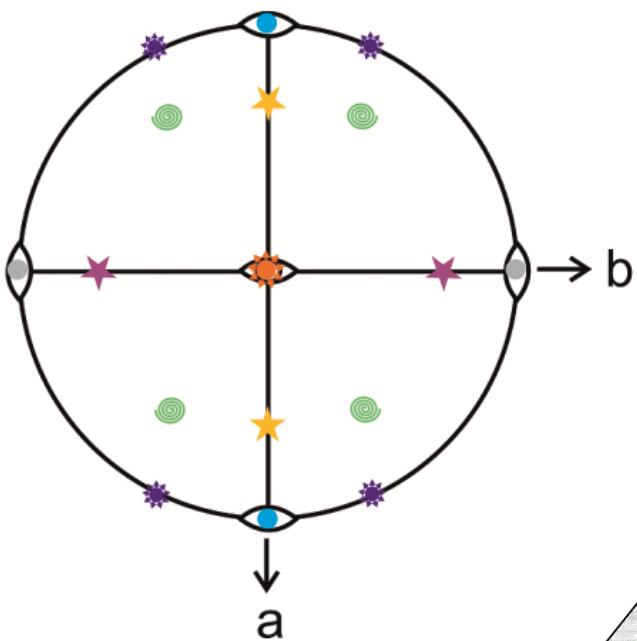
Rompski sustav

Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova			Schönlies-ov simbol
	1	2	3	
1) $\frac{2}{m} \frac{2}{m} \frac{2}{m}$	$\langle 100 \rangle$ 1	$\langle 010 \rangle$ 1	$\langle 001 \rangle$ 1	D_{2h}
2) $m m 2$				C_{2v}
3) $2 \ 2 \ 2$	kristalografska os a	kristalografska os b	kristalografska os c	D_2

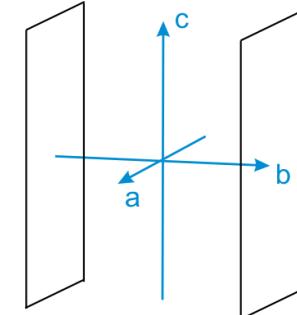
Karakteristike sustava:
 $a \neq b \neq c$
 $\alpha = \beta = \gamma = 90^\circ$



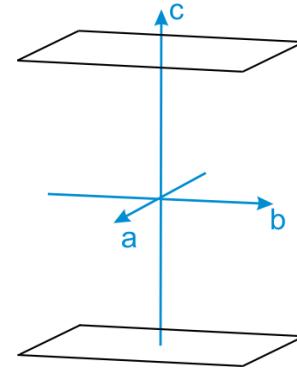
Holoedrija rompskog sustava $\left(\frac{2}{m} \frac{2}{m} \frac{2}{m}\right)$ - forme



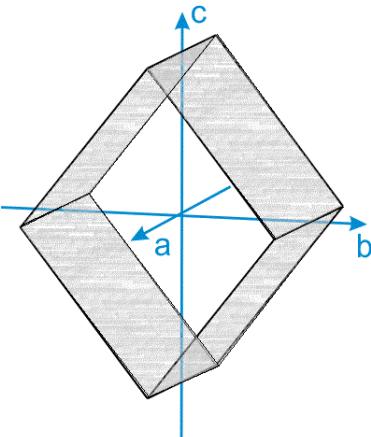
● prednji pinakoid
 $\{100\}$
 2 plohe



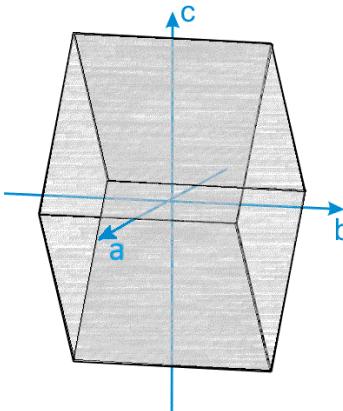
● bočni pinakoid
 $\{010\}$
 2 plohe



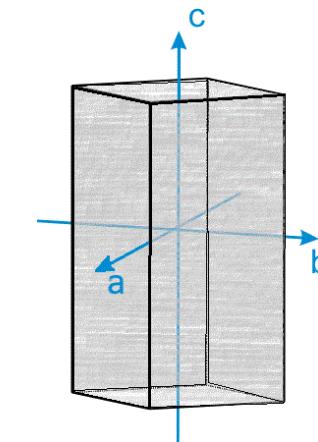
● bazni pinakoid
 $\{001\}$
 2 plohe



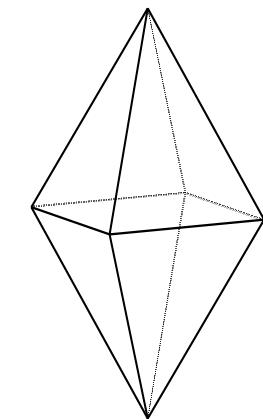
★ rompska prizma
 I. položaja
 $\{0kl\}$ pr. $\{011\}$
 4 plohe



★ rompska prizma
 II. položaja
 $\{h0l\}$ pr. $\{101\}$
 4 plohe

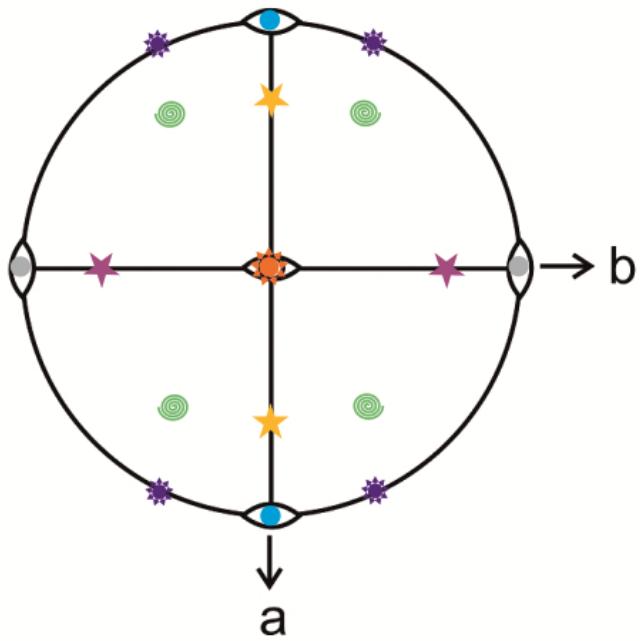


★ rompska prizma
 III. položaja
 $\{hk0\}$ pr. $\{110\}$
 4 plohe



● rompska dipiramida
 $\{hkl\}$ pr. $\{111\}$
 8 ploha

Holoedrija rompskog sustava $\left(\frac{2}{m} \frac{2}{m} \frac{2}{m}\right)$ - forme



- prednji pinakoid, {100}, 2 plohe
- bočni pinakoid, {010}, 2 plohe
- bazni pinakoid, {001}, 2 plohe
- rompska prizma I. položaja, {0kl} pr. {011}, 4 plohe
- rompska prizma II. položaja, {h0l} pr. {101}, 4 plohe
- rompska prizma III. položaja, {hk0} pr. {110}, 4 plohe
- rompska dipiramida, {hkl} pr. {111}, 8 ploha

! 7 mogućih formi unutar svake klase

- 7 različitih položaja u odnosu na kristalografske osi
- 7 različitih tipova Millerovih indeksa

OPĆA FORMA = plohe sijeku sve tri kristalografske osi u različitim udaljenostima

- {hkl}
- najveći mogući broj ploha u toj klasi
- karakteristična za svaku pojedinu kristalnu klasu



Naziv klase prema općoj formi: **rompska dipiramidska klasa**

Monoklinski sustav

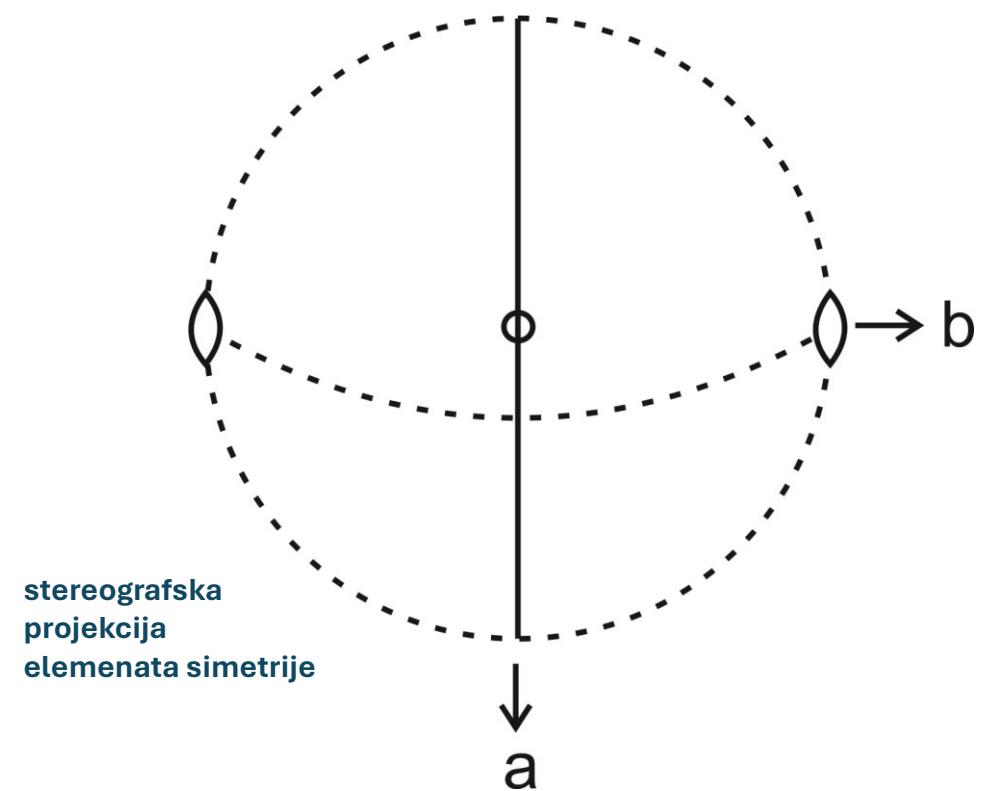
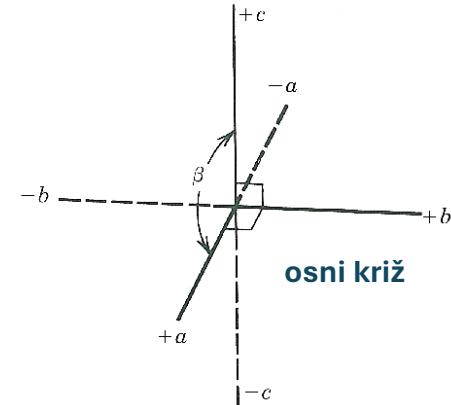
Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova	Schönlies-ov simbol
	1 2 3	
1) $\frac{2}{m}$	$<010>$ 1	C_{2h}
2) m		C_s
3) 2	kristalografska os b	C_2

Karakteristike sustava:

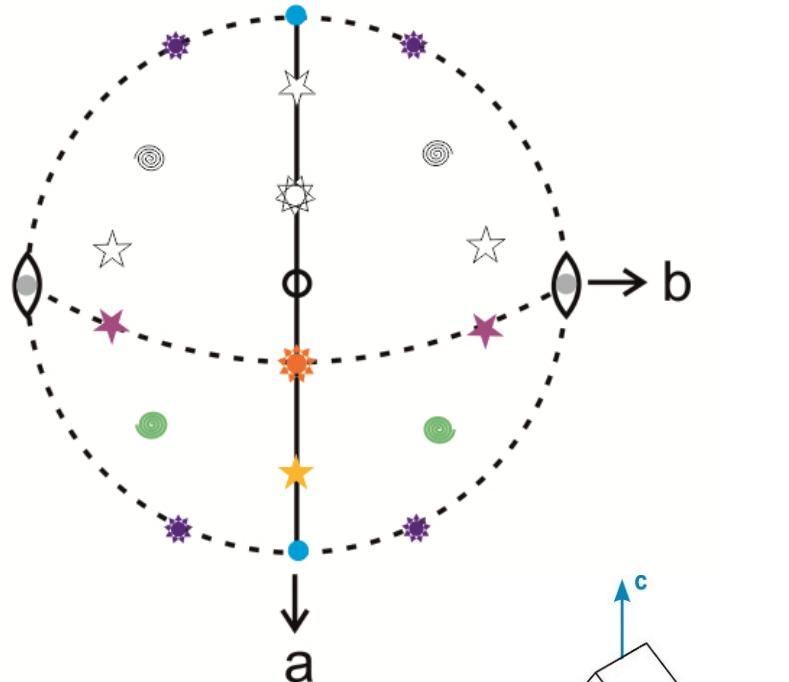
$$a \neq b \neq c$$

$$\alpha = \gamma = 90^\circ$$

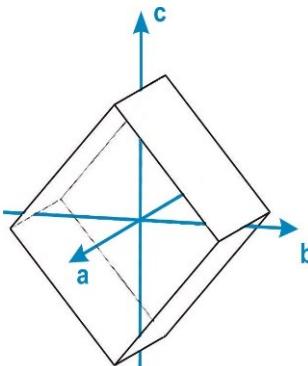
$$\beta > 90^\circ$$



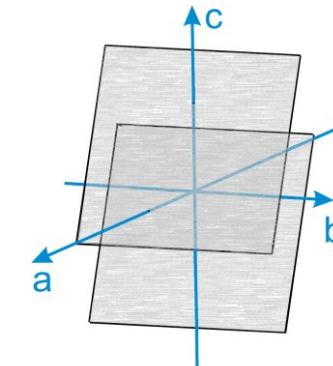
Holoedrija monoklinskog sustava $\left(\frac{2}{m}\right)$ - forme



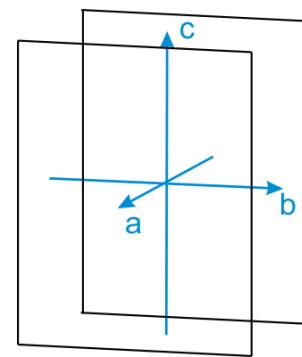
★ monoklinska prizma
I. položaja
 $\{0kl\}$ pr. $\{011\}$
4 plohe



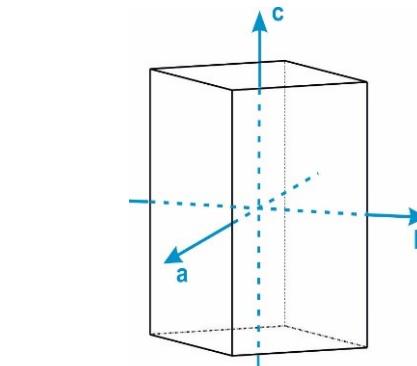
★ pinakoid II. položaja
 $\{h0l\}$ pr. $\{101\}$
2 plohe



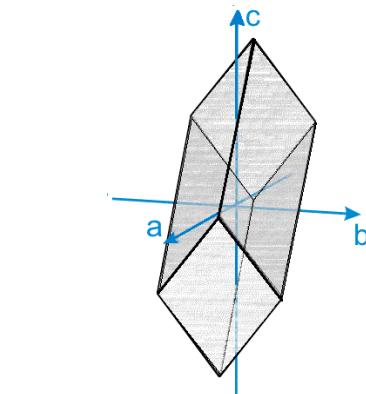
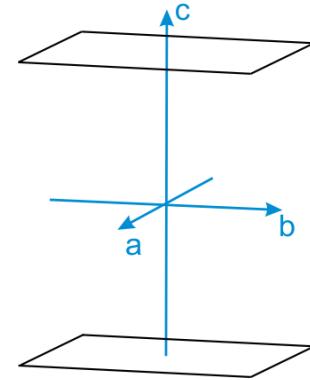
● prednji pinakoid
 $\{100\}$
2 plohe



● bočni pinakoid
 $\{010\}$
2 plohe



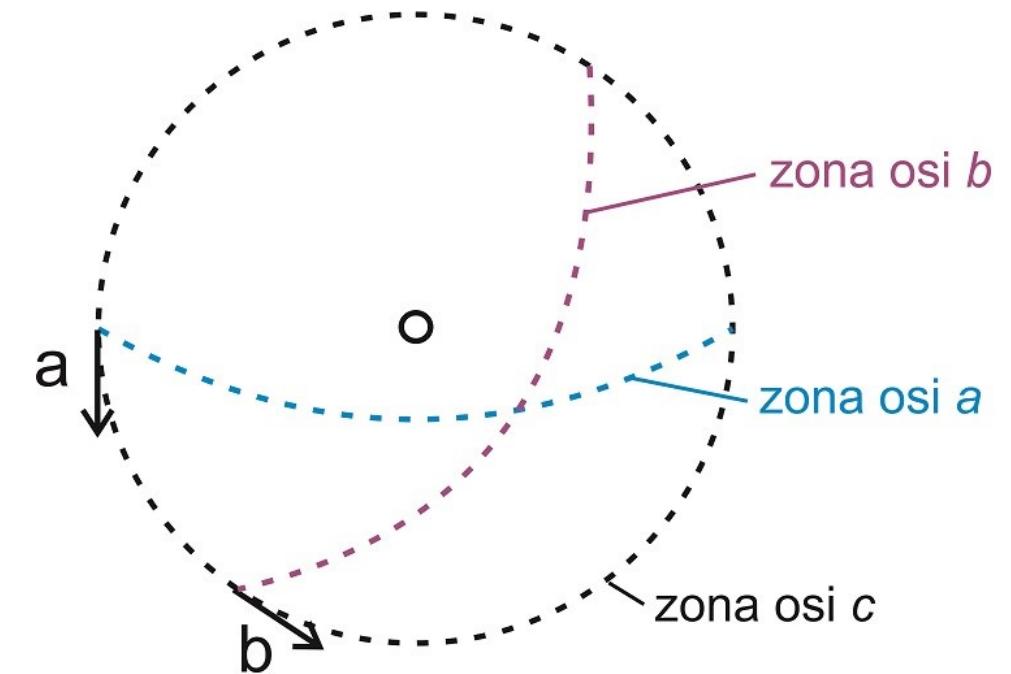
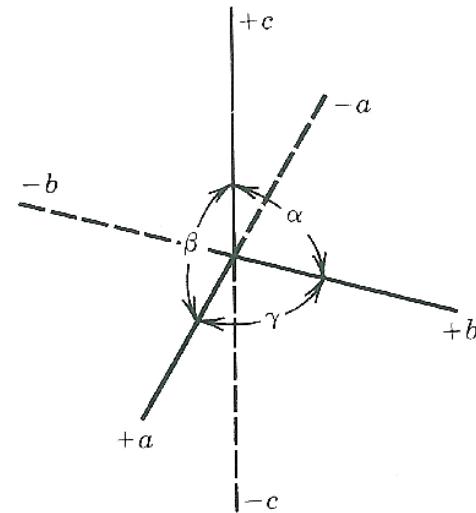
● bazni pinakoid
 $\{001\}$
2 plohe



● monoklinska prizma
IV. položaja
 $\{hkl\}$ pr. $\{111\}$
4 plohe

Triklinski sustav

Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova	Schönflies-ov simbol
	1 2 3	
1) $\bar{1}$		C_i
2) 1		C_1



