

Jednadžbe i nejednadžbe

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U određivanju slika i preslika te domena funkcija pojavljuju se razne jednadžbe i nejednadžbe. Uvjetovano je uobičajeno njihovo rješavanje.

- uz rješavanje jednadžbi i nejednadžbi ponekad komentirati je li sljedeći korak u rješavanju ekvivalentan prethodnom ili samo slijedi iz njega, što to znači za rješavanje, zašto se ponekad ne smije podijeliti s nečim...

Zad. Riješite jednadžbu: $\frac{7}{x^2-1} + \frac{8}{x^2-2x+1} = \frac{25-9x}{x^3-x^2-x+1}$

Rj. $\frac{7}{(x-1)(x+1)} + \frac{8}{(x-1)^2} = \frac{25-9x}{(x-1)^2(x+1)}$ (nakon faktoriacije nazivnika)

uvjeti koji moraju biti zadovoljeni: $x \neq \pm 1$

U tom slučaju možemo pomnožiti s $(x-1)^2(x+1)$.

$$7(x-1) + 8(x+1) = 25-9x \Rightarrow 15x+1 = 25-9x$$

$$\Rightarrow \boxed{x=1}, \text{ što ne može biti rješenje zbog } x \neq \pm 1$$

$\Rightarrow$ jednadžba nema rješenja

Zad. Riješite jednadžbu u ovisnosti o parametrima a i b:

✓ $x(a-x) + (b-x)^2 = a+b^2-4$.

Rj. $ax - x^2 + b^2 - 2bx + x^2 = a + b^2 - 4$

$$(a-2b)x = a-4$$

1) ako je $a-2b \neq 0$, tj. $a \neq 2b$, rješenje je $x = \frac{a-4}{a-2b}$

2) ako je $a-2b=0$ i $a-4 \neq 0$, tj. $a=2b$ i $a \neq 4$, onda jednadžba nema rješenja ($0 \cdot x =$ nešto različito od 0)

3) ako je $a=2b$ i $a=4$, tj. $a=4$ i $b=2$, onda jednadžba glasi:
 $0 \cdot x = 0 \Rightarrow \forall x \in \mathbb{R}$ je rješenje

Zad. Riješite jednadžbu: $|x + |\frac{1}{2}x + |\frac{1}{3}x - 1||| = 1$.

Rj. Rješavanje podijelimo na 2 slučaja: 1° $\frac{1}{3}x - 1 \geq 0$

2° $\frac{1}{3}x - 1 < 0$

$$1^o \Rightarrow |x + |\frac{1}{2}x + \frac{1}{3}x - 1|| = 1 \quad (\text{u } 1^o \text{ imamo } \frac{1}{3}x + 2 \cdot 0 \Leftrightarrow x \geq 3) \quad \text{X}$$

$$|x + |\frac{5}{6}x - 1|| = 1$$

$$\frac{5}{6}x - 1 \geq \frac{5}{6} \cdot 3 - 1 = \frac{3}{2} > 0 \Rightarrow |x + \frac{5}{6}x - 1| = 1$$

$$|\frac{11x}{6} - 1| = 1$$

$$\frac{11x}{6} - 1 = \pm 1 \rightarrow \begin{cases} x_1 = 0 < 3 \\ x_2 = \frac{12}{11} < 3 \end{cases}$$

$$2^o \Rightarrow |x + |\frac{1}{2}x - \frac{1}{3}x + 1|| = 1$$

$$|x + |\frac{1}{6}x + 1|| = 1$$

$$\text{ako } \frac{1}{6}x + 1 < 0 \Rightarrow |x - \frac{1}{6}x + 1| = 1$$

$$\Leftrightarrow x < -6 < 3$$

$$\frac{5}{6}x - 1 = \pm 1 \rightarrow \begin{cases} x_1 = 0 < -6 \\ x_2 = \frac{12}{5} > -6 \end{cases}$$

$$\text{ako } \frac{1}{6}x + 1 \geq 0, \text{ tj.}$$

$$x \geq -6 \text{ i } x < 3$$

$$-6 \leq x < 3$$

$$|x + \frac{1}{6}x + 1| = 1$$

$$\frac{7}{6}x + 1 = \pm 1$$

$$\boxed{x_1 = 0}$$

$$\boxed{x_2 = -\frac{12}{7}}$$

57 Zad. Riješite jednačinu: $\sqrt{1 - \sqrt{x^4 - x^2}} = x - 1$

Rj. Ugeti: $\begin{cases} 1) x^4 - x^2 \geq 0 \\ 2) 1 - \sqrt{x^4 - x^2} \geq 0 \end{cases}$ većinu zapisivati (prođugo je!)

