

$$\frac{f(x)}{g(x)}$$

x, g polinomi

$$1) \text{ st } f \geq \text{st } g \Rightarrow f = q \cdot g + r, \text{ st } r < \text{st } g$$

$$\frac{f}{g} = \frac{q \cdot g + r}{g} = q + \frac{r}{g}$$

$$1) \text{ st } f < \text{st } g$$

g rastavimo na linearne i kvadratne faktore bez
redukih multiplikci

ad. Rastavite na parcijalne razlomke

$$\frac{5x+11}{x^2+3x-10}$$

$$\therefore \frac{5x+11}{x^2+3x-10} = \frac{5x+11}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} \quad | \cdot (x-2)(x+5)$$

$$5x+11 = A(x+5) + B(x-2)$$

$$5x+11 = (A+B)x + (5A-2B)$$

$$\Rightarrow \left. \begin{array}{l} A+B=5 \\ 5A-2B=11 \end{array} \right\} \Rightarrow A=3, B=2$$

$$\frac{5x+11}{x^2+3x-10} = \frac{3}{x-2} + \frac{2}{x+5}$$

Imamo 0 i 1. i 2. multiplikci rastaviti na razlomke.

ver.

Rozložte na parciálne zlomky

$$\frac{1}{x^3 + x^2 - x - 1}$$

$$\frac{1}{x^3 + x^2 - x - 1} = \frac{1}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad | \cdot (x+1)^2(x-1)$$

$$1 = A(x+1)^2 + B(x^2-1) + C(x-1)$$

Teda je ľahšie určiť hodnoty A, B, C od y (možno určiť
bilo koje $x \in \mathbb{R}$):

$$= 1 \dots \quad 1 = A \cdot 4 \quad \Rightarrow \quad A = \frac{1}{4}$$

$$= -1 \dots \quad 1 = C(-2) \quad \Rightarrow \quad C = -\frac{1}{2}$$

$$= 0 \dots \quad 1 = A - B - C \quad \Rightarrow \quad B = -\frac{1}{4}$$

$$\Rightarrow \frac{1}{x^3 + x^2 - x - 1} = \frac{\frac{1}{4}}{x-1} - \frac{\frac{1}{4}}{x+1} - \frac{\frac{1}{2}}{(x+1)^2}$$

d. Rozložte na parciálne zlomky

$$\frac{1}{x^3 + 1}$$

$$\frac{1}{x^3 + 1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \quad | \cdot (x^3+1)$$

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$1 = x^2(A+B) + x(B+C-A) + (A+C)$$

$$\Rightarrow \left. \begin{array}{l} A+B=0 \\ B+C-A=0 \\ A+C=1 \end{array} \right\} \Rightarrow A = \frac{1}{3}, B = -\frac{1}{3}, C = \frac{2}{3}$$

$$\Rightarrow \frac{1}{3} - \frac{1}{3}x + \frac{2}{3}$$

ways

$$\frac{x^7 + x^3 - x + 11}{x^2(x^2+1)^3(x-1)(x^2+2x)^4} = \frac{x^7 + x^3 - x + 11}{x^6(x^2+1)^3(x-1)(x+2)^4} =$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x^5} + \frac{F}{x^6} + \frac{Gx+H}{x^2+1} + \frac{Jx+K}{(x^2+1)^2} + \frac{Lx+M}{(x^2+1)^3} +$$

$$+ \frac{N}{x-1} + \frac{O}{x+2} + \frac{P}{(x+2)^2} + \frac{R}{(x+2)^3} + \frac{S}{(x+2)^4}$$

RASTAV NA PARC RAZLO

1. RAC. FUNKCIONAL JE CILIK f $g \neq 0$

$$\text{Ali je } \deg f \geq g \Rightarrow f = 2g + 1 \Rightarrow 2 + \left(\frac{r}{g}\right)$$

Prova RAC - FJA ako $\deg g > \deg f$

2.