Karakteristike sustava:
 $a \neq b \neq c$
 $\alpha \neq \beta \neq \gamma (\neq 90^\circ)$

Holoedrija triklinskog sustava ($\bar{1}$) – forme

PINAKOID = otvorena forma od 2 paralelne plohe:

- **prednji** – plohe sijeku samo os a , $\{100\}$

- **bočni** – plohe sijeku samo os b , $\{010\}$

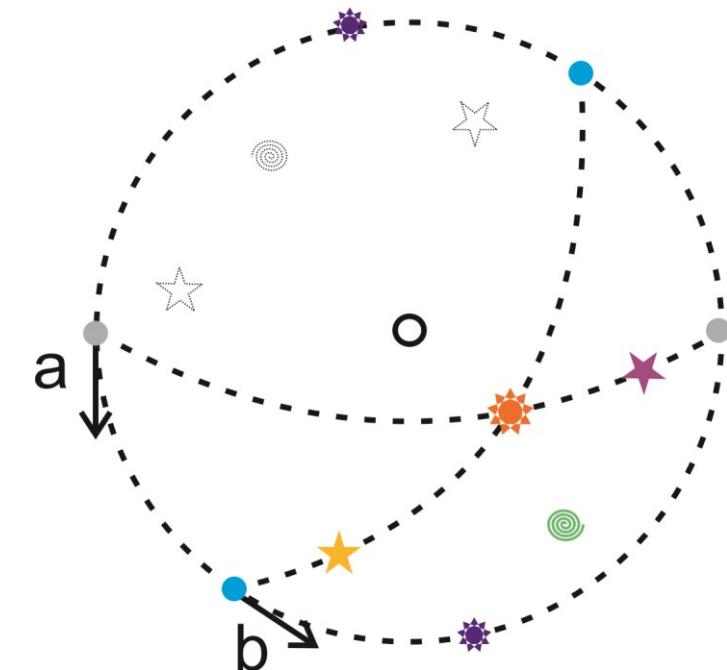
- **bazni** – plohe sijeku samo os c , $\{001\}$

- **I. položaj** – plohe ne sijeku os a (paralelan s a); siječe osi b i c , $\{0kl\}$

- **II. položaj** – plohe ne sijeku os b (paralelan s b); siječe osi a i c , $\{h0l\}$

- **III. položaj** – plohe ne sijeku os c (paralelan s c); siječe osi a i b , $\{hk0\}$

- **IV. (opći) položaj** – plohe sijeku sve 3 osi, $\{hkl\}$

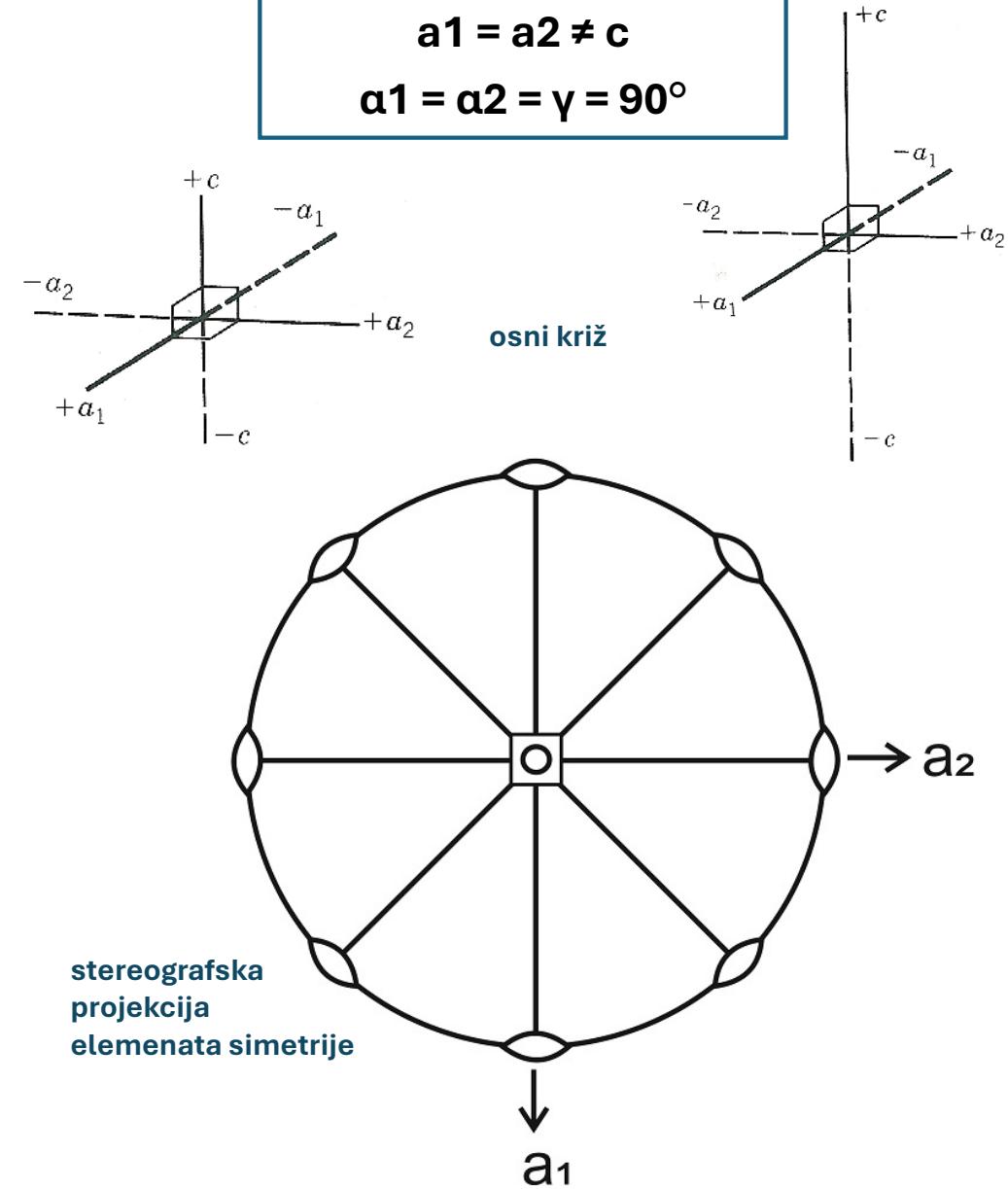


Tetragonski sustav

Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova			Schönflies-ov simbol
	1	2	3	
1) $\frac{4}{m} \frac{2}{m} \frac{2}{m}$	<001>	<100>	<110>	D_{4h}
2) $\bar{4} 2 m$	1	2	2	D_{2d}
3) $4 m m$				C_{4v}
4) $\frac{4}{m}$				C_{4h}
5) $4 2 2$				D_4
6) $\bar{4}$				S_4
7) 4	kristalografska os c	kristalografske osi a_1, a_2	simetrale između kristalografskih osi a_1, a_2	C_4

Karakteristike sustava:

$$\begin{aligned} a_1 &= a_2 \neq c \\ a_1 &= a_2 = \gamma = 90^\circ \end{aligned}$$



Forme

PRIZMA = otvorena forma od 3 ili više (4, 6, 8, 12) paralelnih plohe koje se sijeku u međusobno paralelnim bridovima, tj. paralelne su s nekim pravcem
→ naziv prema **kristalnom sustavu** (presjeku) i **položaju**

! U **tetragonskom i heksagonskom** sustavu:

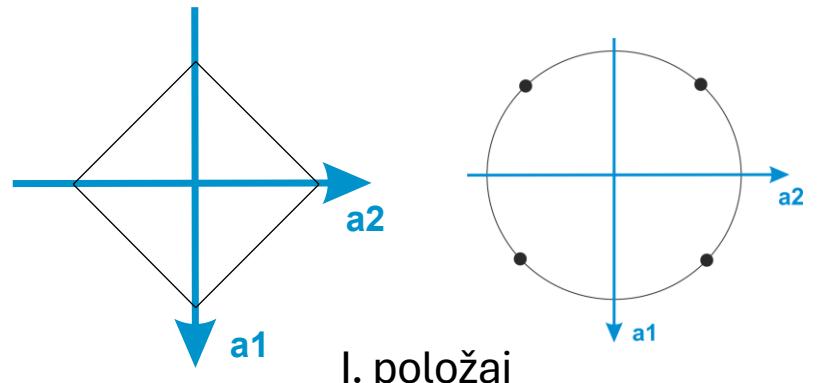
→ **uvijek paralelne sa osi c!**

I. položaj – stereografska projekcija ploha nalazi se
na simetralama između osi a_1 i a_2 (i/ili a_3)

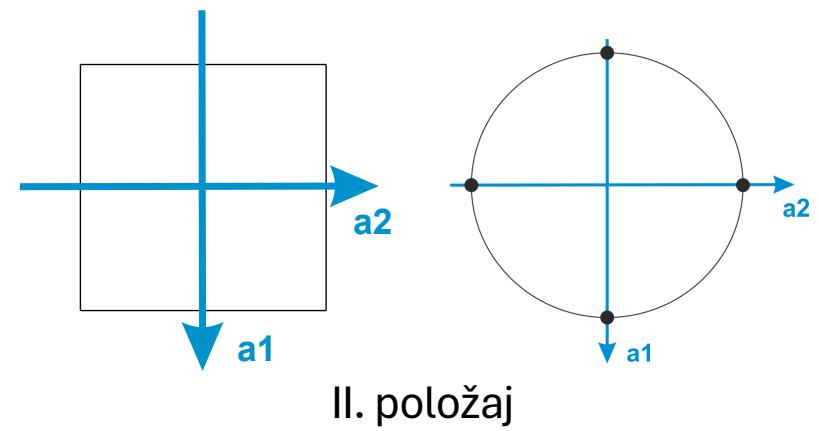
$\{110\}$ tj. $\{2\bar{1}\bar{1}0\}$

II. položaj – stereografska projekcija ploha nalazi se
na osima a_1 i a_2 (i a_3)

$\{100\}$ tj. $\{10\bar{1}0\}$



I. položaj



II. položaj

poprečni presjek
prizme

stereografska
projekcija ploha
prizme

DIPIRAMIDA (BIPIRAMIDA) = zatvorena forma od 6 ili više (8, 12, 16 ili 24) ploha koje su pod jednakim kutom nagnute u odnosu na neki pravac
→ naziv prema **kristalnom sustavu** (presjeku) i **položaju**

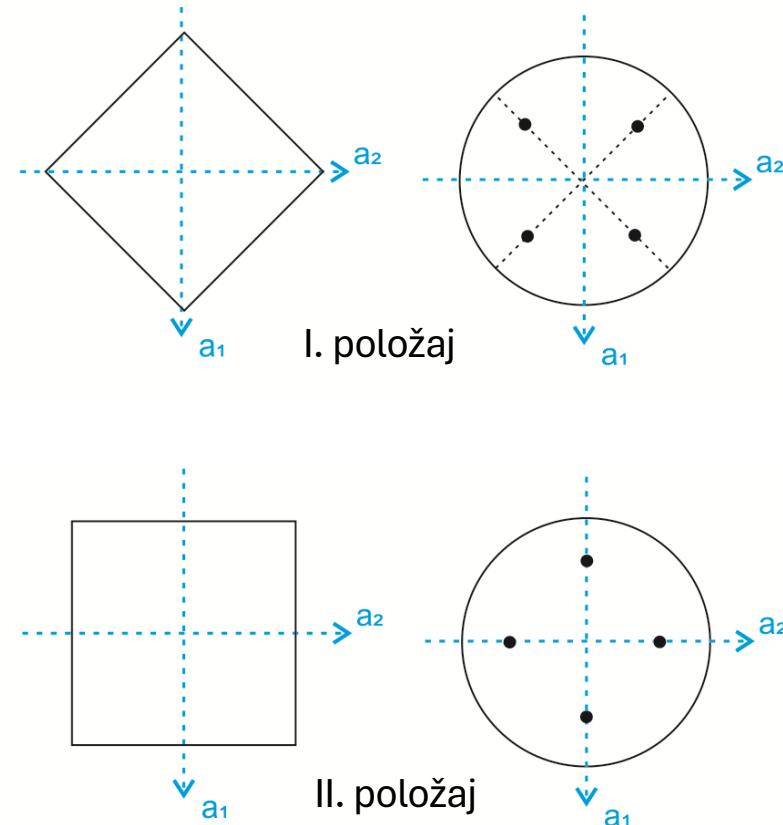
! U **tetragonskom i heksagonskom** sustavu:

I. položaj – stereografska projekcija ploha nalazi se na simetralama između osi a_1 i a_2 (i/ili a_3)

$$\{110\} \text{ tj. } \{2\bar{1}\bar{1}0\}$$

II. položaj – stereografska projekcija ploha nalazi se na osima a_1 , i a_2 (i a_3)

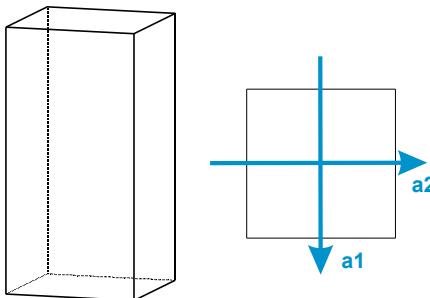
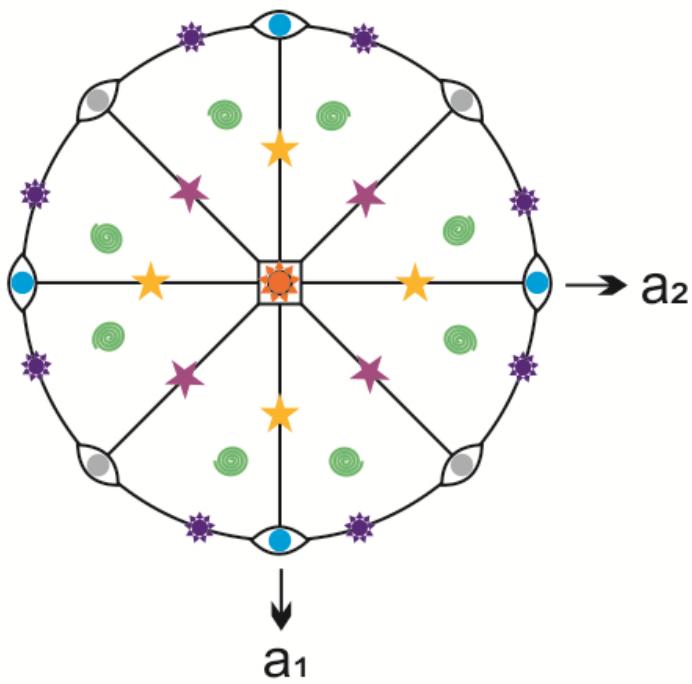
$$\{100\} \text{ tj. } \{10\bar{1}0\}$$



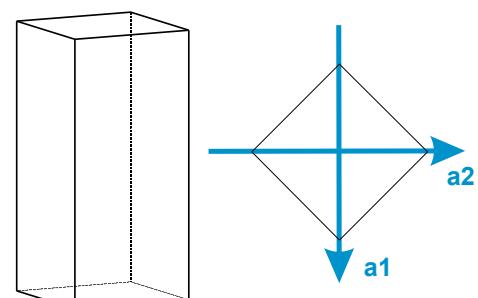
poprečni presjek
dipiramide

stereografska
projekcija ploha
dipiramide

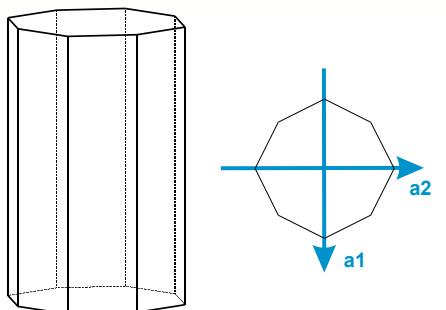
Holoedrija tetragonskog sustava $\left(\frac{4}{m} \frac{2}{m} \frac{2}{m}\right)$ - forme



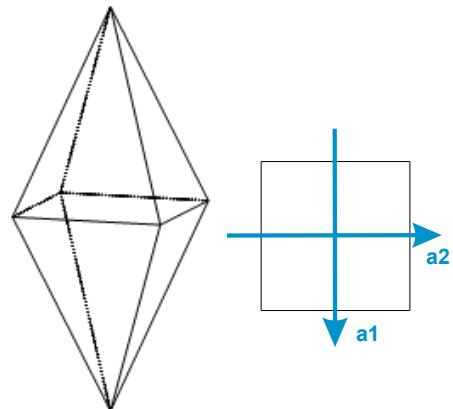
● tetragonska prizma
II. položaja
{100}
4 plohe



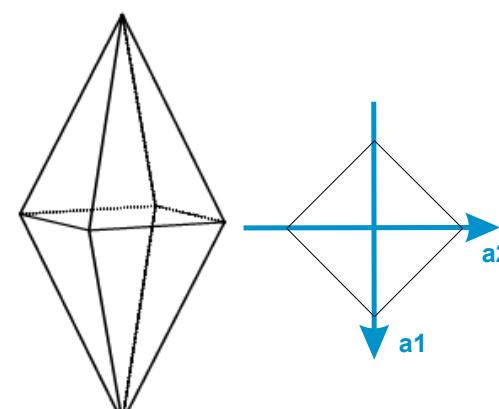
● tetragonska prizma
I. položaja
{110}
4 plohe



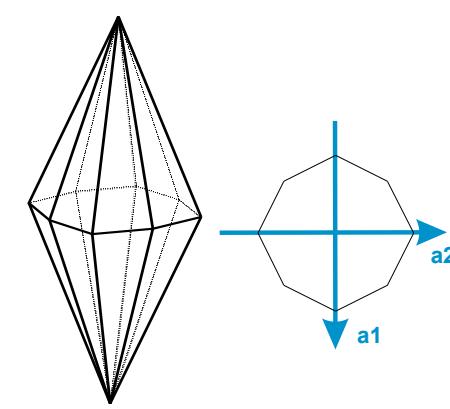
● ditetragonska prizma
{hk0} pr. {210}
8 ploha



● tetragonska dipiramida
II. položaja
{h0l} pr. {101}
8 ploha



● tetragonska dipiramida
I. položaja
{hhl} pr. {111}
8 ploha



● ditetragonska dipiramida
{hkl} pr. {211}
16 ploha

Heksagonski sustav (dio I.)

Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova			Schönlies-ov simbol
	1	2	3	
1) $\frac{6}{m} \frac{2}{m} \frac{2}{m}$	<001>	<100>	<210>	D_{6h}
2) $\bar{6} m 2$	1	3	3	D_{3h}
3) $6 m m$				C_{6v}
4) $6\bar{2}2$				D_6
5) $\frac{6}{m}$				C_{6h}
6) $\bar{6}$				C_{3h}
7) 6	kristalografska os c	kristalografske osi a_1, a_2, a_3	simetrale između kristalografskih osi a_1, a_2, a_3	C_6

Bravais-Millerovi indeksi {hkl}

$$\mathbf{h} + \mathbf{k} + \mathbf{l} = \mathbf{0}$$

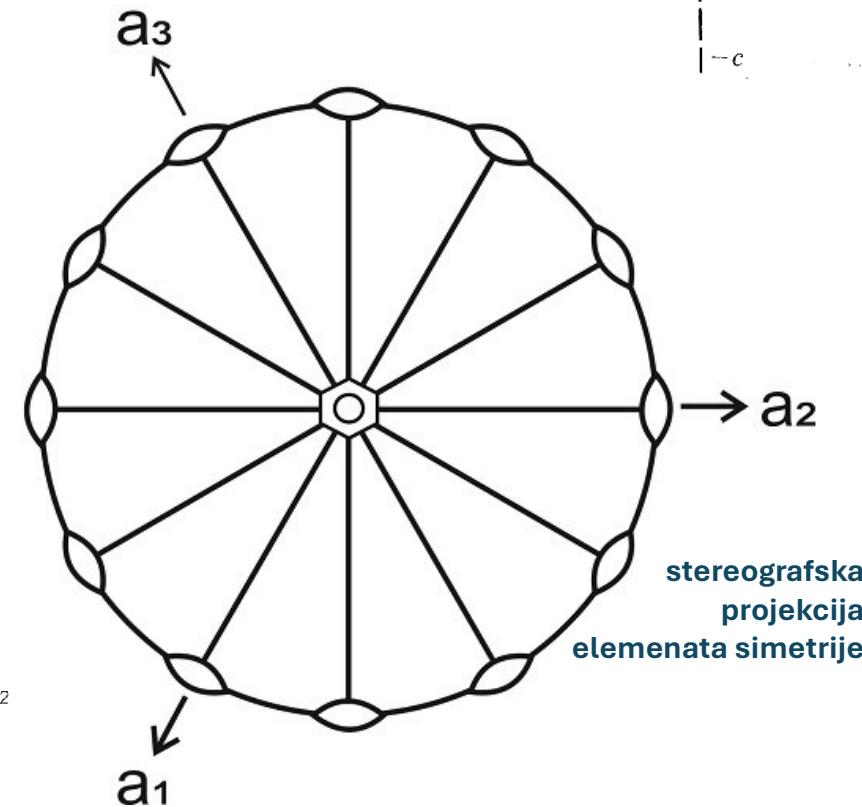
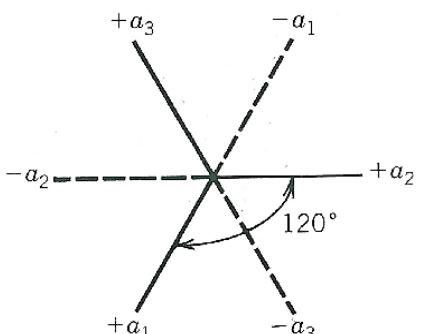
*ponekad se piše samo (hkl) ili (hk.l)

Karakteristike sustava:

$$a_1 = a_2 = a_3 \neq c$$

$$\alpha_1 = \alpha_2 = \alpha_3 = 120^\circ$$

$$\delta = 90^\circ$$



Forme

PRIZMA = otvorena forma od 3 ili više (4, 6, 8, 12) paralelnih plohe koje se sijeku u međusobno paralelnim bridovima, tj. paralelne su s nekim pravcem
→ naziv prema **kristalnom sustavu** (presjeku) i **položaju**

! U tetragonskom i heksagonskom sustavu:

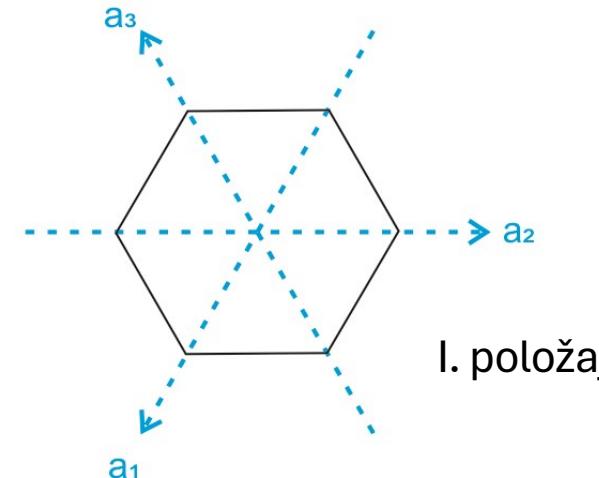
→ **uvijek paralelne sa osi c!**

I. položaj – stereografska projekcija ploha nalazi se
na simetralama između osi a_1 i a_2 (i/ili a_3)

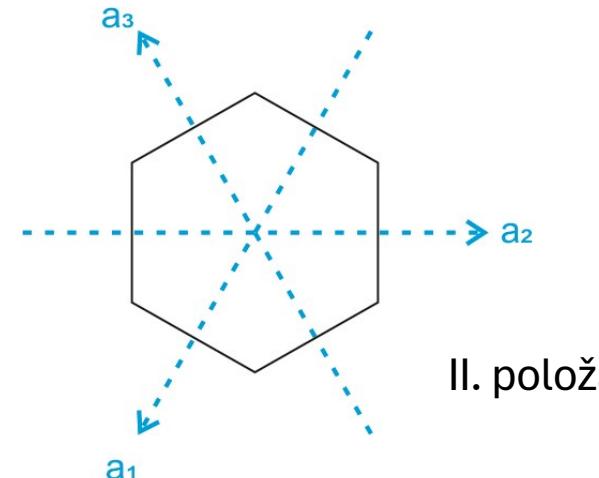
$\{110\}$ tj. $\{2\bar{1}\bar{1}0\}$

II. položaj – stereografska projekcija ploha nalazi se
na osima a_1 , a_2 (i a_3)

$\{100\}$ tj. $\{10\bar{1}0\}$

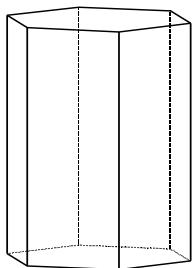
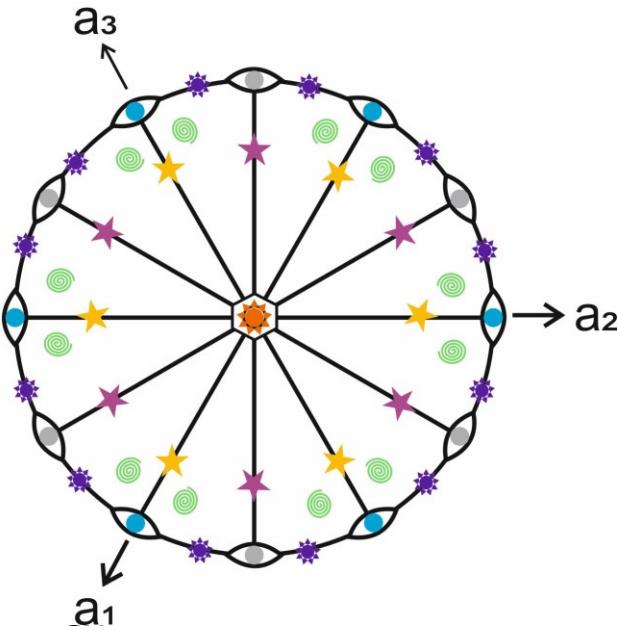


I. položaj

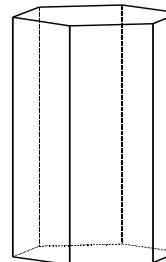
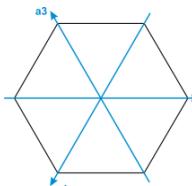


II. položaj

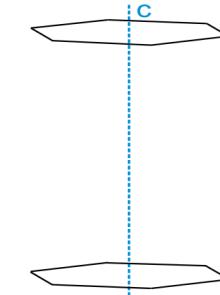
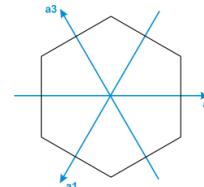
Holoedrija heksagonskog sustava $\left(\frac{6}{m} \frac{2}{m} \frac{2}{m}\right)$ - forme



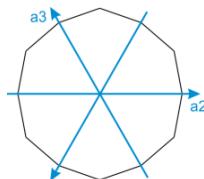
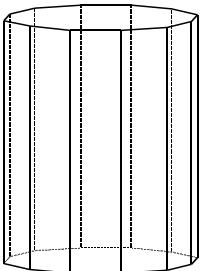
● heksagonska prizma
I. položaja
 $\{10\bar{1}0\}$
6 ploha



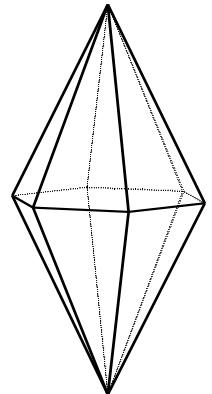
● heksagonska prizma
II. položaja
 $\{11\bar{2}0\}$
6 ploha



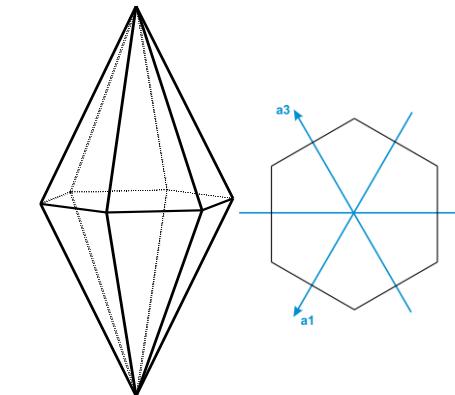
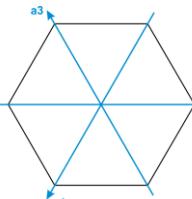
● bazni pinakoid
 $\{0001\}$
2 plohe



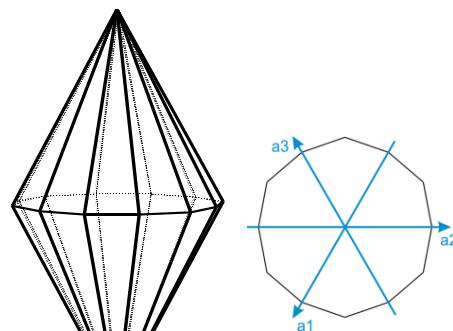
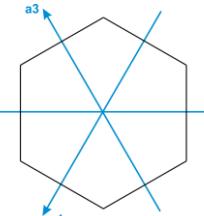
● diheksagonska prizma
 $\{hk\bar{i}0\}$, pr. $\{21\bar{3}0\}$
12 ploha



● heksagonska dipiramida
I. položaja
 $\{h0\bar{h}l\}$ pr. $\{10\bar{1}1\}$
12 ploha



● heksagonska dipiramida
II. položaja
 $\{hh\bar{2}hl\}$ pr. $\{11\bar{2}1\}$
12 ploha



● diheksagonska
dipiramida
 $\{hk\bar{l}l\}$ pr. $\{21\bar{3}1\}$
24 ploha

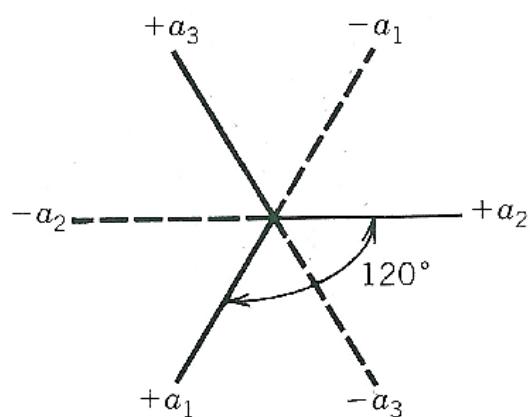
Heksagonski sustav (dio II.)

Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova			Schönflies-ov simbol
	1	2	3	
1) $\bar{3} \frac{2}{m}$	<001>	<100>	<210>	D_{3d}
2) $3m$	1	3	3	C_{3v}
3) 32				D_3
4) $\bar{3}$				C_{3i}
5) 3	kristalografska os c	kristalografske osi a_1, a_2, a_3	simetrale između kristalografskih osi a_1, a_2, a_3	C_3

Bravais-Millerovi indeksi {hklil}

$$\mathbf{h+k+l=0}$$

*ponekad se piše samo (hkl) ili (hk.l)

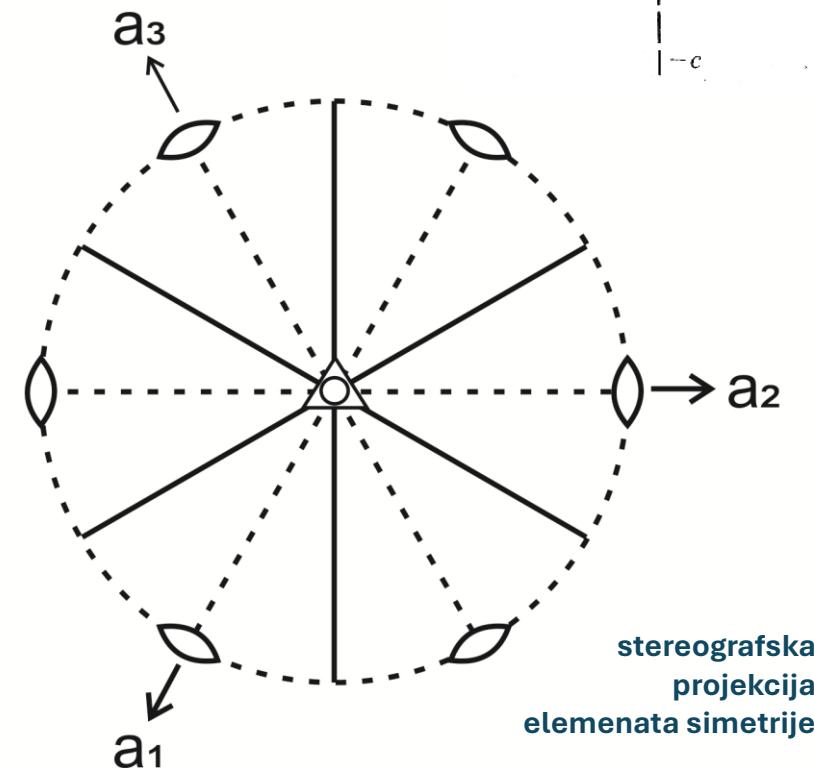
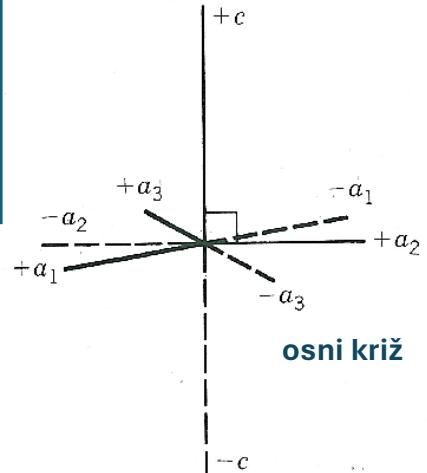


Karakteristike sustava:

$$a_1 = a_2 = a_3 \neq c$$

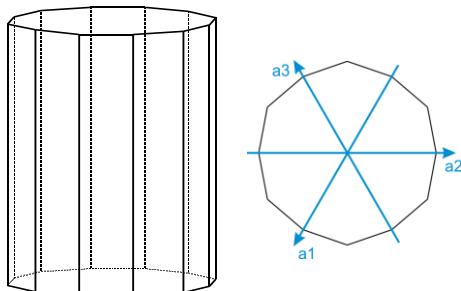
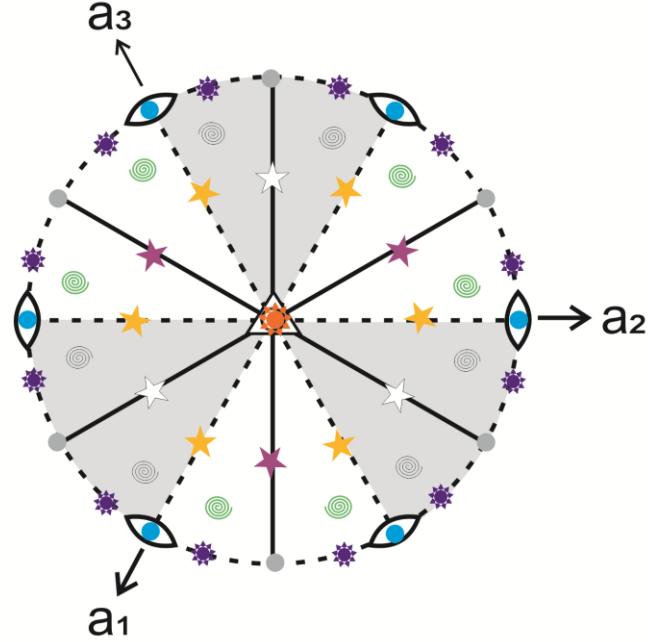
$$a_1 = a_2 = a_3 = 120^\circ$$

$$\delta = 90^\circ$$

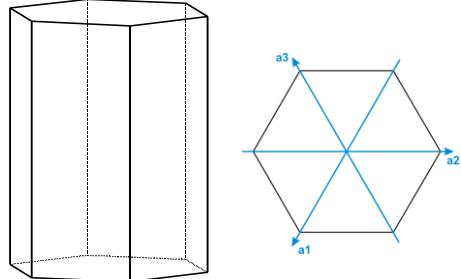


stereografska projekcija elemenata simetrije

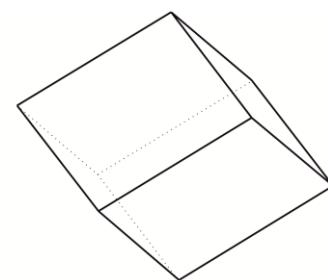
Romboedrijska hemiedrija heksagonskog sustava $(\bar{3} \frac{2}{m})$ - forme



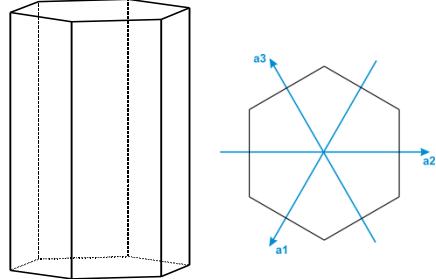
diheksagonska prizma
 $\{hk\bar{0}\}$, pr. $\{21\bar{3}0\}$
 12 ploha



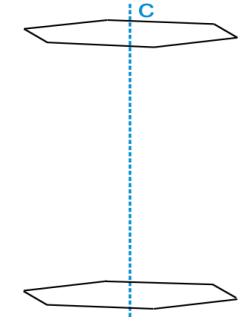
heksagonska prizma
 I. položaja
 $\{10\bar{1}0\}$
 6 ploha



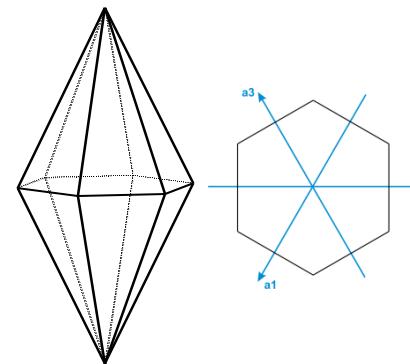
romboedar (+)
 $\{h0\bar{h}l\}$ pr. $\{10\bar{1}1\}$
 6 ploha