$$\text{Kvadriramo: } 1 - \sqrt{x^4 - x^2} = x^2 - 2x + 1$$

$$\sqrt{x^4 - x^2} = 2x - x^2 \quad |^2$$

$$x^4 - x^2 = 4x^2 - 4x^3 + x^4$$

$$4x^3 - 5x^2 = 0$$

$$x^2(4x - 5) = 0 \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = \frac{5}{4} \end{cases}$$

Da bismo provjerili jesu li to stvarno rješenja, uvrstimo ih u početnu jednačinu: $\sqrt{1 - \sqrt{0}} = 0 - 1 \Leftrightarrow 1 = -1 \quad \text{tj.} \Rightarrow x_1 = 0 \text{ nije r.}$

$$\sqrt{1 - \sqrt{\frac{625}{256} - \frac{25}{16}}} = \frac{5}{4} - 1 \Leftrightarrow \sqrt{1 - \sqrt{\frac{225}{256}}} = \frac{1}{4} \Leftrightarrow \sqrt{1 - \frac{15}{16}} = \frac{1}{4} \Leftrightarrow \frac{1}{4} = \frac{1}{4} \quad \checkmark$$

$\Rightarrow$ jedinstveno rješenje je $\boxed{x = \frac{5}{4}}$

Zad. Riješite uz diskusiju po $a \in \mathbb{R}$: $\sqrt[3]{a+x} + \sqrt[3]{a-x} = \sqrt[3]{2a}$ (14)

Rj. Treći korijen je definiran na cijelom $\mathbb{R} \Rightarrow$ nema ujeta

$f(t) = t^3$ je bijekcija pa kod kubiranja ne trebamo paziti na izgled

Kubiramo: $a+x + 3\sqrt[3]{(a+x)^2(a-x)} + 3\sqrt[3]{(a+x)(a-x)^2} + a-x = 2a$

$$3\sqrt[3]{(a+x)(a-x)} [\sqrt[3]{a+x} + \sqrt[3]{a-x}] = 0$$

$$1) \sqrt[3]{(a+x)(a-x)} = 0 \quad \begin{cases} a+x=0 \Rightarrow x=-a \\ (a+x)(a-x)=0 \\ a-x=0 \Rightarrow x=a \end{cases}$$

$$2) \sqrt[3]{a+x} = -\sqrt[3]{a-x}$$

$$a+x = -a-x$$

$$a = -a \in \text{ne postoji } 0 \text{ } x \text{ : točno je za } a=0 \Rightarrow \text{g. } \mathbb{R}$$

$$2a=0 \quad \text{netočno za } a \neq 0 \Rightarrow \text{g. } \emptyset$$

Konačno rješenje: za $a \neq 0 \Rightarrow x = \pm a$ (dva rješenja)

za $a=0 \Rightarrow$ cijeli $\mathbb{R}$

Zad. Riješite jednačinu: a) $9^x - 15^x - 2 \cdot 25^x = 0$

$$b) 4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$$

Rj. a) ideja: svđenje na zajedničku bazu

$$3^{2x} - 3^x \cdot 5^x - 2 \cdot 5^{2x} = 0 \quad | : 5^{2x} (\neq 0)$$

$$\left(\frac{3}{5}\right)^{2x} - \left(\frac{3}{5}\right)^x - 2 = 0 \quad t = \left(\frac{3}{5}\right)^x$$

$$t^2 - t - 2 = 0$$

$$\Rightarrow t_1 = -1$$

$$t_2 = 2$$

$$\left(\frac{3}{5}\right)^x = -1$$

$$\left(\frac{3}{5}\right)^x = 2 \quad | \log$$

nema g.

$$x(\log 3 - \log 5) = \log 2$$

$$\boxed{x = \frac{\log 2}{\log 3 - \log 5}}$$

$$\text{ili } x = \log_{\frac{3}{5}} 2$$

b) ideja: svodjenje na zajednički eksponent

$$2^{2x} - \frac{3^x}{\sqrt{3}} = \sqrt{3} \cdot 3^x - \frac{2^{2x}}{2} \Leftrightarrow 2^{2x} + \frac{2^{2x}}{2} = \sqrt{3} \cdot 3^x + \frac{3^x}{\sqrt{3}}$$

$$\Leftrightarrow \frac{3}{2} \cdot 2^{2x} = \frac{4}{\sqrt{3}} \cdot 3^x \cdot \frac{1}{2} \Leftrightarrow \frac{2^{2x}}{8} = \frac{3^x}{3\sqrt{3}}$$

$$\Leftrightarrow 2^{2x-3} = \frac{3^x}{3\sqrt{3}} \Leftrightarrow 4^{\frac{1}{2}(2x-3)} = 3^{x-\frac{3}{2}}$$