PRIMA RAC FJA $\deg f < \deg g$ $\rightarrow$ taut. g m
lin. i lin. i lin.

$$f(1)=2 \quad f(2)=5 \quad \leadsto \text{ i hodnoty}$$

$$g(-1)=2 \quad g(1)=5 \quad \text{ } \} \text{ Porozumění ne shoduje}$$

$$\frac{f(x)}{g(x)} \quad \text{Roc. funkce}$$

fig. polynomu

Def. Racionální funkce

RASSTAV NA PÁRECH

Vše 2 x 2

$$\frac{f}{g} \quad 1^{\circ} \quad \left(\frac{f}{g} = 2 + \frac{5}{g} \right)$$

NA LINEARNE FAKTORIZACE
1. KURZEM BEZ NÍŽE N.S.
2. Faktorizace g(x) = 0

3. $\downarrow$

ČLÁN V RASSTAVU

FAKTOR
V NA 2

$ax+b$

$$\frac{A}{ax+b}$$

$(ax+b)^k$

$$\frac{A_1}{ax+b} + \dots + \frac{A_k}{(ax+b)^k}$$

ax^2+bx+c

$$\frac{Ax+b}{ax^2+bx+c}$$

$(ax^2+bx+c)^k$

$$\frac{A_k + B_k}{ax^2+bx+c} + \dots + \frac{A_k + B_k}{(ax^2+bx+c)^k}$$

Rastav na parcijalne razlomke

$$f, g \in \mathbb{R}[x]$$

$\frac{f}{g}$ racionalna funkcija

$$f(x) = q(x)g(x) + r(x), \quad \deg r < \deg g$$

$$\frac{f(x)}{g(x)} = \frac{q(x)g(x) + r(x)}{g(x)} = q(x) + \boxed{\frac{r(x)}{g(x)}}$$

Želimo $\frac{r(x)}{g(x)}$ prikazati kao sumu rac. funkcija s nazivnicima oblika $x-a$ ili x^2+bx+c

g rastavimo na linearne i kvadratne faktore

$$g(x) = \underbrace{(x-a_1)^{k_1} \dots (x-a_m)^{k_m}}_{\substack{a_1, \dots, a_m \\ \text{realne nt.}}} \underbrace{(x^2+b_1x+c_1)^{r_1} \dots (x^2+b_lx+c_l)^{r_l}}_{\substack{\text{parovi kompleksnih} \\ \text{multiplikacija}}}$$

$$\begin{aligned} \frac{r(x)}{g(x)} &= \frac{A_{1,1}}{x-a_1} + \frac{A_{1,2}}{(x-a_1)^2} + \dots + \frac{A_{1,k_1}}{(x-a_1)^{k_1}} + \dots \\ &+ \frac{A_{m,1}}{x-a_m} + \frac{A_{m,2}}{(x-a_m)^2} + \dots + \frac{A_{m,k_m}}{(x-a_m)^{k_m}} \\ &+ \frac{B_{1,1}x+C_{1,1}}{x^2+b_1x+c_1} + \dots + \frac{B_{1,r_1}x+C_{1,r_1}}{(x^2+b_1x+c_1)^{r_1}} + \dots \\ &+ \frac{B_{l,1}x+C_{l,1}}{x^2+b_lx+c_l} + \dots + \frac{B_{l,r_l}x+C_{l,r_l}}{(x^2+b_lx+c_l)^{r_l}} \end{aligned}$$

$$A_{i,j}, B_{i,j}, C_{i,j} \in \mathbb{R}$$

Rastav na
parcijalne razlomke

PRIMER:

$$\frac{r(x)}{x^2(x^2+1)^3(x-1)(x^2+2x)^4} = \frac{r(x)}{x^6(x^2+1)^3(x-1)(x+2)^4}$$

" $(x(x+2))^4$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x^5} + \frac{F}{x^6}$$

$$+ \frac{Gx+H}{x^2+1} + \frac{Ix+J}{(x^2+1)^2} + \frac{Kx+L}{(x^2+1)^3} + \frac{M}{x-1}$$

$$+ \frac{N}{x+2} + \frac{O}{(x+2)^2} + \frac{P}{(x+2)^3} + \frac{Q}{(x+2)^4}$$

Zad 31 Rastavite na parcijalne razlomke

$$\frac{5x+11}{x^2+3x-10}$$

Rj

$$x^2+3x-10=(x-2)(x+5)$$

$$\frac{5x+11}{x^2+3x-10} = \frac{A}{x-2} + \frac{B}{x+5} \quad | \cdot (x-2)(x+5)$$