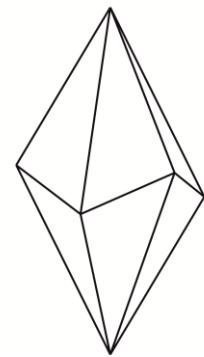
heksagonska prizma
 II. položaja
 $\{11\bar{2}0\}$
 6 ploha



bazni pinakoid
 $\{0001\}$
 2 plohe



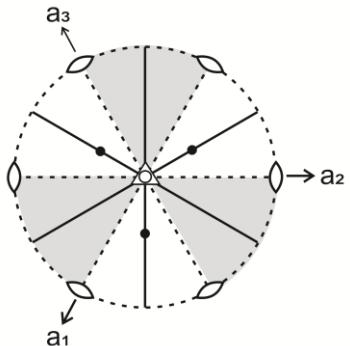
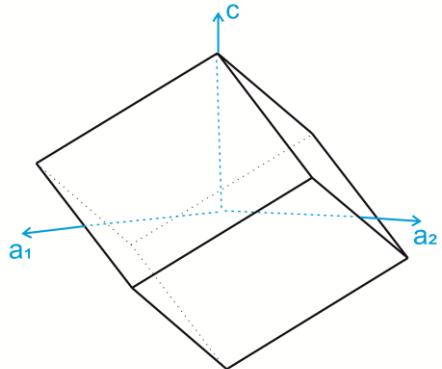
heksagonska dipiramida
 II. položaja
 $\{hh\bar{2}hl\}$ pr. $\{11\bar{2}1\}$
 12 ploha



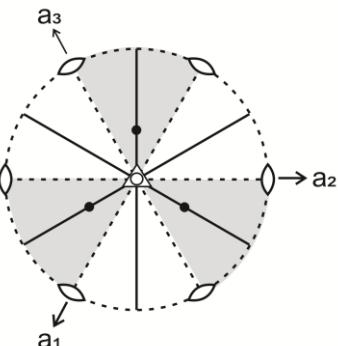
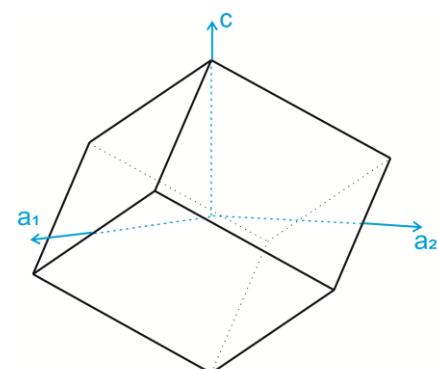
ditrigonski skalenoedar (+)
 $\{hk\bar{l}l\}$ pr. $\{21\bar{3}1\}$
 12 ploha

Romboedrijska hemiedrija heksagonskog sustava $(\bar{3} \frac{2}{m})$

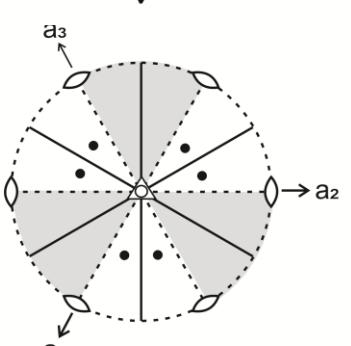
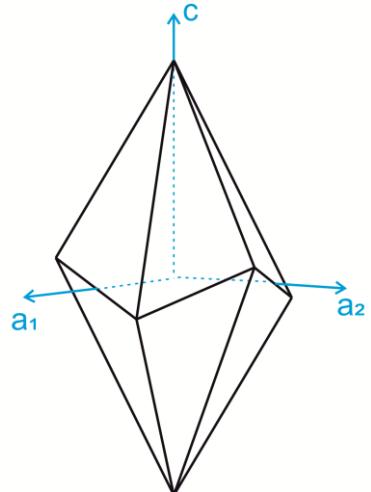
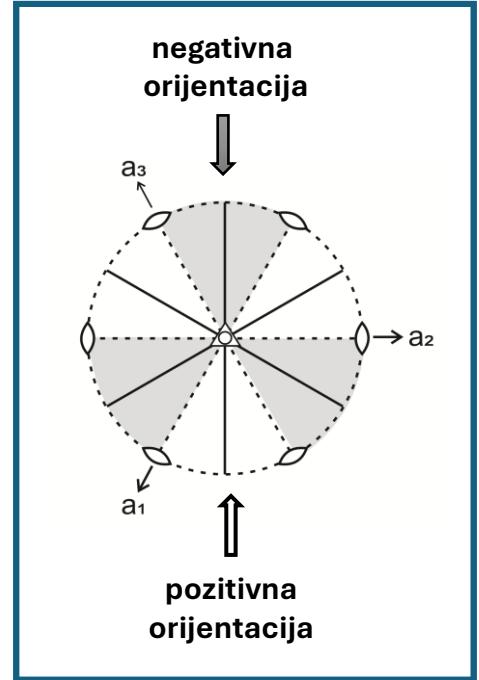
- pozitivne (+) i negativne (-) orijentacije formi



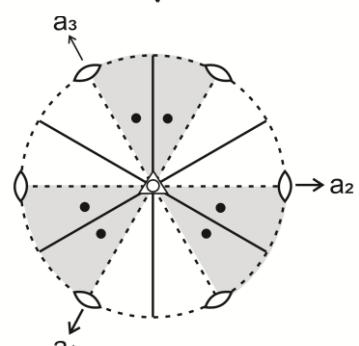
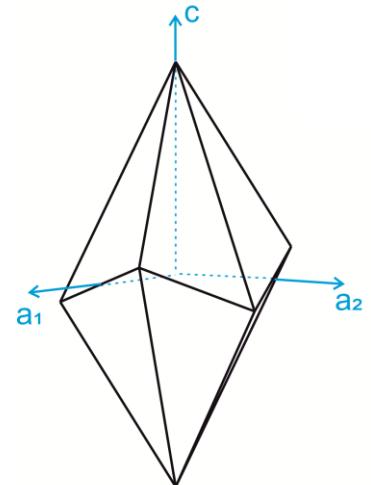
romboedar (+)
 $\{h0\bar{h}\}$ pr. $\{10\bar{1}1\}$
6 ploha



romboedar (-)
 $\{0h\bar{h}l\}$ pr. $\{01\bar{1}1\}$
6 ploha



**ditrigonski
skalenoedar (+)**
 $\{hk\bar{l}\}$ pr. $\{21\bar{3}1\}$
12 ploha



**ditrigonski
skalenoedar (-)**
 $\{hk\bar{l}\}$ pr. $\{12\bar{3}1\}$
12 ploha

Heksagonski sustav - forme

	6/m 2/m 2/m		$\bar{3}$ 2/m	
	Ime forme	Indeks forme	Ime forme	Indeks forme
1	Bazni pinakoid	{0001}	Bazni pinakoid	{0001}
2	Heksagonska prizma I. položaja	{10\bar{1}0}	Heksagonska prizma I. položaja	{10\bar{1}0}
3	Heksagonska prizma II. položaja	{11\bar{2}0}	Heksagonska prizma II. položaja	{11\bar{2}0}
4	Diheksagonska prizma	{hki0}	Diheksagonska prizma	{hki0}
5	Heksagonska dipiramida I. položaja	{h0\bar{h}l}	Romboedar +/-	{h0\bar{h}l} {0h\bar{h}l}
6	Heksagonska dipiramida II. položaja	{hh\bar{2}h\bar{l}}	Heksagonska dipiramida II. položaja	{hh\bar{2}h\bar{l}}
7	Diheksagonska dipiramida	{hkil}	Ditrigonski skalenoedar +/-	{hkil}

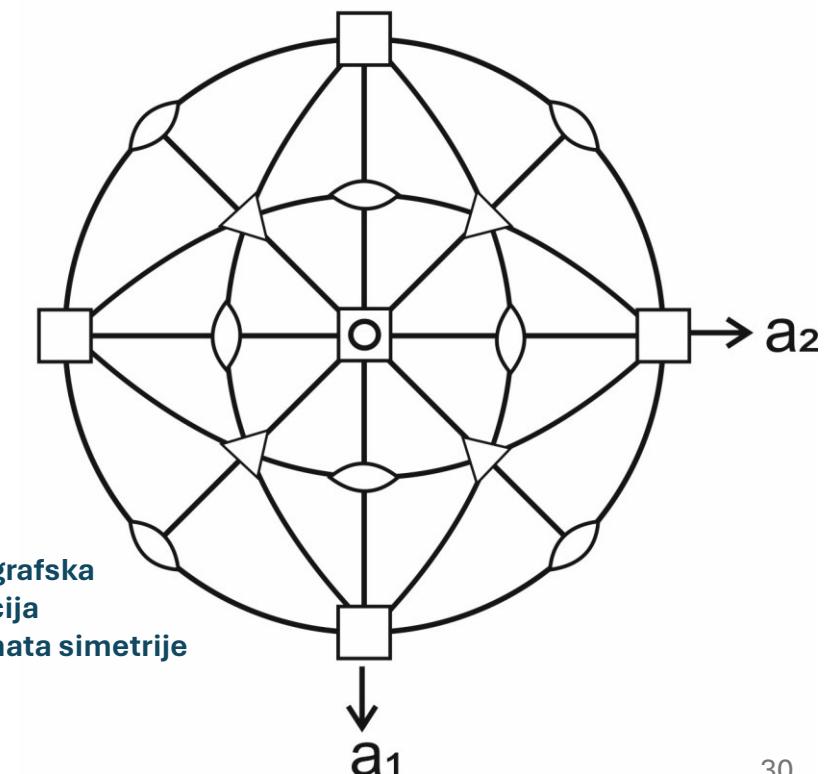
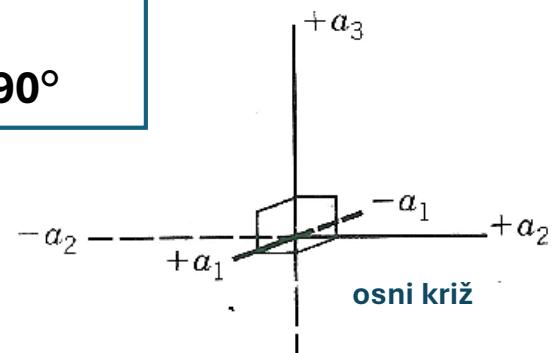
Kubični sustav

Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova	Schönlies-ov simbol
	1 2 3	
1) $\frac{4}{m} \bar{3} \frac{2}{m}$	$<100>$	O_h
2) $\bar{4}3m$	3	T_d
3) 432	$<111>$	O
4) $\frac{2}{m} \bar{3}$	smjerovi prostornih dijagonala	T_h
5) 23	kristalografske osi a_1, a_2, a_3	T

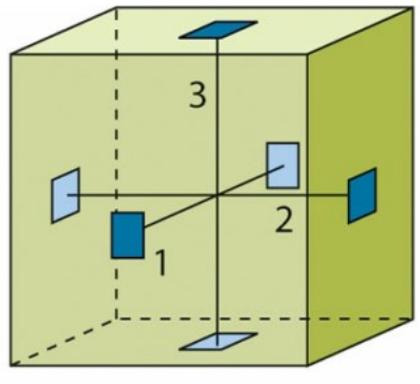
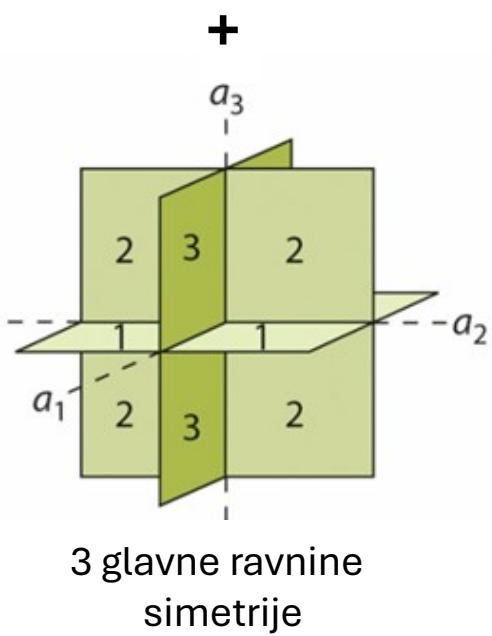
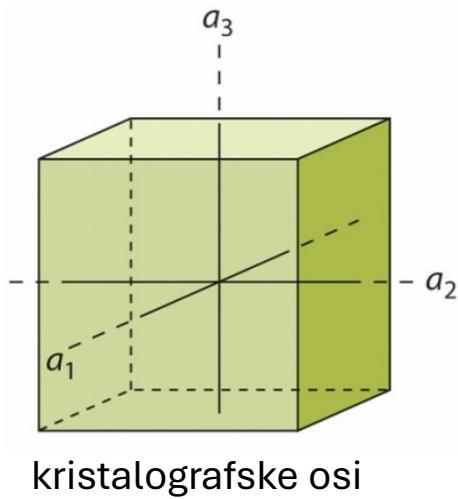
Karakteristike sustava:

$$a_1 = a_2 = a_3$$

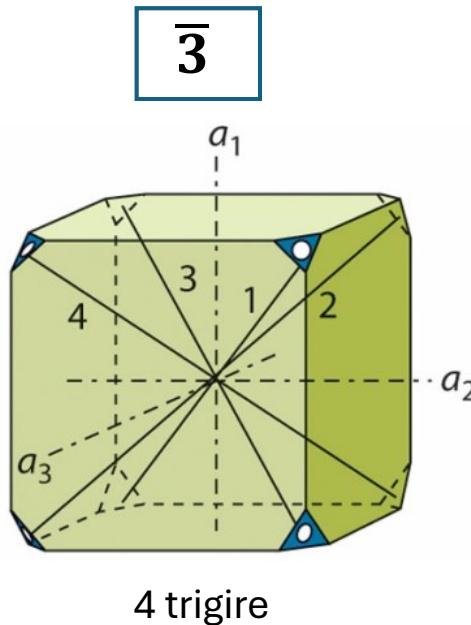
$$\alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$$



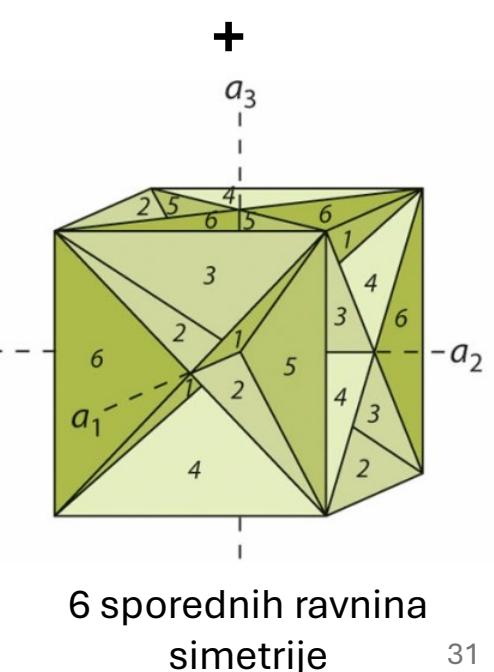
Holoedrija kubičnog sustava $\left(\frac{4}{m} \overline{3} \frac{2}{m}\right)$



+

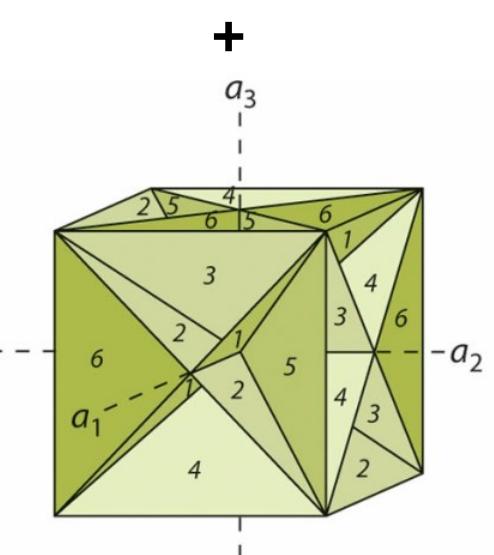


$\overline{3}$

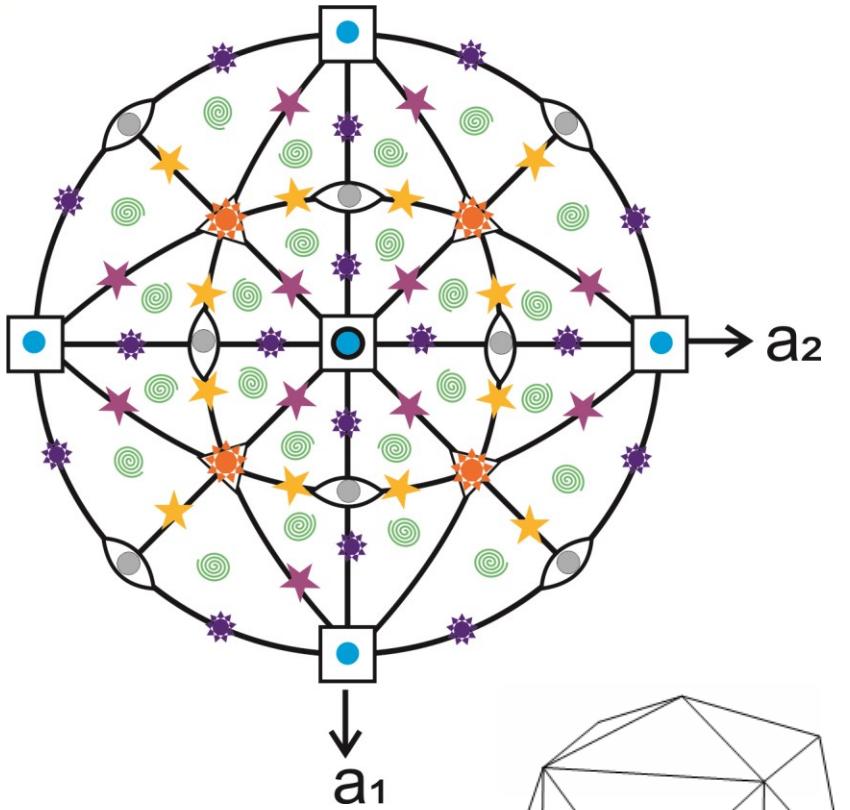


+

$4/m$

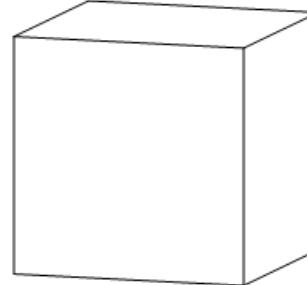


Holoedrija kubičnog sustava $\left(\frac{4}{m} \overline{3} \frac{2}{m}\right)$ - forme

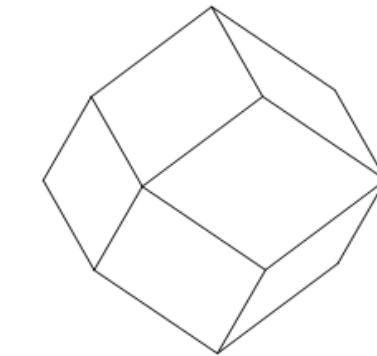
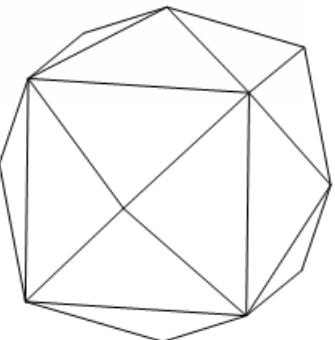


tetrakisheksaedar
 $\{hk0\}$ pr. $\{210\}$
 24 plohe

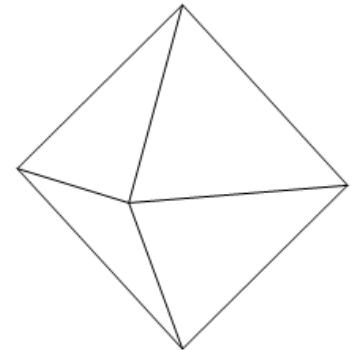
deltoidski ikozitetraedar
 $\{hll\}$, $h>l$ pr. $\{211\}$
 24 plohe