$$4^{\frac{2x-3}{2}} = 3^{\frac{2x-3}{2}}$$

$$2x-3=0$$

$$\boxed{x = \frac{3}{2}}$$

Zad. Riješite jednačinu: a) $\log_{\sqrt{2}}(\sqrt{2x+3} + \sqrt{x+1}) = \log_2 9$

b) $(x+1)^{\log(x+1)} = 100(x+1)$

R. a) uvjet: $2x+3 \geq 0$, $x+1 \geq 0$
 $x \geq -\frac{3}{2}$, $x \geq -1$

$$\sqrt{2x+3} + \sqrt{x+1} \geq 0$$

oba su ≥ 0 i ne mogu biti istovremeno bitni $\Rightarrow$ oboje uvijek

$$\Rightarrow x \in [-\frac{3}{2}, +\infty)$$

$$2 \log_2(\sqrt{2x+3} + \sqrt{x+1}) = \log_2 3^2$$

$$2 \log_2(\sqrt{2x+3} + \sqrt{x+1}) = \log_2 3$$

$$\sqrt{2x+3} + \sqrt{x+1} = 3/2$$

$$2x+3 + 2\sqrt{(2x+3)(x+1)} + x+1 = 9$$

$$2\sqrt{2x^2+3x+4} = -3x+1/2$$

$$4(2x^2+3x+4) = 9x^2 - 6x + 1$$

$$x^2 - 58x + 15 = 0 \Rightarrow x_1 = -1$$

$$x_2 = 19$$

provera:

$$x = -1 \quad 1+2=3$$

$$x = 19 \quad 1+18=19$$

$$\Rightarrow \text{konacno: } \boxed{x = -1}$$

b) uvjet: $x+1 > 0 \Leftrightarrow x > -1$

logaritmiramo: $\log(x+1) - \log(x+1) = \log 100 + \log(x+1)$

$$t_1 = \log(x+1) \Rightarrow t^2 - 6t - 2 = 0 \Rightarrow t_{1,2} = \frac{1 \pm 3}{2}$$

$$t_1 = -1, t_2 = 2$$

$$\log(x+1) = -1 \Rightarrow x+1 = 10^{-1} \Rightarrow x = -\frac{9}{10} > -1$$

$$\log(x+1) = 2 \Rightarrow x+1 = 100 \Rightarrow x = 99 > -1$$

Konacno:

$$\boxed{x \in \{-\frac{9}{10}, 99\}}$$

Zad. Riješite nejednačinu: $|x-2|^{1-|x|} < |x-2|^x$.

(7) (25)

Rj.

- potencije su s istom bazom, $|x-2| \geq 0$ i

$|x-2|=0$ i $|x-2|=1$ nisu gledište $\Rightarrow x \neq 2, 1, 3$

- logaritmirat ćemo s bazom $|x-2|$

1) $|x-2| > 1$, tj. $x \in (-\infty, 1) \cup (3, +\infty) \Rightarrow \log_{|x-2|}$ raste

$$\Rightarrow 1-|x| < x : a) x \geq 0 \Rightarrow 1-x < x$$

$$1 < 2x$$

$$x > \frac{1}{2} ; x \geq 0 \Rightarrow x > \frac{1}{2}$$

$$b) x < 0 \Rightarrow 1-(-x) < x$$

$$1 < 0 \text{ nemoguće, tj. } \emptyset$$

$$\Rightarrow x \in (\frac{1}{2}, +\infty) \text{ i ujed. } x \in (-\infty, 1) \cup (3, +\infty)$$

$$\Rightarrow \text{tj. od 1) je } \underline{x \in (\frac{1}{2}, 1) \cup (3, +\infty)}$$

2) $|x-2| < 1$, tj. $x \in (1, 3) \Rightarrow \log_{|x-2|}$ pada

$$\Rightarrow 1-|x| > x$$

- (kao je $x \in (1, 3)$, slijedi $x > 0 \Rightarrow |x| = x \Rightarrow$)

$$\Rightarrow 1-x > x$$

$$x < \frac{1}{2} \text{ i ujed. } x \in (1, 3) \Rightarrow \emptyset \rightarrow \text{rješenje od 2)}$$

$$\text{Konačno: } 1) \cup 2) \Rightarrow \boxed{x \in (\frac{1}{2}, 1) \cup (3, +\infty)}$$

Zad. Riješite nejednačinu: $\frac{1}{\log_{x-3}(x+1)} + \frac{1}{\log_x(x^2+2x+1)} < 1$.