$$5x+11 = A(x+5) + B(x-2)$$

$$= (A+B)x + (5A-2B)$$

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$$A+B=5$$

$$5A-2B=11$$

$$A=3, B=2$$

$$\frac{5x+11}{x^2+3x-10} = \frac{3}{x-2} + \frac{2}{x+5}$$

Zad 32 Rastavite na parcijalne razlomke

$$\frac{1}{x^3+x^2-x-1}$$

Rj

$$x^3+x^2-x-1 = x^2(x+1)-(x+1)$$

$$= (x^2-1)(x+1) = (x-1)(x+1)^2$$

$$\frac{1}{x^3+x^2-x-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad | \cdot (x-1)(x+1)^2$$

$$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$$= (A+B)x^2 + (2A+C)x + (A-B-C)$$

{

$$A+B=0$$

$$2A+C=0$$

$$A-B-C=1$$

{

$$-2B+C=0$$

$$-2B-C=1$$

$$-4B=1 \Rightarrow B=-\frac{1}{4}$$

$$C=-\frac{1}{4}$$

$$A=\frac{1}{2}$$

Zad 33 Rastavite na parcijalne razlomke

$$\frac{1}{x^3+1}$$

Rj

$$x^3+1 = (x+1)(x^2-x+1)$$

$$(-1)^2 - 4 \cdot 1 \cdot 1 = 1 - 4 < 0$$

$\Rightarrow$ nema realnih
nultočka

$$\frac{1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \quad | \cdot (x+1)(x^2-x+1)$$

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$1 = (A+B)x^2 + (-A+B+C)x + (A+C)$$

$$\begin{cases} A+B = 0 \\ -A+B+C = 0 \\ A+C = 1 \end{cases} \Leftrightarrow \begin{cases} A+B = 0 \\ 2B+C = 0 \\ -B+C = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} A+B = 0 \\ -B+C = 1 \\ 3C = 2 \end{cases}$$

$$C = \frac{2}{3}$$

$$A = \frac{1}{3}$$

$$B = -\frac{1}{3}$$

$$\frac{1}{x^3+1} = \frac{1}{3} \cdot \frac{1}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}$$

Zad 34 Rastavite na parcijalne razlomke

$$\frac{x-1}{x^4 - 4x^3 + 5x^2 - 4x + 4}$$

Rj

$$g(x) = x^4 - 4x^3 + 5x^2 - 4x + 4$$

Kandidati za racionalne n.t. od g :

$$\pm 1, \pm 2, \pm 4$$

	1	-4	5	-4	4
1	1	-3	2	-2	2
-1	1	-5	10	-14	18
2	1	-2	1	-2	0
2	1	0	1	0	

$$g(x) = (x-2)(x^3 - 2x^2 + x - 2)$$

$$= (x-2)^2(x^2 + 1)$$

$$\frac{x-1}{g(x)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+1} \quad | \cdot (x-2)^2(x^2+1)$$

$$x-1 = A(x-2)(x^2+1) + B(x^2+1) + (Cx+D)(x-2)^2$$

$$= A(x^3 - 2x^2 + x - 2) + B(x^2+1)$$

$$+ C(x^3 - 4x^2 + 4x - 2) + D(x^2 - 4x + 4)$$

$$\left\{ \begin{array}{l} A + C = 0 \\ -2A + B - 4C + D = 0 \\ A + 4C - 4D = 1 \\ -2A + B + 4D = -1 \end{array} \right\} \begin{array}{l} + \\ + \end{array}$$

$$3C - 4D = 1 \quad | \cdot 3$$

$$4C + 3D = -1 \quad | \cdot 4$$

$$25C = -1$$

$$C = -\frac{1}{25}$$

$$\begin{aligned} D &= \frac{1}{3}(-1 - 4C) \\ &= \frac{1}{3}\left(-1 + \frac{4}{25}\right) \\ &= \frac{1}{3}\left(-\frac{21}{25}\right) \\ &= -\frac{7}{25} \end{aligned}$$

$$A = -C = \frac{1}{25}$$

$$\begin{aligned} B &= -1 + 2A - 4D \\ &= -1 + \frac{2}{25} + \frac{28}{25} \\ &= \frac{5}{25} = \frac{1}{5} \end{aligned}$$

$$\frac{x-1}{g(x)} = \frac{1}{25(x-2)} + \frac{1}{5(x-2)^2} + \frac{-x-7}{25(x^2+1)}$$