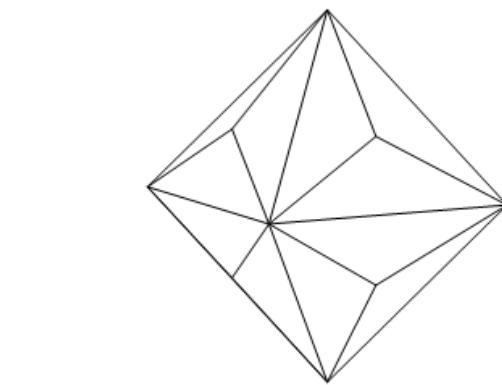
heksaedar
 $\{100\}$
 6 ploha



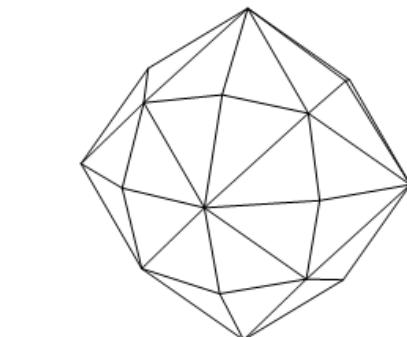
rompski dodekaedar
 $\{110\}$
 12 ploha



oktaedar
 $\{111\}$
 8 ploha



heksakisoktaedar
 $\{hkl\}$ pr. $\{321\}$
 48 plohe



trisoktaedar
 $\{hhl\}$, $h>l$ pr. $\{221\}$
 24 plohe

Kubični sustav

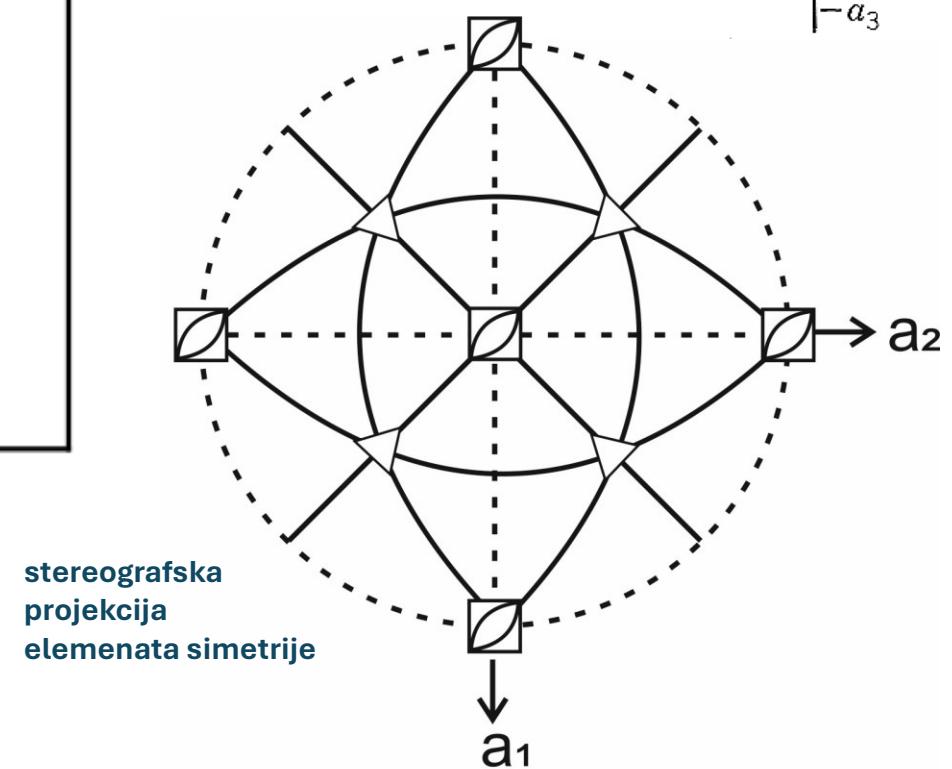
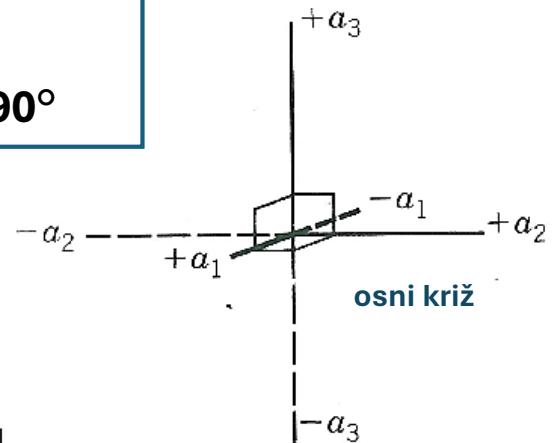
Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova	Schönlies-ov simbol
	1 2 3	
1) $\frac{4}{m} \bar{3} \frac{2}{m}$	$<100>$	O_h
2) $\boxed{\bar{4}3m}$	3	T_d
3) 432	$<111>$	O
4) $\frac{2}{m} \bar{3}$	$<110>$	T_h
5) 23	kristalografske osi a_1, a_2, a_3	T

smjerovi prostornih dijagonala
simetrale između kristalografskih osi a_1, a_2, a_3

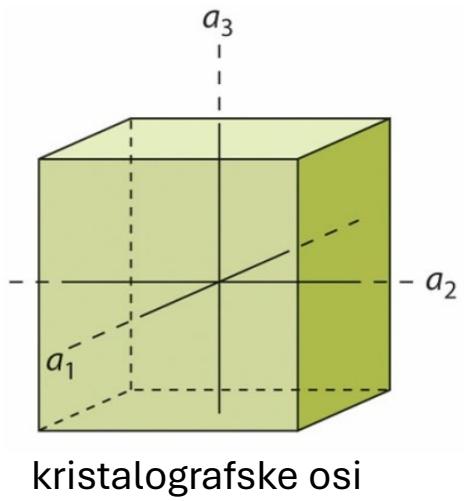
Karakteristike sustava:

$$a_1 = a_2 = a_3$$

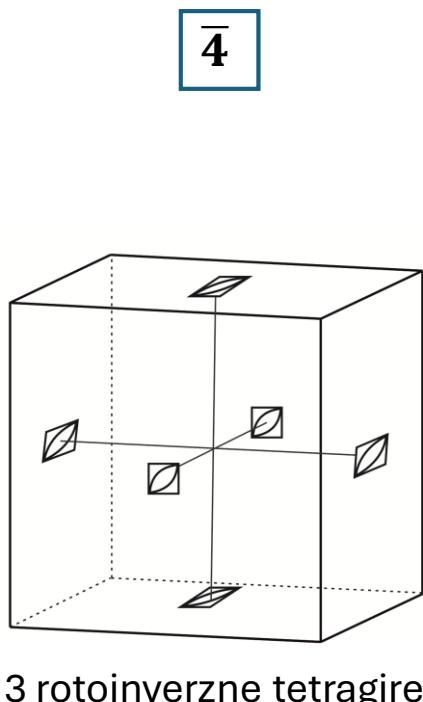
$$\alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$$



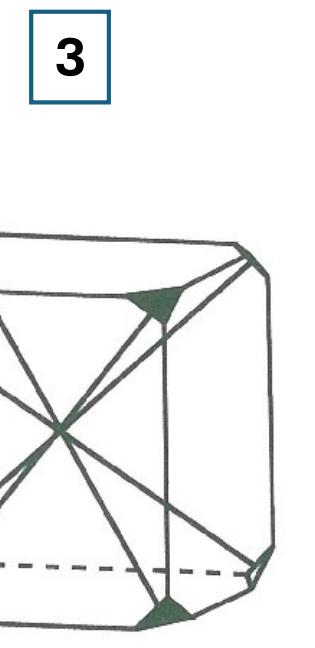
Tetraedarska hemiedrija kubičnog sustava ($\bar{4} \ 3 \ m$)



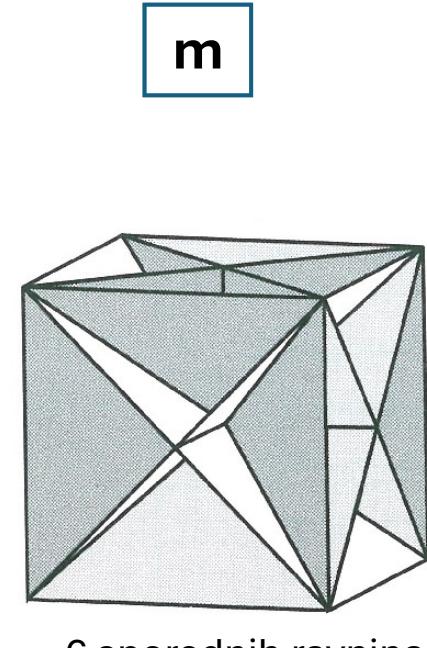
kristalografske osi



3 rotoinverzne tetragire

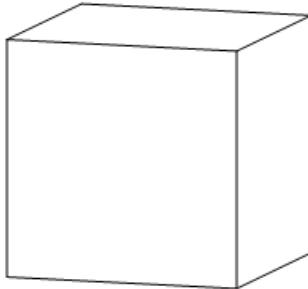
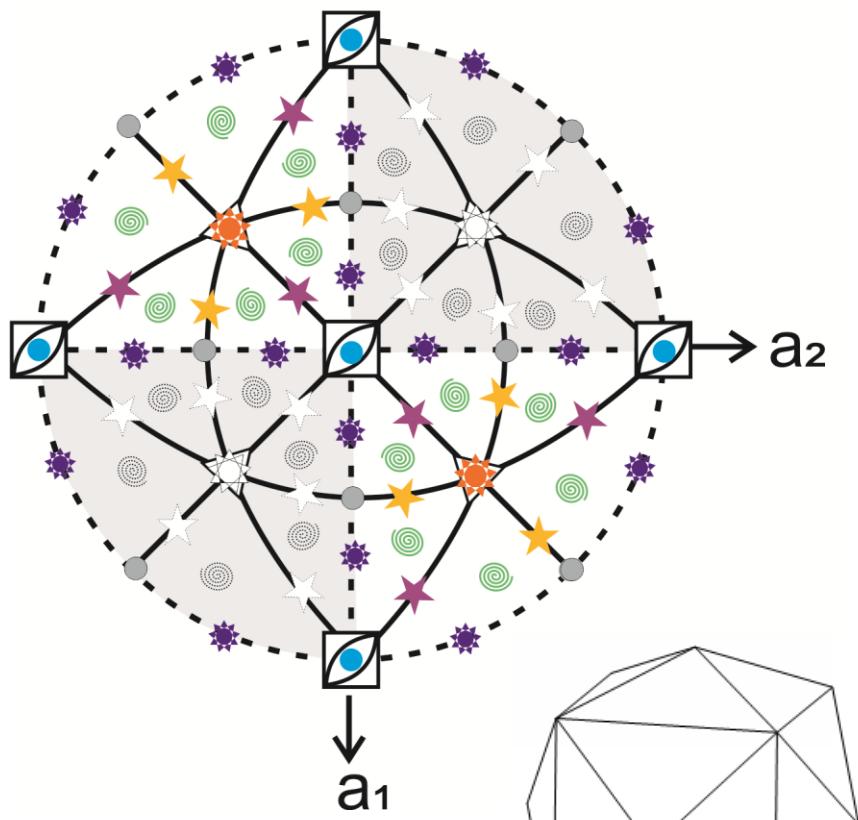


4 trigire

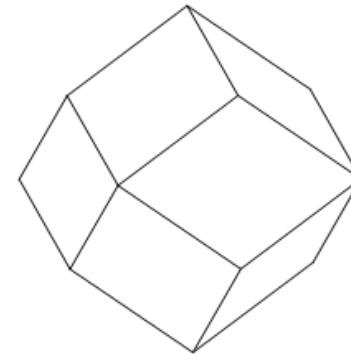


6 sporednih ravnina simetrije

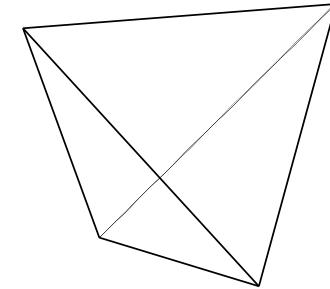
Tetraedarska hemiedrija kubičnog sustava ($\bar{4} 3 m$) - forme



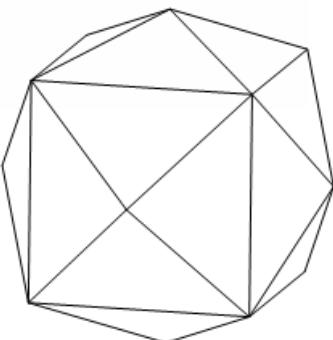
● heksaedar
{100}
6 ploha



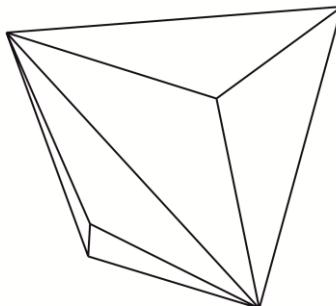
● rompski dodekaedar
{110}
12 ploha



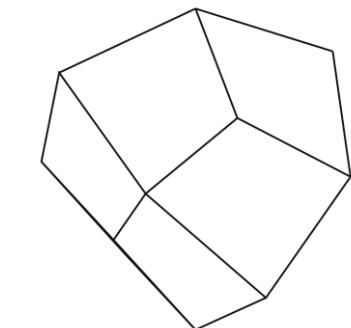
● tetrataedar (+)
{111}
4 plohe



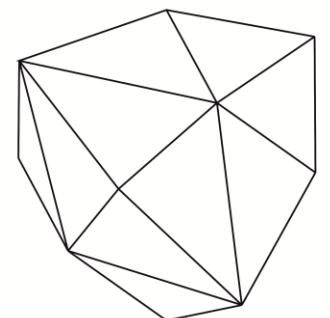
● tetrakisheksaedar
{hk0} pr. {210}
24 plohe



● tristetraedar (+)
{hll}, h>l pr. {211}
12 ploha



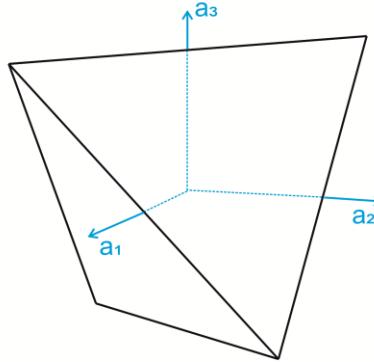
● deltoidski dodekaedar (+)
{hhl}, h>l pr. {221}
12 ploha



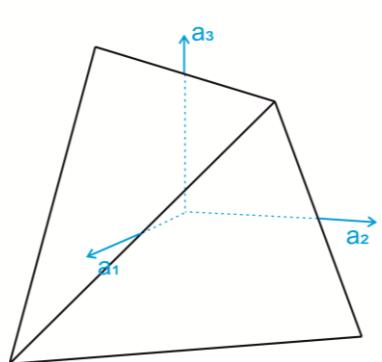
● heksakistetraedar (+)
{hkl} pr. {321}
24 plohe

Tetraedarska hemiedrija kubičnog sustava ($\bar{4} 3 m$) - forme

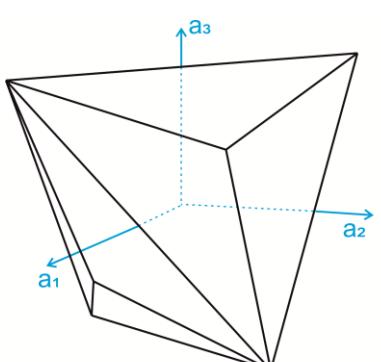
- pozitivne (+) i negativne (-) orijentacije formi



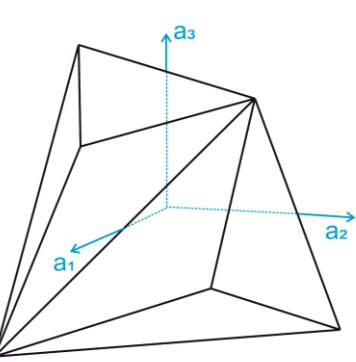
tetraedar (+)
 $\{111\}$
 4 plohe



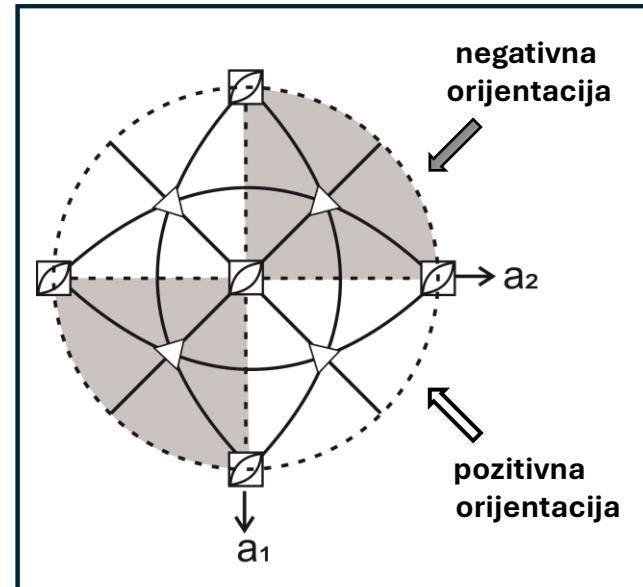
tetraedar (-)
 $\{1\bar{1}\bar{1}\}$
 4 plohe



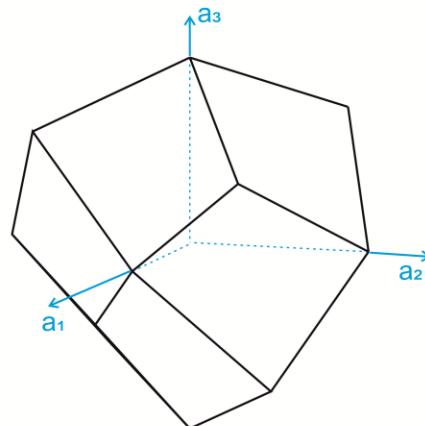
tristetraedar (+)
 $\{hll\}, h>l$, pr. $\{211\}$
 12 ploha



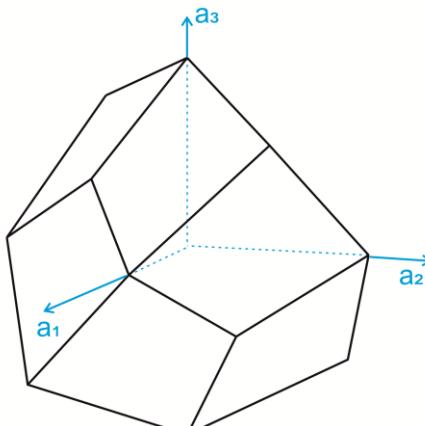
tristetraedar (-)
 $\{hll\}, h>l$, pr. $\{2\bar{1}1\}$
 12 ploha