Rj.

$$\text{ujed. : } x-3 > 0, x > 0 \Rightarrow x > 3$$

$$x-3 \neq 1, x \neq 4$$

$$x+1 > 0, x^2+2x+1 = (x+1)^2 \geq 0$$

$$\left. \begin{array}{l} x > 3 \\ x \neq 4 \end{array} \right\} \Rightarrow x > 3, x \neq 4$$

$$\frac{1}{\log_{x-3}(x+1)} + \frac{1}{\log_x(x+1)^2} < 1$$

$$\frac{1}{\log_{x-3}(x+1)} + \frac{1}{2 \log_x(x+1)} < 1$$

$$\frac{1}{\log_{x-3}(x+1)} + \frac{1}{2} < 1$$

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$$\log_{x+1}(x-3) + \frac{1}{2} \log_{x+1} x < 1$$

$$\log_{x+1}(x-3) + \log_{x+1} \sqrt{x} < 1$$

$$\log_{x+1}(x-3)\sqrt{x} < 1$$

$$\log_{x+1}(x-3)\sqrt{x} < \log_{x+1}(x+1) \quad (x \neq -1)$$

$$1^0 \quad x+1 > 1, \text{ f. } x > 0$$

$$(x-3)\sqrt{x} < (x+1)^2 \quad (\text{obje strane su pozitivne})$$

$$x^3 - 6x^2 + 9x < x^2 + 2x + 1$$

$$x^3 - 7x^2 + 7x - 1 < 0$$

$$(x-1)(x^2 + x + 1) - 7x(x-1) < 0$$

$$(x-1)(x^2 - 6x + 1) < 0$$

$$x-1 > 0, \text{ jer je } x > 3 \rightarrow x^2 - 6x + 1 < 0$$

$$x_{1,2} = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$$\Rightarrow x \in (3-2\sqrt{2}, 3+2\sqrt{2})$$

$$\text{tjeret } \Rightarrow x \in (3-2\sqrt{2}, 3+2\sqrt{2}) \cap \{4\}$$

$$2^0 \quad x+1 < 1, \text{ f. } x < 0$$

$$\text{neimaqer, jer je } x > 3$$

$$\text{Konačno rješenje: } 1^0 \cup 2^0 \Rightarrow x \in (3-2\sqrt{2}, 3+2\sqrt{2}) \cap \{4\}$$

~~Zad.~~ Rijesite vjeđnadžbu po x u ovisnosti o $a \in \mathbb{R}$:

$$\frac{3a^2}{2x-a} \leq 2x-3a.$$

Rj: $\frac{3a^2}{2x-a} - 2x + 3a \leq 0$

$$\frac{a \cdot (2a-x)}{2x-a} \leq 0 \quad \text{nultočke: } 0, \frac{a}{2}, 2a$$

1° $a > 0$

	$-\infty$	0	$\frac{a}{2}$	$2a$	$+\infty$
$4x$		-	+	+	
$2a-x$	+	+	+	0	-
$2x-a$	-	-	0	+	+
	$\oplus$	$\ominus$	$\oplus$	$\ominus$	

$$\Rightarrow x \in [0, \frac{a}{2}] \cup [2a, +\infty)$$

2° $a = 0$

$$\frac{0}{2x} \leq 2x : \boxed{x > 0}$$

3° $a < 0$

	$-\infty$	$2a$	$\frac{a}{2}$	0	$+\infty$
$4x$	-	-	-	0	+
$2a-x$	+	0	-	-	-
$2x-a$	~~+~~	-	0	+	+
	$\oplus$	$\ominus$	$\oplus$	$\ominus$	

$$\Rightarrow x \in [2a, \frac{a}{2}] \cup [0, +\infty)$$

Zad. Rijesite vjeđnadžbu: $|x^3 - x^2| < |x^2 + x|$.

Rj:

Konstano: $|xy| = |x| \cdot |y|, |x^2| = |x|^2 = x^2$

$$x^2|x-1| < |x| \cdot |x+1|$$

1° $x \leq 1$ $x^2(1-x) < -x(-1-x)$

$$x^2 - x^3 < x + x^2$$

$$-x^3 - x < 0 \quad | \cdot (-1)$$

$$x(x^2+1) > 0$$

$$\Rightarrow x \neq 0$$

3° $x \in [0, 1]$

$$x^2(1-x) < x(x+1)$$

$$x^2 - x^3 < x^2 + x$$

$$-x^3 - x < 0$$

$$-x(x^2+1) < 0$$

$$\Rightarrow x > 0$$

$$\Rightarrow x \in (0, 1]$$

2° $x \in [-1, 0]$ $x^2(1-x) < -x(x+1)$

$$x^2 - x^3 < -x^2 - x$$

$$-x^3 + 2x^2 + x < 0$$

$$x(x^2 - 2x - 1) < 0$$

$$> 0 \text{ (jer je } x \in [-1, 0])$$

$$\Rightarrow x^2 - 2x - 1 < 0$$

$$x_{1,2} = 1 \pm \sqrt{2}$$

na intervalu

$$x \in (-1-\sqrt{2}, 1+\sqrt{2}) \Rightarrow x \in (-1-\sqrt{2}, 1)$$

~ 4° $x \geq 1$

$$x^2(1-x) < x(x+1)$$

kao pod 2°: $x \in [1, 1+\sqrt{2})$

$$\Rightarrow x \in (-1-\sqrt{2}, 1+\sqrt{2}) \setminus \{0\}$$

Zad. Riješite nejednadžbu: $\sqrt{4-x^2} > 2-2x$.

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Rje. ujet: $4-x^2 > 0$, tj. $x^2 \leq 4$, tj. $x \in [-2, 2]$

Želimo kvadrirati da bismo se riješili korijena. Smijemo li?

Na nejednakosti smijemo primijeniti rastuću i padajuću (uz promjenu znaka nejednakosti) funkcije.

$f(t) > t^2$ uje uiti rastuću uiti padajuću. No, za $t < 0$ to je strogo rastuća $\Rightarrow$ ako su obje strane pozitivne, tada smijemo kvadrirati.

Lijeva strana je uvijek ≥ 0 , a desna za $2-2x \geq 0$, tj. $x \leq 1$

Mi sve gledamo na $[-2, 2]$ pa imamo dva slučaja:

1° $x \in [-2, 1] \rightarrow$ desna strana ≥ 0

2° $x \in (1, 2] \rightarrow$ desna strana negativna

1° $x \in [-2, 1] \Rightarrow$ kvadriramo

$$4-x^2 > 4-8x+4x^2$$

$$5x^2-8x < 0$$

$$x_1=0, x_2=\frac{8}{5}$$

$$\cancel{x \in [-2, \frac{8}{5}]} \Rightarrow \boxed{x \in [0, 1]}$$

2° $x \in (1, 2] \Rightarrow \underbrace{\sqrt{4-x^2}}_{\geq 0} > \underbrace{2-2x}_{\text{neg.}}$ $\leftarrow$ ovo uvijek uipadi

$\Rightarrow x \in (1, 2]$ je prazno

Konačno: 1° $\cup$ 2° : $\boxed{x \in [0, 1]}$

Zad. Riješite nejednadžbu: $\sqrt{2(5^x+24)} + \sqrt{5^x-7} \geq \sqrt{5^x+7}$.

Rje. ujeti: 1) $5^x+24 \geq 0 \checkmark$

2) $5^x-7 \geq 0 \Leftrightarrow 5^x \geq 7 \Leftrightarrow x \geq \log_5 7 \approx 1.209$

3) $5^x+7 \geq 0 \checkmark$

Htjeli bismo kvadrirati da se riješimo korijena, no ne smijemo jer je 2. strana možda neg pa zato prebacimo na desnu stranu:

$$\sqrt{2(5^x+24)} \geq \sqrt{5^x+7} + \sqrt{5^x+7}$$

$$2(5^x+24) \geq 5^x+7 + 2\sqrt{(5^x-7)(5^x+7)} + 5^x+7$$

$$2\sqrt{25^x-49} \leq 48 - 2 \quad | :2$$

$$25^x - 49 \leq 5^x - 7$$

$$25^x \leq 625 - 25^x$$

$$\Rightarrow x \leq 2 \quad \left. \begin{array}{l} \\ \text{+ ujet} \end{array} \right\} x \in [\log_5 7, 2]$$

JEDNAŽBE I NEJEDNAŽBE

Zad 1 Riješite jednačinu: $\frac{7}{x^2-1} + \frac{8}{x^2-2x+1} = \frac{25-9x}{x^3-x^2-x+1}$

Rj $\frac{7}{(x+1)(x-1)} + \frac{8}{(x-1)^2} = \frac{25-9x}{(x-1)^2(x+1)} \quad | \cdot (x-1)^2(x+1)$

$\Rightarrow$ Rješenje mora zadovoljavati uslov $x \neq \pm 1$

$$7x-7+8x+8 = 25-9x$$

$$24x = 24 \Leftrightarrow \underline{x=1} \quad \text{to ne može biti rješenje zbog } x \neq \pm 1$$

Jednačina nema rješenja

Zad 2 Riješite jednačinu u ovisnosti o parametrima a i b :

$$x(a-x) + (b-x)^2 = a + b^2 - 4$$

Rj $ax - x^2 + b^2 - 2bx + x^2 = a + b^2 - 4$
 $x(a-2b) = a-4$

1°) Ako je $a-2b \neq 0 \Leftrightarrow a \neq 2b$, onda je rješenje $x = \frac{a-4}{a-2b}$