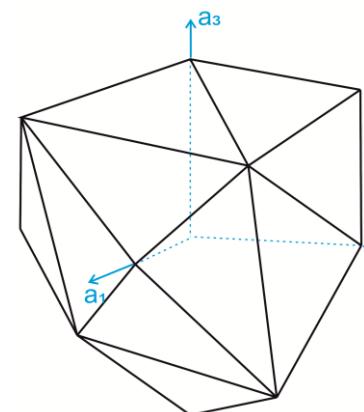
! polarna trigira



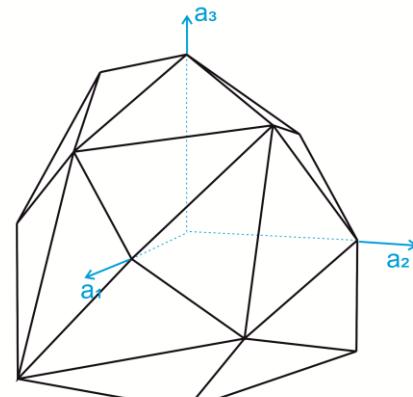
deltoidski dodekaedar (+)
 $\{hh\}, h>l$, pr. $\{221\}$
 12 ploha



deltoidski dodekaedar (-)
 $\{hh\}, h>l$, pr. $\{2\bar{2}1\}$
 12 ploha



heksakistetraedar (+)
 $\{hkl\}$ pr. $\{321\}$
 24 plohe



heksakistetraedar (-)
 $\{hkl\}$ pr. $\{3\bar{2}1\}$
 24 plohe

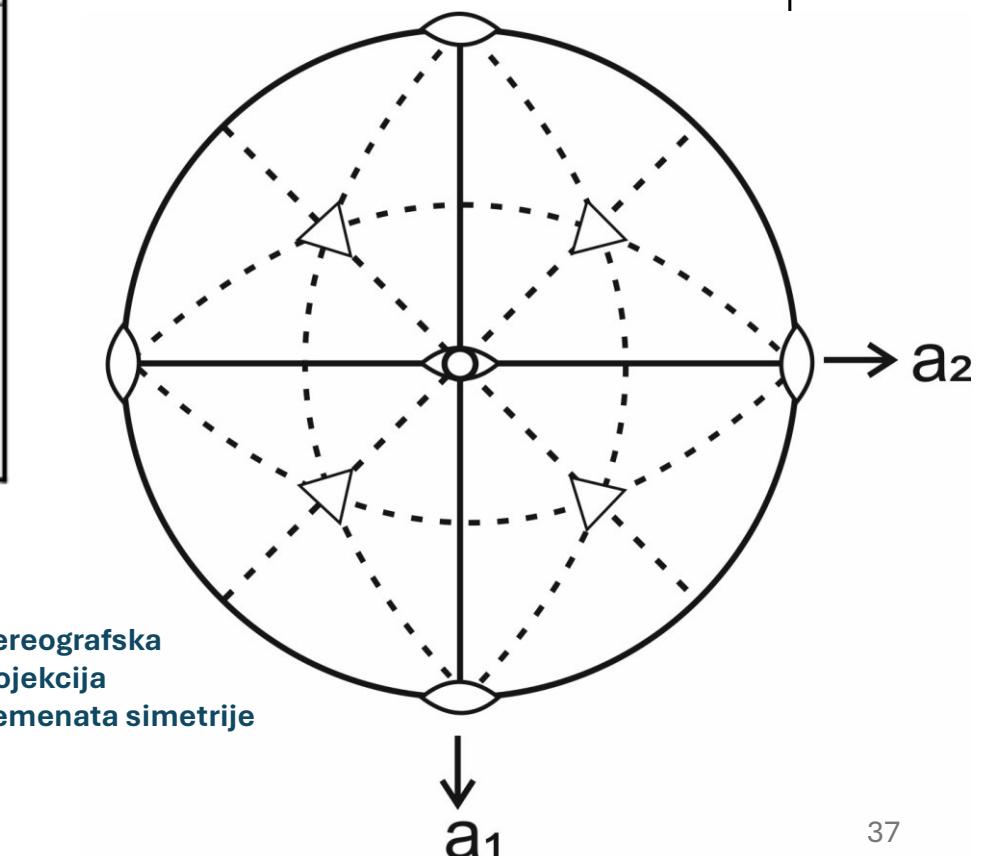
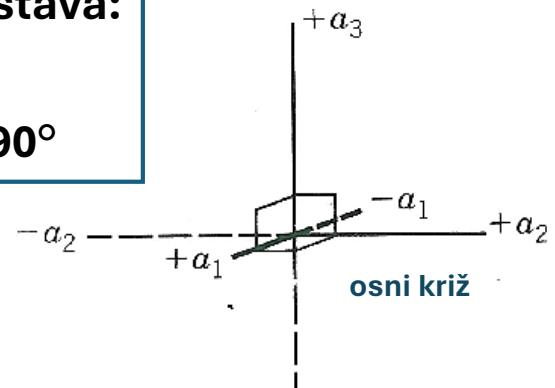
Kubični sustav

Hermann-Mauguin-ov simbol	smjer na koji se odnosi simbol na odgovarajućem mjestu u Hermann-Mauguin-ovom simbolu i broj ekvivalentnih smjerova	Schönlies-ov simbol
	1 2 3	
1) $\frac{4}{m} \bar{3} \frac{2}{m}$	$<100>$	O_h
2) $\bar{4}3m$	3	T_d
3) 432	4	O
4) $\frac{2}{m} \bar{3}$	smjerovi prostornih dijagonala	T_h
5) 23	kristalografske osi a_1, a_2, a_3	T

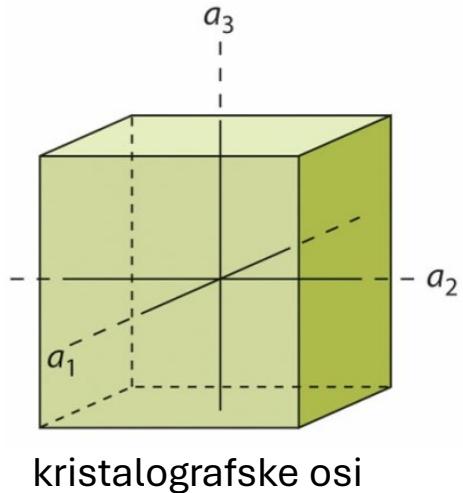
Karakteristike sustava:

$$a_1 = a_2 = a_3$$

$$\alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$$

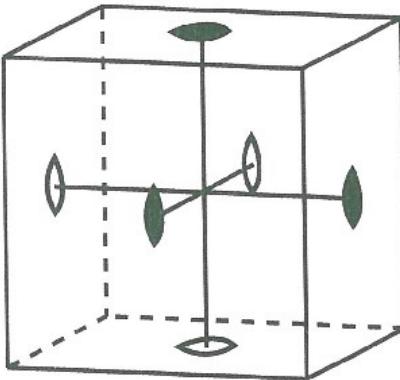


Pentagonska hemiedrija tetragonskog sustava ($\frac{2}{m} \bar{3}$)



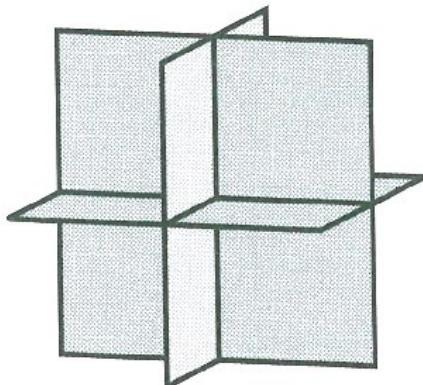
kristalografske osi

$$\frac{2}{m}$$



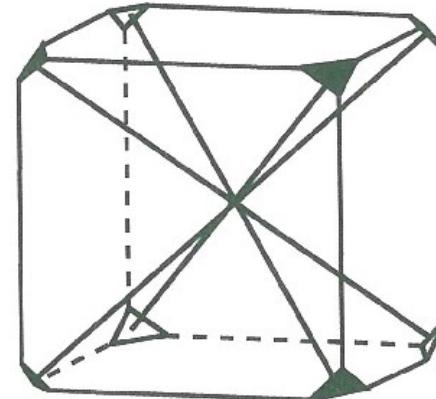
3 digire

+



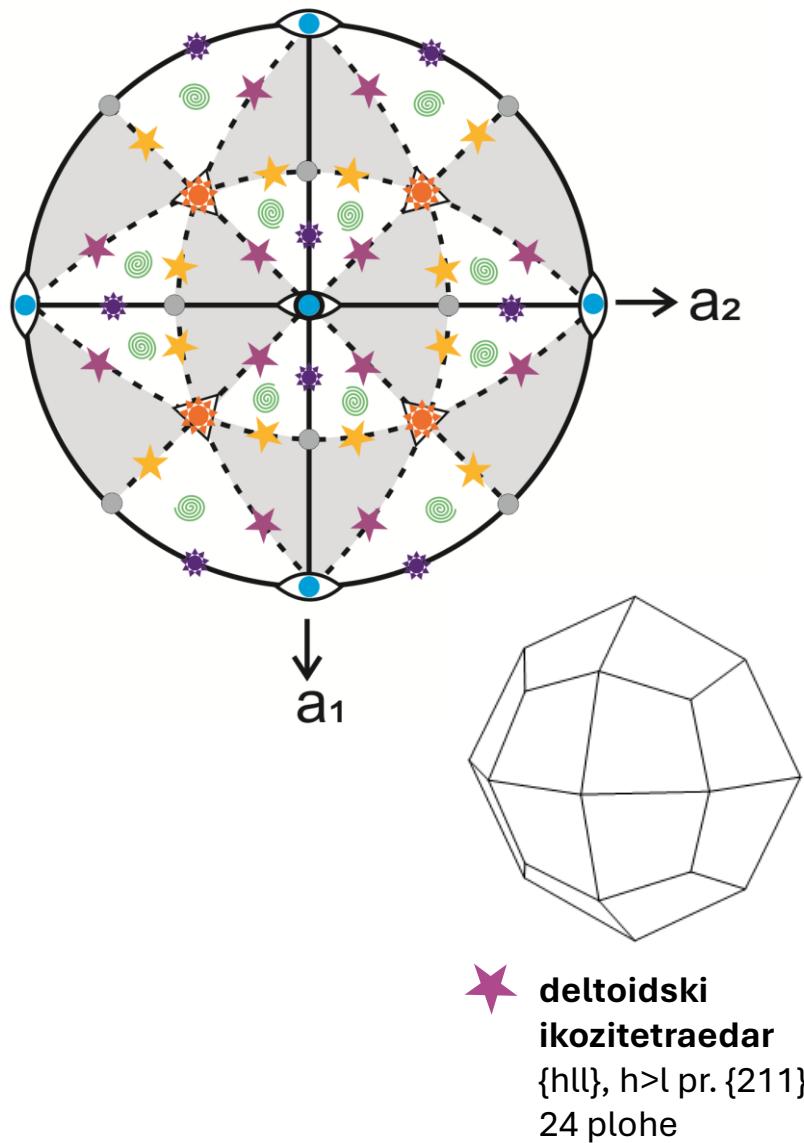
3 glavne ravnine simetrije

$$\bar{3}$$



4 trigire

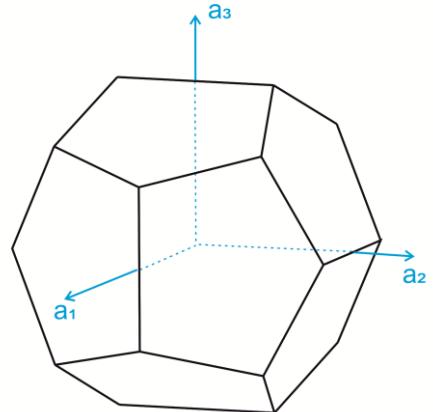
Pentagonska hemiedrija kubičnog sustava $\left(\frac{2}{m}\bar{3}\right)$ - forme



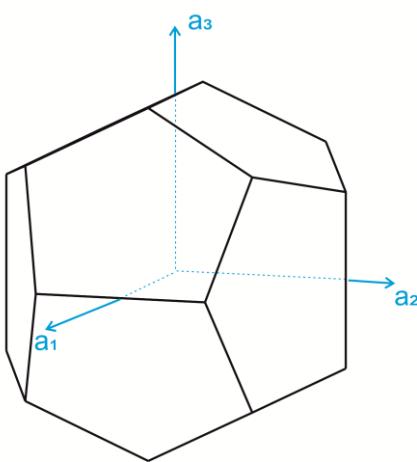
- heksaedar**
 $\{100\}$
6 ploha
- rompski dodekaedar**
 $\{110\}$
12 ploha
- oktaedar**
 $\{111\}$
8 ploha
- pentagonski dodekaedar (+)**
 $\{hl0\}$, pr. {210}
12 ploha
- trisoktaedar**
 $\{hhl\}$, $h>l$ pr. {221}
24 plohe
- disdodekaedar (-)**
 $\{hkl\}$, pr. {312}
24 plohe
- deltoidski ikozitetraedar**
 $\{hll\}$, $h>l$ pr. {211}
24 plohe

Pentagonska hemiedrija kubičnog sustava $(\frac{2}{m}\bar{3})$ - forme

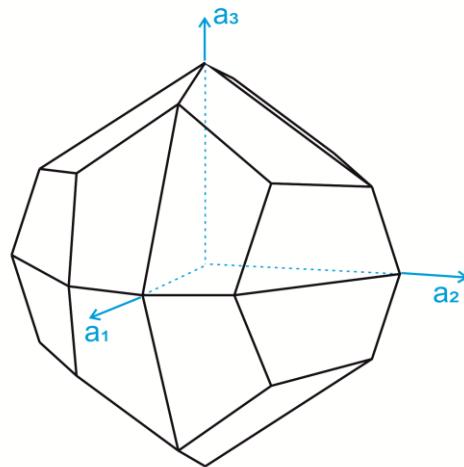
- pozitivne (+) i negativne (-) orijentacije formi



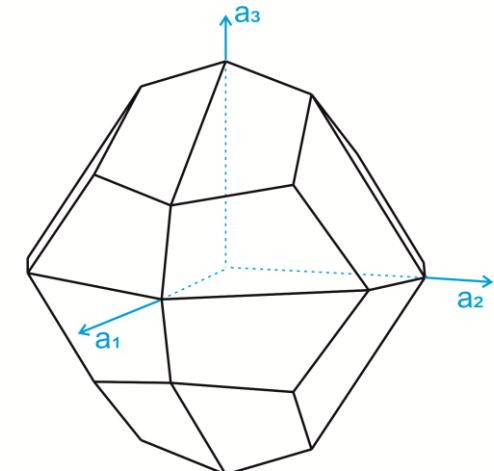
pentagonski dodekaedar (+)
 $\{hl0\}$ pr. $\{210\}$
12 ploha



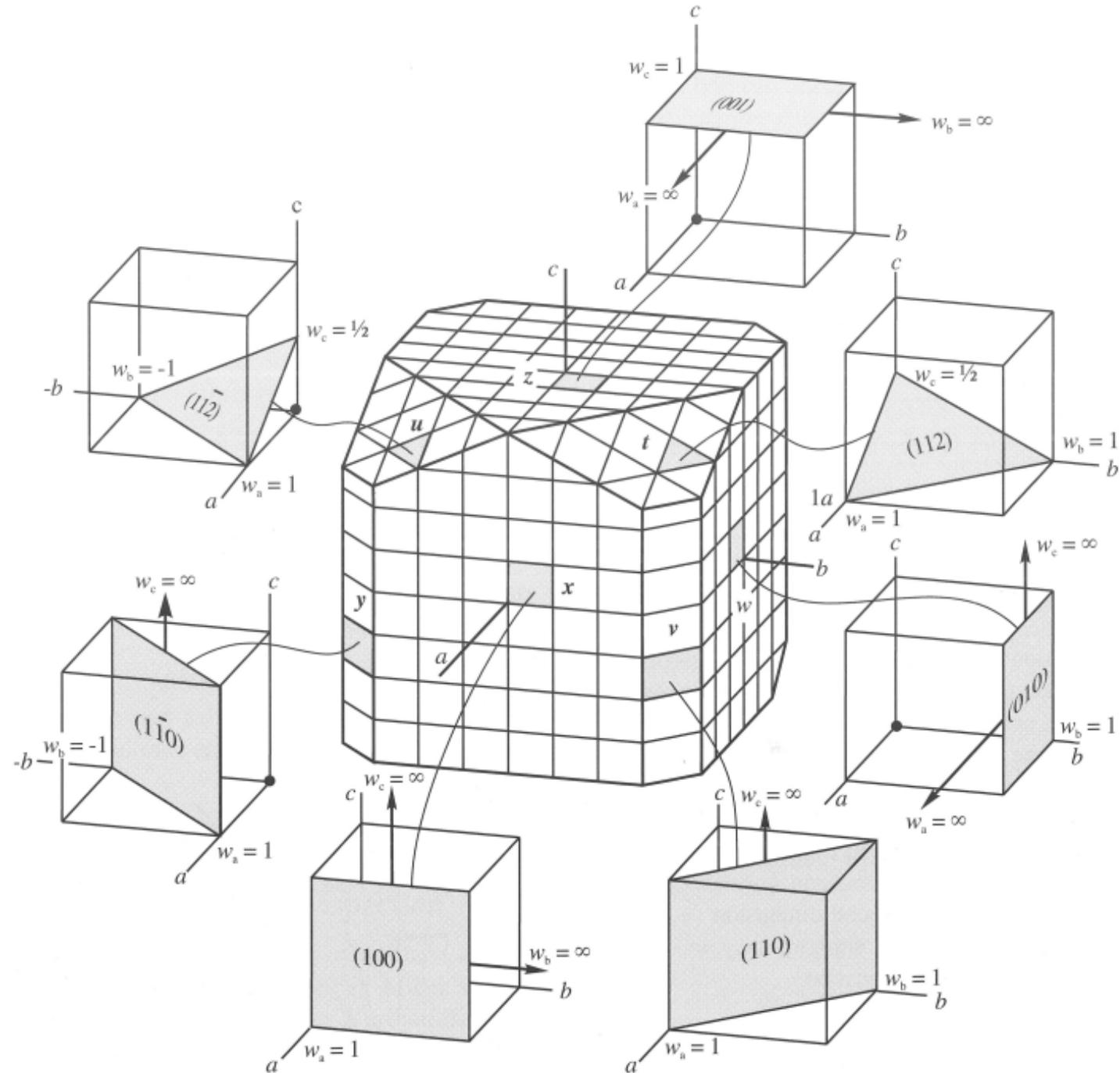
pentagonski dodekaedar (-)
 $\{hl0\}$ pr. $\{120\}$
12 ploha



disdodekaedar (+)
 $\{hkl\}$ pr. $\{312\}$
24 plohe



disdodekaedar (-)
 $\{hkl\}$ pr. $\{321\}$
24 plohe



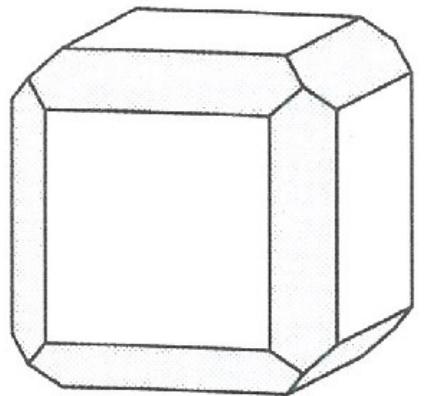
Kubični sustav - forme

	4/m $\bar{3}$ 2/m		$\bar{4}$ 3 m		2/m $\bar{3}$	
	Ime forme	Indeks forme	Ime forme	Indeks forme	Ime forme	Indeks forme
1	Heksaedar (6)	{100}	Heksaedar (6)	{100}	Heksaedar (6)	{100}
2	Rompski dodekaedar (12)	{110}	Rompski dodekaedar (12)	{110}	Rompski dodekaedar (12)	{110}
3	Oktaedar (8)	{111}	Tetraedar +/- (4)	{111} {1̄1̄1}	Oktaedar (8)	{111}
4	Tetrakisheksaedar (24)	{hk0}	Tetrakisheksaedar (24)	{hk0}	Pentagonski dodekaedar +/- (12)	{hk0} {kh0}
5	Deltoidski ikozitetraedar (24)	{hll}	Tristetraedar +/- (12)	{hll} {h̄l̄l}	Deltoidski ikozitetraedar (24)	{hll}
6	Trisoktaedar (24)	{hh̄l}	Deltoidski dodekaedar +/- (12)	{hh̄l} {h̄h̄l}	Trisoktaedar (24)	{hh̄l}
7	Heksakisoktaedar (48)	{hkl}	Heksakistetraedar (24)	{hkl} {h̄k̄l}	Disdodekaedar +/- (24)	{hkl} {kh̄l}