2°) Ako je $a-2b = 0 \Leftrightarrow a = 2b$, imamo $x \cdot 0 = a-4$

2.1°) za $a \neq 4$ nema rješenja ($x \cdot 0 \neq 0$)

2.2°) za $a = 4$ svaki $x \in \mathbb{R}$ je rješenje ($x \cdot 0 = 0$)

Zad 3 Riješite jednačinu: $|x + \frac{1}{2}x + \frac{1}{3}x - 1| = 1$

Rj 1°) $\frac{1}{3}x - 1 \geq 0 \Leftrightarrow x \geq 3$

$$|x + \frac{1}{2}x + \frac{1}{3}x - 1| = 1$$

$$|x + \frac{5}{6}x - 1| = 1$$

$$\frac{5}{6}x - 1 \geq \frac{5}{6} \cdot 3 - 1 = \frac{3}{2} > 0$$

$$\Rightarrow |x + \frac{5}{6}x - 1| = 1$$

$$|\frac{11}{6}x - 1| = 1 \quad (\frac{11}{6}x > \frac{11}{2} > 5)$$

$$\frac{11}{6}x - 1 = 1$$

$$\frac{11}{6}x = 2 \Rightarrow x = \frac{12}{11} < 3$$

$$2^{\circ}) \quad \frac{1}{3}x - 1 < 0 \Leftrightarrow x < 3$$

$$|x + |\frac{1}{2}x - \frac{1}{3}x + 1|| = 1$$

$$|x + |1 + \frac{1}{6}x|| = 1$$

2.1°)

$$\frac{1}{6}x + 1 \geq 0 \Leftrightarrow x \geq -6$$

$$\boxed{-6 \leq x < 3}$$

$$\Rightarrow |x + \frac{1}{6}x + 1| = 1 \Leftrightarrow \frac{7}{6}x + 1 = \pm 1$$

$$\left\{ \begin{array}{l} x = 0 \quad \checkmark \\ x = -\frac{12}{7} \quad \checkmark \end{array} \right.$$

$$\left\{ \begin{array}{l} x = \frac{12}{5} \\ x = 0 \end{array} \right.$$

$$2.2^{\circ}) \quad \frac{1}{6}x + 1 < 0 \Leftrightarrow \boxed{x < -6} \rightarrow |x - \frac{1}{6}x - 1| = 1 \Leftrightarrow \frac{5}{6}x - 1 = \pm 1$$

$$\left\{ \begin{array}{l} x = \frac{12}{5} \\ x = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} x = \frac{12}{5} \\ x = 0 \end{array} \right.$$

Rješenja su $x_1 = 0$ i $x_2 = -\frac{12}{7}$

zad 3 Riješite jednačinu: $\sqrt{1 - \sqrt{x^4 - x^2}} = x - 1$

2.1. uvjeti: 1) $x^4 - x^2 \geq 0$

2) $1 - \sqrt{x^4 - x^2} \geq 0$

Kvadriranjem dobivamo: $1 - \sqrt{x^4 - x^2} = x^2 - 2x + 1$

$$\sqrt{x^4 - x^2} = 2x - x^2 \quad |^2$$

$$x^4 - x^2 = 4x^2 - 4x^3 + x^4$$

$$4x^3 - 5x^2 = 0$$

$$x^2(4x - 5) = 0$$

$$x = 0 \quad x = \frac{5}{4}$$

Za provjeru, uvrstimo u početnu jednačinu:

$$\sqrt{1 - \sqrt{0 - 0}} = \sqrt{1 - 0} = 1 \neq 0 - 1 \quad x = 0 \text{ nije rj.}$$

$$\sqrt{1 - \sqrt{\frac{625}{256} - \frac{25}{16}}} = \sqrt{1 - \sqrt{\frac{25(25-16)}{256}}} = \sqrt{1 - \sqrt{\frac{25 \cdot 9}{256}}}$$

$$= \sqrt{1 - \frac{15}{16}} = \sqrt{\frac{1}{16}} = \frac{1}{4} = \frac{5}{4} - 1 \quad \checkmark$$

Rješenje jednačine je $x = \frac{5}{4}$ (2)

Zad 4 U ovisnosti o parametru $a \in \mathbb{R}$ riješite jednačinu:

$$\sqrt[3]{a+x} + \sqrt[3]{a-x} = \sqrt[3]{2a}$$

Rj. Treći korijen je definiran na cijelom $\mathbb{R} \Rightarrow$ nema uvjeta!