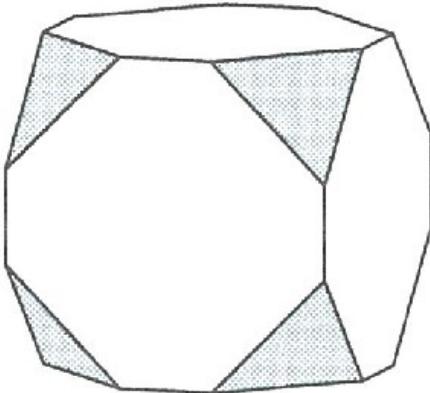
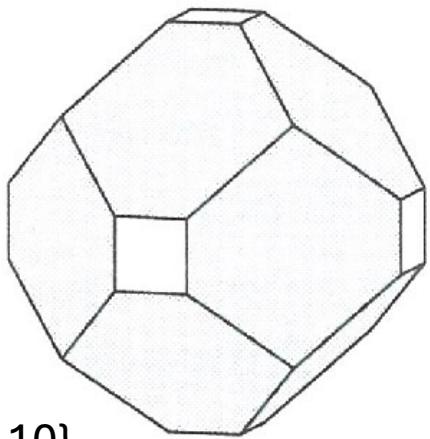
$h > k > l$

*u zagradi označen broj ploha svake forme

Razvoj formi



$\{100\} + \{110\}$

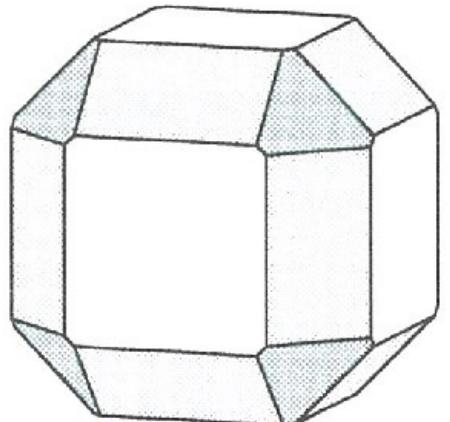


$\{100\} + \{111\}$

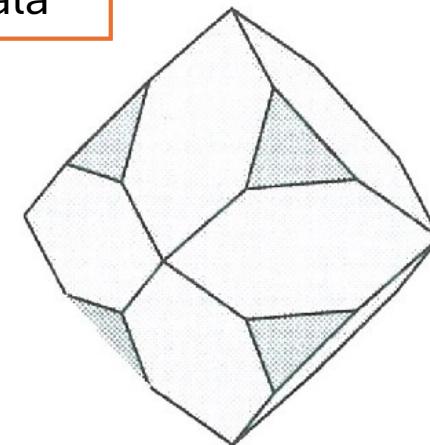
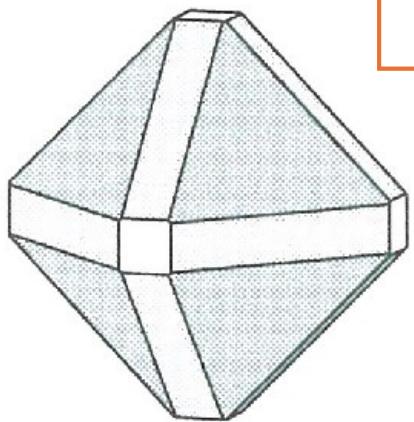
iste forme, ali različito razvijene



različita morfologija kristala



$\{100\} + \{110\} + \{111\}$

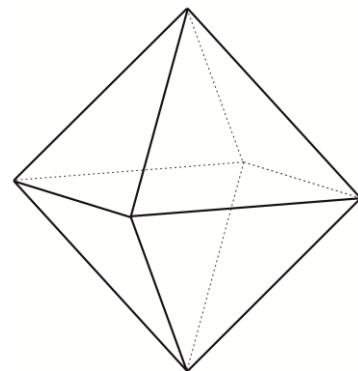


$\{110\} + \{111\}$

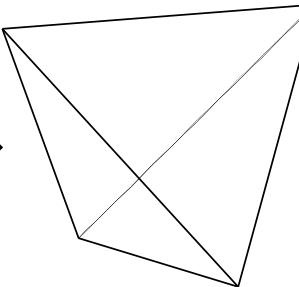
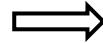
Komplementarne forme

! Gubitak elemenata simetrije → smanjivanje broja ekvivalentnih ploha.

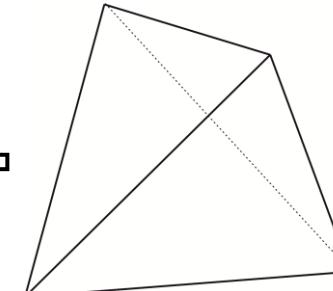
→ Umjesto forme iz klase s višim stupnjem simetrije moguće su dvije forme, koje se dopunjaju u formu iz koje su nastale = **komplementarne forme**



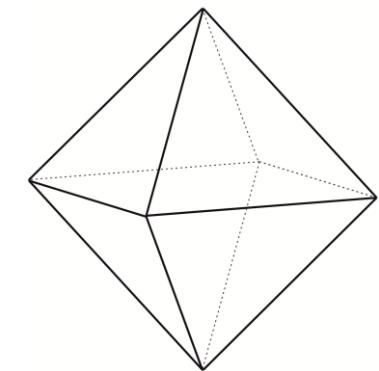
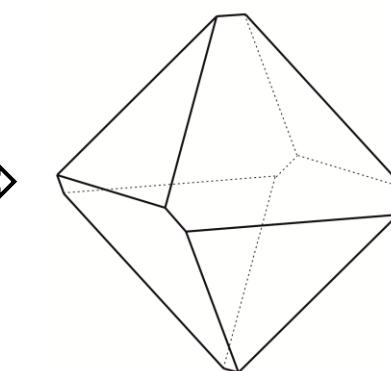
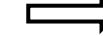
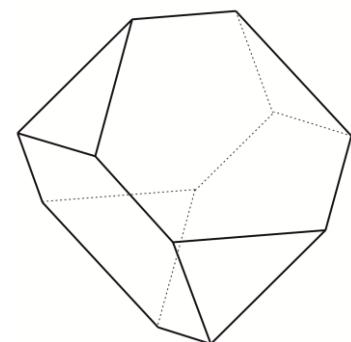
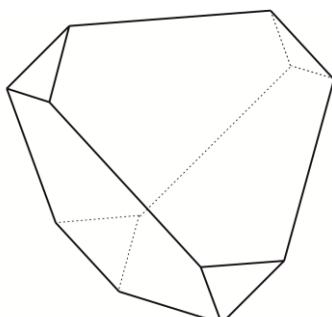
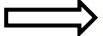
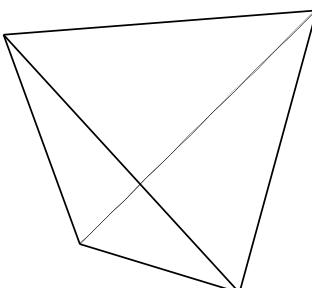
oktaedar
 $\{111\}$
8 ploha



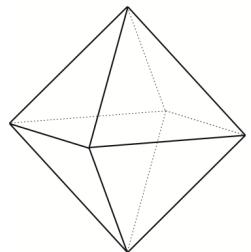
tetraedar (+)
 $\{111\}$
4 plohe



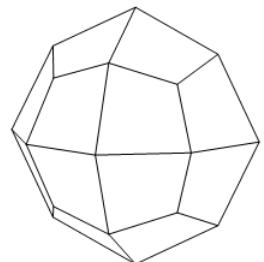
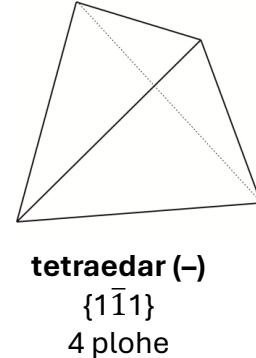
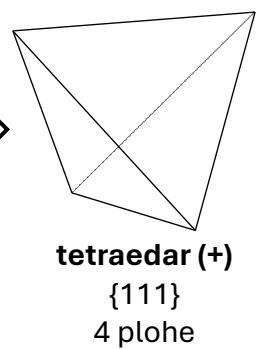
tetraedar (-)
 $\{1\bar{1}\bar{1}\}$
4 plohe



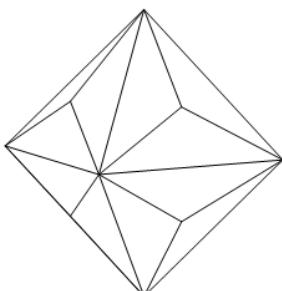
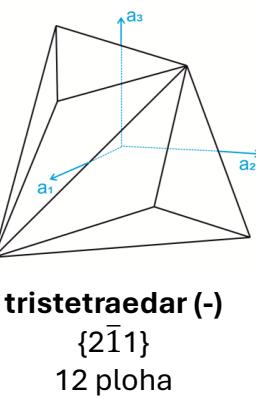
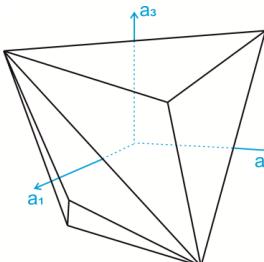
Komplementarne forme (kubični sustav)



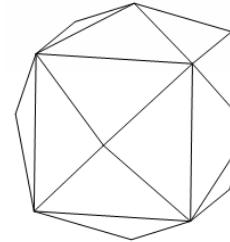
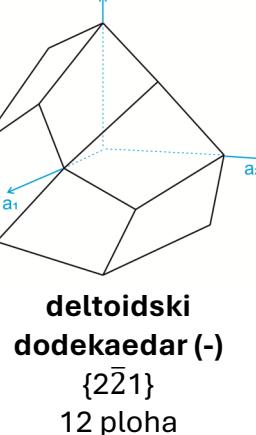
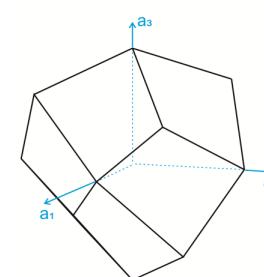
oktaedar
 $\{111\}$
 8 ploha



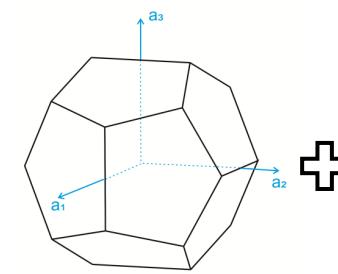
deltoidski ikozitetraedar
 $\{211\}$
 24 plohe



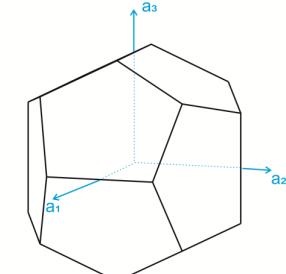
trisoktaedar
 $\{221\}$
 24 plohe



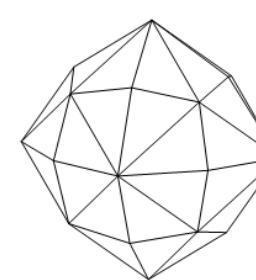
tetrakisheksaeder
 $\{210\}$
 24 plohe



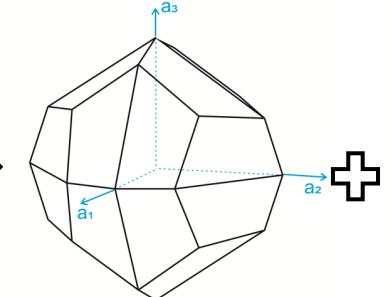
pentagonski
 dodekaedar (+)
 $\{210\}$
 12 ploha



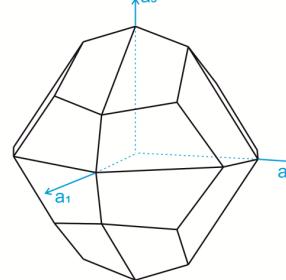
pentagonski
 dodekaedar (-)
 $\{120\}$
 12 ploha



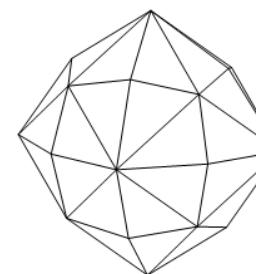
heksakisoktaedar
 $\{321\}$
 48 ploha



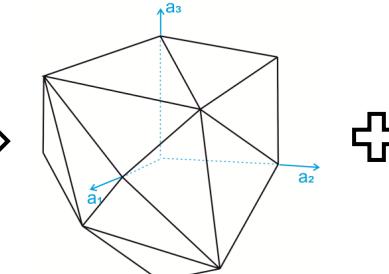
disdodekaedar (+)
 $\{321\}$
 24 plohe



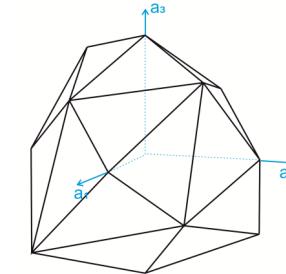
disdodekaedar (-)
 $\{231\}$
 24 ploha



heksakisoktaedar
 $\{321\}$
 48 ploha

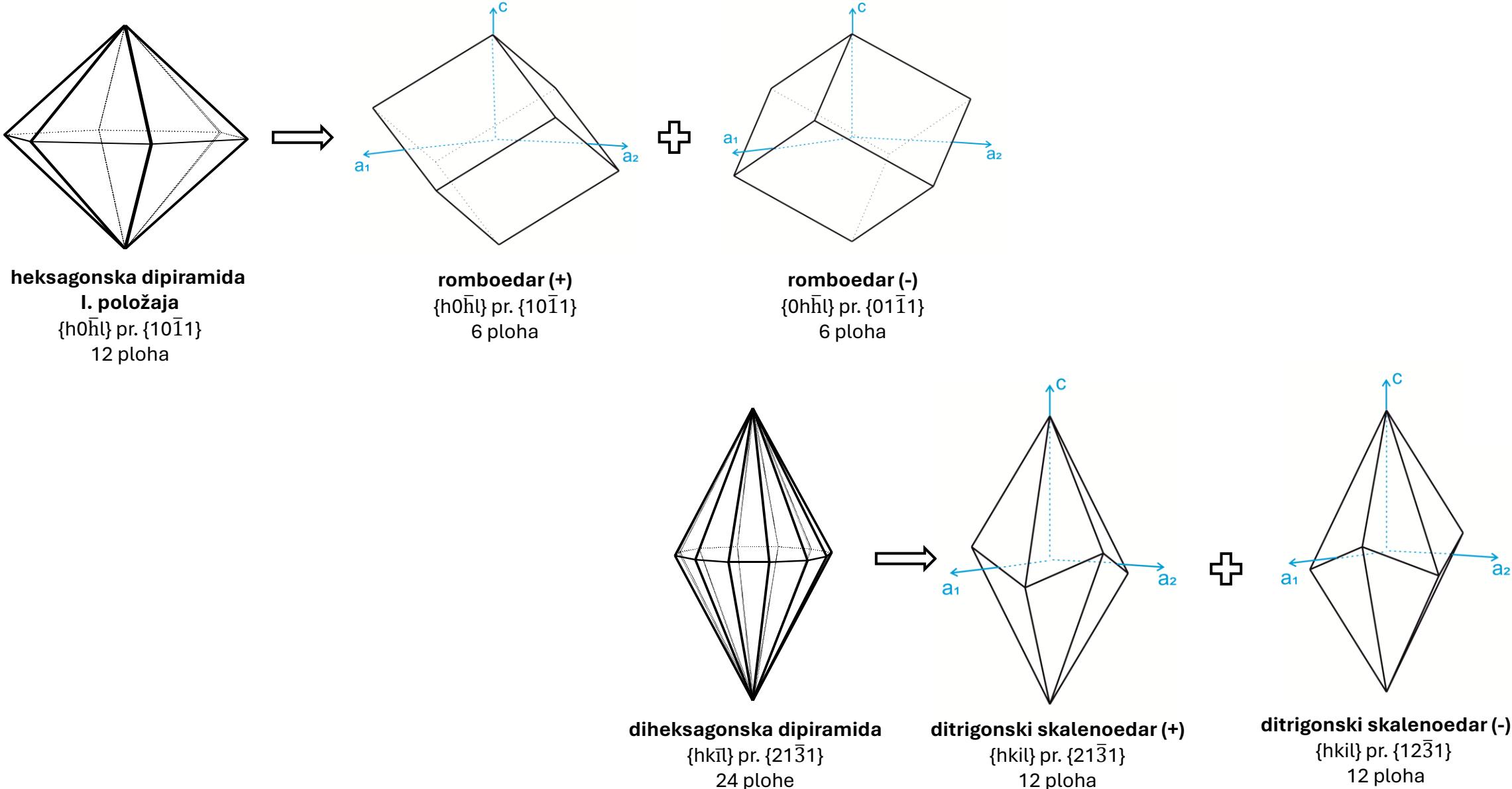


heksakistetraedar (+)
 $\{hkl\}$ pr. $\{3\bar{2}1\}$
 24 plohe



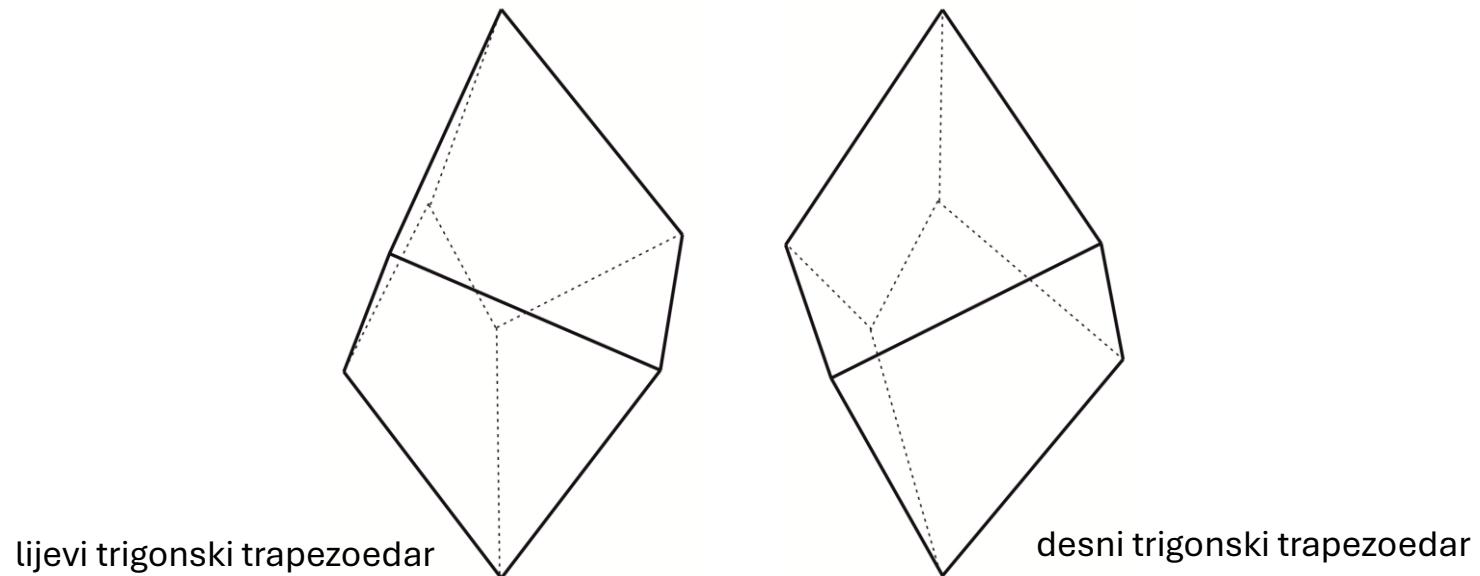
heksakistetraedar (-)
 $\{hkl\}$ pr. $\{3\bar{2}\bar{1}\}$
 24 plohe

Komplementarne forme (heksagonski sustav)



Komplementarne forme

- (1) **Pozitivne i negativne** - dvije forme se zakretanjem može dovesti do potpunog preklapanja, međusobno su **potpuno jednake**, samo se razlikuju po odnosu ploha prema kristalografskim osima
- (2) **Lijeve i desne** - dvije forme se više zakretanjem ne može dovesti do preklapanja = **enantiomorfne forme**



Pseudoforme

Forme koje su geometrijski iste onima iz holedrije, ali su kristalografski različite; **njihova prava simetrija je niža.**



problem određivanja simetrije

→ prepoznatljivo iz prutanja, izjedina, fizikalnih svojstava



primjer: pirit (FeS_2), klasa: $\frac{2}{m} \bar{3}$

Kubični sustav - pseudoforme

	4/m $\bar{3}$ 2/m		$\bar{4}$ 3 m		2/m $\bar{3}$	
	Ime forme	Indeks forme	Ime forme	Indeks forme	Ime forme	Indeks forme
1	Heksaedar (6)	{100}	Heksaedar (6)	{100}	Heksaedar (6)	{100}
2	Rompski dodekaedar (12)	{110}	Rompski dodekaedar (12)	{110}	Rompski dodekaedar (12)	{110}
3	Oktaedar (8)	{111}	Tetraedar +/- (4)	{111} {1̄1̄1}	Oktaedar (8)	{111}
4	Tetrakisheksaedar (24)	{hk0}	Tetrakisheksaedar (24)	{hk0}	Pentagonski dodekaedar +/- (12)	{hk0} {kh0}
5	Deltoidski ikozitetraedar (24)	{hll}	Tristetraedar +/- (12)	{hll} {h̄l̄l}	Deltoidski ikozitetraedar (24)	{hll}
6	Trisoktaedar (24)	{hh̄l}	Deltoidski dodekaedar +/- (12)	{hh̄l} {h̄h̄l}	Trisoktaedar (24)	{hh̄l}
7	Heksakisoktaedar (48)	{hkl}	Heksakistetraedar (24)	{hkl} {h̄k̄l}	Disdodekaedar +/- (24)	{hkl} {kh̄l}

$h > k > l$

* u zagradi označen broj ploha svake forme

** narančasto označene pseudoforme

Heksagonski sustav - pseudoforme

	6/m 2/m 2/m		$\bar{3}$ 2/m	
	Ime forme	Indeks forme	Ime forme	Indeks forme
1	Bazni pinakoid (2)	{0001}	Bazni pinakoid (2)	{0001}
2	Heksagonska prizma I. položaja (6)	{10 $\bar{1}$ 0}	Heksagonska prizma I. položaja (6)	{10 $\bar{1}$ 0}
3	Heksagonska prizma II. položaja (6)	{11 $\bar{2}$ 0}	Heksagonska prizma II. položaja (6)	{11 $\bar{2}$ 0}
4	Diheksagonska prizma (12)	{hki0}	Diheksagonska prizma (12)	{hki0}
5	Heksagonska dipiramida I. položaja (12)	{h0 $\bar{h}l$ }	Romboedar +/- (6)	{h0 $\bar{h}l$ } {0h $\bar{h}l$ }
6	Heksagonska dipiramida II. položaja (12)	{hh $\bar{2}h$ l}	Heksagonska dipiramida II. položaja (12)	{hh $\bar{2}h$ l}
7	Diheksagonska dipiramida (24)	{hkil}	Ditrigonski skalenoedar +/- (12)	{hkil}

$h > k > l$

* u zagradi označen broj ploha svake forme

** narančasto označene pseudoforme

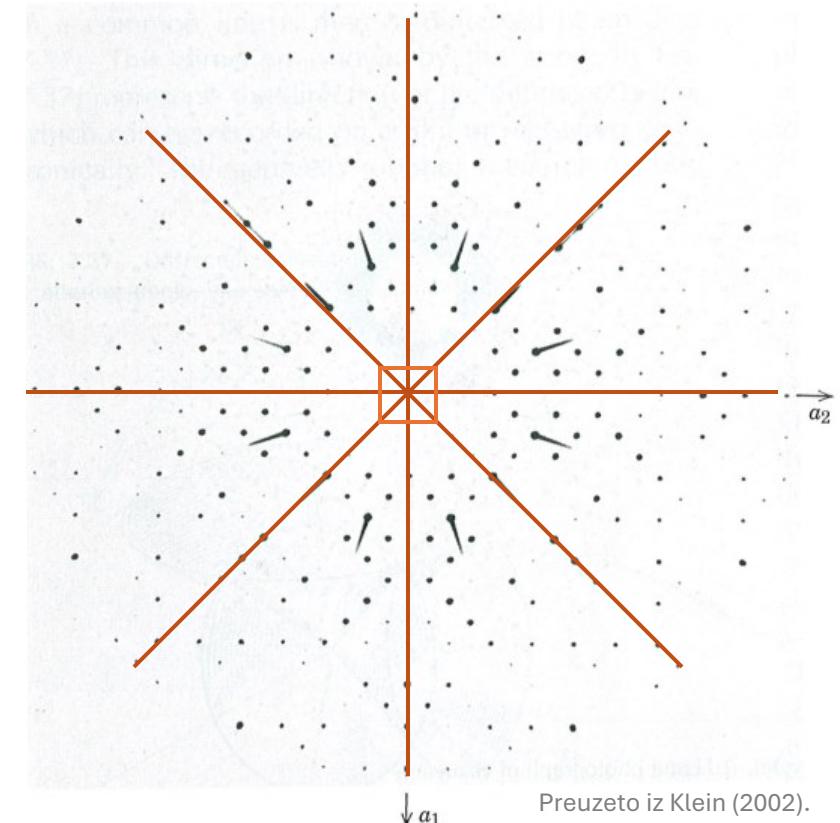
Određivanje simetrije

! Ukoliko je obasjana rendgenskim zrakama, kristalizirana materija daje pravilnu difrakcijsku sliku.

→ difrakcijska slika za određivanje kristalne strukture

!!! Fridelov zakon: na snimkama dobivenim primjenom difrakcije X-zraka u pravilu je prisutan centar simetrije.

→ moguće razlikovati samo 11 klasa = Laueove grupe



Preuzeto iz Klein (2002).

Primjer:

$$\bar{4} \ 3 \ m + \bar{1} = \frac{4}{m} \ \bar{3} \ \frac{2}{m}$$

$$4 \ 3 \ 2 + \bar{1} = \frac{4}{m} \ \bar{3} \ \frac{2}{m}$$

} 3 različite klase → ista difrakcijska slika

Određivanje simetrije

Laueova grupa	Kristalne klase
$\bar{1}$	$\bar{1}, 1$
$\frac{2}{m}$	$\frac{2}{m}, m, 2$
$\frac{2}{m} \frac{2}{m} \frac{2}{m}$	$\frac{2}{m} \frac{2}{m} \frac{2}{m}, mm2, 222$
$\bar{3}$	$\bar{3}, 3$
$\bar{3} 2m$	$\bar{3} \frac{2}{m}, 3m, 32$
$\frac{6}{m}$	$\frac{6}{m}, \bar{6}, 6$
$\frac{6}{m} \frac{2}{m} \frac{2}{m}$	$\frac{6}{m} \frac{2}{m} \frac{2}{m}, \bar{6}m2, 6mm, 622$
$\frac{4}{m}$	$\frac{4}{m}, \bar{4}, 4$
$\frac{4}{m} \frac{2}{m} \frac{2}{m}$	$\frac{4}{m} \frac{2}{m} \frac{2}{m}, \bar{4}2m, 4mm, 422$
$\frac{2}{m} 3$	$\frac{2}{m} 3, 23$
$\frac{4}{m} \bar{3} \frac{2}{m}$	$\frac{4}{m} \bar{3} \frac{2}{m}, \bar{4}3m, 432$

Određivanje simetrije

Svojstva uvjetovana simetrijom kristala koja mogu pomoći pri određivanju prave simetrije kristala:

- **Prutanja** = rebraste tvorbe na plohamu nekih kristala



Određivanje simetrije

Svojstva uvjetovana simetrijom kristala koja mogu pomoći pri određivanju prave simetrije kristala:

- **Prutanja** = rebraste tvorbe na ploham nekih kristala
- **Izjedine** (figure jetkanja) = nastaju tretiranjem ploha odgovarajućim otapalom



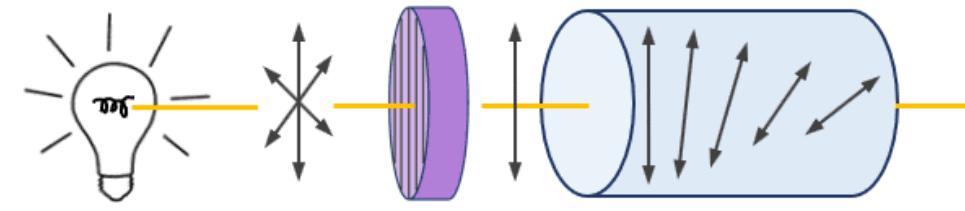
Izvor fotografije: <https://www.mindat.org>

Primjer: Widmanstättenove figure na Fe-Ni-meteoritu

Određivanje simetrije

Svojstva uvjetovana simetrijom kristala koja mogu pomoći pri određivanju prave simetrije kristala:

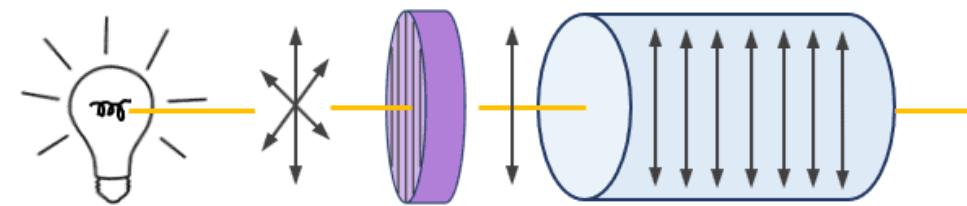
- **Prutnja** = rebraste tvorbe na ploham nekih kristala
- **Izjedine** (figure jetkanja) = nastaju tretiranjem ploha odgovarajućim otapalom
- **Optička aktivnost** = svojstvo kristala da zakreće ravni polarizacije udesno ili uljevo (samo kod klase bez centra simetrije)
→ ovisi o valnoj duljini svjetlosti i debljini minerala (deblji mineral → veći zaokret)



Light source

Polarizer

An **optically active** compound



An **optically inactive** compound

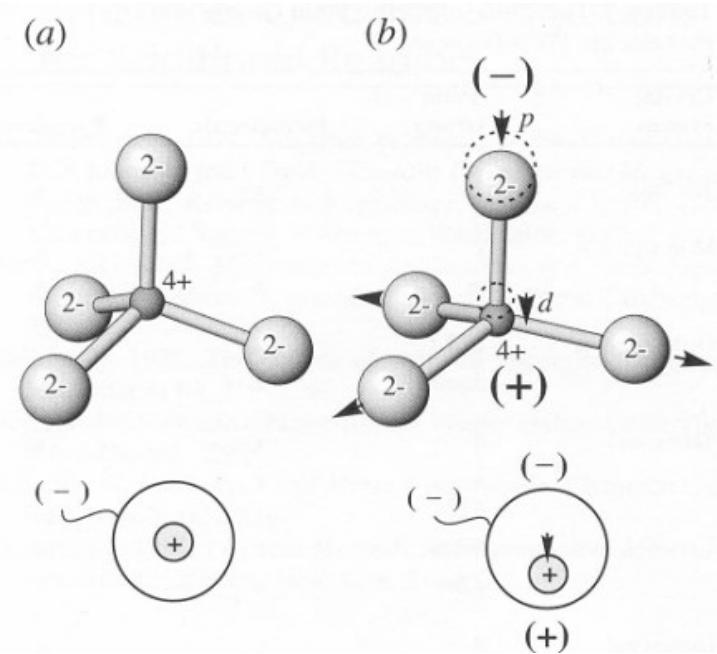
Izvor fotografije: <https://www.chemistrysteps.com>

Određivanje simetrije

Svojstva uvjetovana simetrijom kristala koja mogu pomoći pri određivanju prave simetrije kristala:

- **Prutanja** = rebraste tvorbe na ploham nekih kristala
- **Izjedine** (figure jetkanja) = nastaju tretiranjem ploha odgovarajućim otapalom
- **Optička aktivnost** = svojstvo kristala da zakreće ravni polarizacije udesno ili uljevo (samo kod klase bez centra simetrije)
→ ovisi o valnoj duljini svjetlosti i debljini minerala (deblji mineral → veći zaokret)
- **Piezoelektricitet** = svojstvo kristala da se uslijed usmjerenih pritisaka **duž polarnih smjerova** polarno nabija (samo kod klase bez centra simetrije, osim 432)
→ Takvi kristali se u električnom polju rastežu ili stežu (obratni piezoelektrični efekt)

Primjer: kvarcni satovi



Preuzeto iz Nesse (2000)



<https://www.youtube.com/watch?app=desktop&v=3jfgQF3jX7A>

Određivanje simetrije

Svojstva uvjetovana simetrijom kristala koja mogu pomoći pri određivanju prave simetrije kristala:

- **Prutanja** = rebraste tvorbe na plohami nekih kristala
- **Izjedine** (figure jetkanja) = nastaju tretiranjem ploha odgovarajućim otapalom
- **Optička aktivnost** = svojstvo kristala da zakreće ravninu polarizacije udesno ili uljevo (samo kod klasa bez centra simetrije)
 - ovisi o valnoj duljini svjetlosti i debljini minerala (deblji mineral → veći zaokret)
- **Piezoelektricitet** = svojstvo kristala da se uslijed usmjerenih pritisaka **duž polarnih smjerova** polarno nabija (samo kod klasa bez centra simetrije, osim 432)
 - Takvi kristali se u električnom polju rastežu ili stežu (obratni piezoelektrični efekt)

Primjer: kvarcni satovi

- **Piroelektricitet** = svojstvo kristala da se električki nabije duž polarnih smjerova uslijed promjene temperature

Određivanje simetrije

Kristalna klasa	Optička aktivnost	Piroelektricitet	Piezoelektricitet
1	+	+	+
2	+	+	+
m	+	+	+
222	+		+
mm2	+	+	+
4	+	+	+
$\bar{4}$	+		+
422	+		+
4mm		+	+
$\bar{4}2m$	+		+
3	+	+	+
32	+		+
3m		+	+
6	+	+	+
$\bar{6}$			+
622	+		+
6mm		+	+
$\bar{6}2m$			+
23	+		+
432	+		
$\bar{4}3m$			+
broj klasa	15	10	20

PUNI SIMBOL	SKRAĆENI SIMBOL
$\frac{2}{m} \frac{2}{m} \frac{2}{m}$	m m m
$\frac{4}{m} \frac{2}{m} \frac{2}{m}$	$\frac{4}{m}$ m m
$\overline{3} \frac{2}{m}$	$\overline{3}$ m
$\frac{6}{m} \frac{2}{m} \frac{2}{m}$	$\frac{6}{m}$ m m
$\frac{4}{m} \overline{3} \frac{2}{m}$	m 3 m
$\frac{2}{m} \overline{3}$	m 3



! Prisutnost nekih elemenata simetrije nužno zahtijeva prisutnost dodatnih elemenata simetrije:

- presjecištem dviju međusobno okomitih ravnina simetrije ide digira ($m \perp m \rightarrow 2$)

Sažetak

Kristalni sustav	Osni sustav
Kubični	$a_1 = a_2 = a_3$ $a_1 = a_2 = a_3 = 90^\circ$
Tetragonski	$a_1 = a_2 \neq c$ $a_1 = a_2 = \gamma = 90^\circ$
Heksagonski	$a_1 = a_2 = a_3 \neq c$
Trigonski	$a_1 = a_2 = a_3 = 120^\circ$ $\delta = 90^\circ$
Rompski	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
Monoklinski	$a \neq b \neq c$ $\alpha = \gamma = 90^\circ; \beta > 90^\circ$
Triklinski	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma (\neq 90^\circ)$

Kristalni sustav	Broj razreda	Karakteristični elem. simetrije
Triklinski	2	c ili 1
Monoklinski	3	L^2 i/ili P duž 1 smjera
Rompski	3	$3L^2$ i/ili 3P duž tri okomita smjera
Tetragonski	7	L^4 ili $L_{(4)}^2$
Heksagonski	12	L^6 ili L^3
Kubični	5	$4L^3$

Sustav	Položaj u Hermann-Mauguinovom simbolu		
	1	2	3
Triklinski		-	
Monoklinski	-	b	-
Rompski	a	b	c
Tetragonski	c	a_1, a_2	$\langle 110 \rangle$
Heksagonski	c	a_1, a_2, a_3	$\langle 10\bar{1}0 \rangle$
Kubični	a_1, a_2, a_3	$\langle 111 \rangle$	$\langle 110 \rangle$