Kubiranjem dobivamo:

$$a+x + 3\sqrt[3]{(a+x)^2(a-x)} + 3\sqrt[3]{(a+x)(a-x)^2} + a-x = 2a$$

$$3\sqrt[3]{(a+x)(a-x)} (\sqrt[3]{a+x} + \sqrt[3]{a-x}) = 0$$

$$1^0) \sqrt[3]{(a+x)(a-x)} = 0 \Leftrightarrow a+x=0 \text{ ili } a-x=0 \Leftrightarrow x=\pm a \quad \text{Newtonov } 0 \text{ a!}$$

$$2^0) \sqrt[3]{a+x} = -\sqrt[3]{a-x} \Leftrightarrow a+x = -(a-x) \Leftrightarrow x+a = x-a$$

↓
za $a=0$ ovo je točno za svaki $x \in \mathbb{R}$
za $a \neq 0$ ovo nije točno ni za koji $x \in \mathbb{R}$

Diskusija: ako je $a \neq 0$, jednačina ima 2 rješenja: $x_1 = a$ i $x_2 = -a$
ako je $a = 0$, svaki $x \in \mathbb{R}$ je rješenje jedne

Zad 5 Riješite jednačinu: (a) $9^x - 15^x - 2 \cdot 25^x = 0$
(b) $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

Rj. (a) Ideja: svodenje na zajedničku bazu

$$3^{2x} - 3^x \cdot 5^x - 2 \cdot 5^{2x} = 0 \quad | : 5^{2x} \neq 0 !$$

$$\left(\frac{3}{5}\right)^{2x} - \left(\frac{3}{5}\right)^x - 2 = 0$$

substitucija: $t = \left(\frac{3}{5}\right)^x$

$$t^2 - t - 2 = 0 \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$$

$$t_1 = 2 \quad t_2 = -1$$

$$\left(\frac{3}{5}\right)^{x_1} = 2 \Rightarrow \underline{\underline{x_1 = \log_{\frac{3}{5}} 2}}$$

$$\text{Rješenje: } x = \log_{\frac{3}{5}} 2$$

$$\left(\frac{3}{5}\right)^{x_2} = -1 \Rightarrow \text{nema rj.}$$

(b) Ideja: svodjenje na zajednički eksponent:

$$2^{2x} - \frac{3^x}{\sqrt{3}} = \sqrt{3} \cdot 3^x - \frac{2^{2x}}{2}$$

$$\Leftrightarrow 2^{2x} + \frac{2^{2x}}{2} = \sqrt{3} \cdot 3^x + \frac{1}{\sqrt{3}} \cdot 3^x$$

$$\Leftrightarrow \frac{3}{2} \cdot 2^{2x} = \frac{4}{\sqrt{3}} \cdot 3^x \quad | :12$$

$$4^{x-\frac{3}{2}} = 3^{x-\frac{3}{2}}$$

$$\Leftrightarrow \frac{1}{9} 2^{2x} = \frac{1}{3\sqrt{3}} \cdot 3^x$$

$$\left(\frac{4}{3}\right)^{x-\frac{3}{2}} = 1$$

$$2^{2x-3} = 3^{x-\frac{3}{2}}$$

$$\Rightarrow x - \frac{3}{2} = 0 \quad \Rightarrow \underline{\underline{x = \frac{3}{2}}}$$

$$2^{2(x-\frac{3}{2})} = 3^{x-\frac{3}{2}}$$

Zad 6 Riješite jednačinu: (a) $\log_{\sqrt{2}} (\sqrt{2x+3} + \sqrt{x+5}) = \log_2 9$

(b) $(x+1)^{\log(x+1)} = 100(x+1)$

Rj (a) uvjeti: $2x+3 \geq 0 \Leftrightarrow x \geq -\frac{3}{2}$

$x+5 \geq 0 \Rightarrow x \geq -5 \rightarrow$ vrijedi kad je $x \geq -\frac{3}{2}$

$\sqrt{2x+3} + \sqrt{x+5} > 0 \rightarrow$ oba su ≥ 0 i ne mogu oba istovremeno biti 0, pa ovo vrijedi

Sve zajedno: $\boxed{x \geq -\frac{3}{2}}$

$$\frac{\log_2 (\sqrt{2x+3} + \sqrt{x+5})}{\log_2 \sqrt{2}} = \log_2 3^2$$

$$2 \log_2 (\sqrt{2x+3} + \sqrt{x+5}) = 2 \log_2 3$$

$$\sqrt{2x+3} + \sqrt{x+5} = 3 \quad |^2$$

$$2x+3 + 2\sqrt{(2x+3)(x+5)} + x+5 = 9$$

$$2\sqrt{(2x+3)(x+5)} = 1-3x \quad |^2$$

$$4(2x^2 + 13x + 15) = 1 - 6x + 9x^2$$

$$x^2 - 59x - 59 = 0$$

$$x^2 + x - 59x - 59 = 0$$

$$(x-59)(x+1) = 0$$

$$\begin{array}{cc} -\frac{3}{2} & -\frac{3}{2} \\ \wedge & \wedge \\ x_1 = -1 & x_2 = 59 \end{array} \quad \text{ok}$$

provjera:

$$\sqrt{-2+3} + \sqrt{-1+5} = 1+2 = 3 \quad \checkmark$$

$$\sqrt{118+3} + \sqrt{59+5} = 11+8 \neq 3 \quad \times$$

Rješenje: x = -1

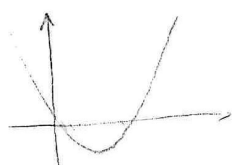
$$1^o) x \in [-2, 1]$$

$$\sqrt{4-x^2} > 2-2x \quad |^2$$

$$4-x^2 > 4-8x+4x^2$$

$$5x^2 - 8x < 0$$

$$x(5x-8) < 0$$



$$\Rightarrow x \in <0, \frac{8}{5}>$$

$$\underline{x \in <0, 1]}$$

$$2^o) x \in <1, 2]$$

$$\underbrace{\sqrt{4-x^2}}_{\geq 0} > \underbrace{2-2x}_{< 0} \rightarrow \text{kvijene vrijedi}$$

$$\underline{x \in <1, 2]}$$

$$\text{konačno rješenje: } (1^o) \cup 2^o) \Rightarrow x \in <0, 2]$$

Zad 8 Riješite nejednačinu

$$\sqrt{2(5^x+24)} - \sqrt{5^x-7} \geq \sqrt{5^x+7}$$

Rj

wjeh:

$$5^x+24 \geq 0 \quad \checkmark$$

$$5^x-7 \geq 0 \Leftrightarrow 5^x \geq 7 \Leftrightarrow x \geq \log_5 7 \quad (\approx 1.209...)$$

$$5^x+7 \geq 0 \quad \checkmark$$

$$\underbrace{\sqrt{2(5^x+24)}}_{\geq 0} \geq \underbrace{\sqrt{5^x-7} + \sqrt{5^x+7}}_{\geq 0} \quad |^2$$

$$2(5^x+24) \geq 5^x-7 + 2\sqrt{(5^x-7)(5^x+7)} + 5^x+7$$

$$2(5^x+24) \geq 2 \cdot 5^x + 2\sqrt{5^{2x}-49}$$

$$\sqrt{5^{2x}-49} \leq 24 \quad |^2$$

$$5^{2x} - 49 \leq 576$$

$$5^{2x} \leq 625$$

$$x \leq 2$$

$$\Rightarrow x \in [\log_5 7, 2]$$

Žad 8. Riješite nejednačinu: $|x^3 - x^2| < |x^2 + x|$

R: $x^2|x-1| < |x||x+1|$

1°) $x < -1$

$$x^2(1-x) < -x(-x-1)$$

$$\cancel{x^2} - x^3 < \cancel{x^2} + x$$

$$x^3 + x > 0$$

$$x(\underbrace{x^2+1}_{>1}) > 0$$

$$\Rightarrow x > 0 \quad \downarrow$$

2°) $-1 \leq x < 0$

$$-x^2(x-1) < -x(x+1)$$

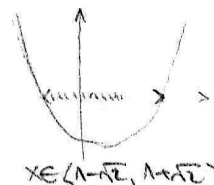
$$-x^3 + x^2 < -x^2 - x$$

$$x^3 - 2x^2 - x > 0$$

$$\underbrace{x(x^2 - 2x - 1)}_{<0} > 0$$

$$x_{1,2} = 1 \pm \sqrt{2}$$

$$\Rightarrow x \in (1-\sqrt{2}, 0)$$



3°) $0 \leq x < 1$

$$-x^2(x-1) < x(x+1)$$

$$-x^3 + \cancel{x^2} < \cancel{x^2} + x$$

$$x(\underbrace{x^2+1}_{>1}) > 0$$

$$\Rightarrow x \in (0, 1)$$

4°) $x \geq 1$

$$x^2(x-1) < x(x+1)$$

$$x^3 - x^2 < x^2 + x$$

$$\underbrace{x(x^2 - 2x - 1)}_{<0} < 0$$

$$x \in [1, 1+\sqrt{2})$$

$$\Rightarrow x \in (1-\sqrt{2}, 1+\sqrt{2}) \setminus \{0\}$$