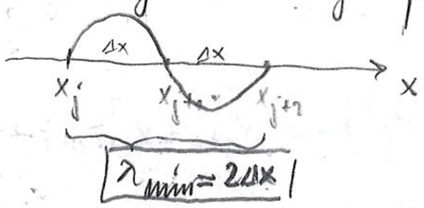
POGREŠKA RASPOZNAVANJA (ALIASING)

- promotriamo nelinearni oblik advektivne jdžbe u 1D: $\frac{\partial \phi}{\partial t} = -u \frac{\partial \phi}{\partial x}$
- njen raspis: $\frac{\partial \phi_j}{\partial t} = -u_j \frac{\phi_{j+1} - \phi_{j-1}}{2\Delta x}$ (NAJLJEPŠI NASLOV)
- na desnoj strani se prilikom razvoja ne mogu prikazati valovi valnih duljina manjih od $\lambda_{min} = 2\Delta x$;



- razvoj u red: $u(x) = \frac{a_0}{2} + \sum_{k\geq 1} \left(a_k \cos k \frac{2\pi}{L} x + b_k \sin k \frac{2\pi}{L} x\right)$

- ovo mogu pisati: $u(x) = \frac{a_0}{2} + \sum_{k\geq 1} \left(a_k \cos \frac{2\pi}{\frac{L}{k}} x + b_k \sin \frac{2\pi}{\frac{L}{k}} x\right) =$

$$= \frac{a_0}{2} + \sum_{k\geq 1} \left(a_k \cos \frac{2\pi}{\lambda_k} x + b_k \sin \frac{2\pi}{\lambda_k} x\right) \quad ; \quad \lambda_k = \frac{L}{k}$$

  - opaska: ovaj k nema nikakve veze s valnim brojem $(\frac{1}{\lambda})$
- sada: $k_{max} = \frac{L}{\lambda_{min}} = \frac{N\Delta x}{2\Delta x} \Rightarrow \boxed{k_{max} = \frac{N}{2}}$

- prilikom rješavanja dolazi do interakcije fja u i ϕ pa se mogu pojaviti i $\lambda < 2\Delta x$, a naš model će to prepoznati kao neke λ koje su moguće, a ne kao greške $\Rightarrow$ to je pogreška prepoznavanja

<u>Primjer aliasinga</u>: ~~p~~ oblik fja: $u(x) = \sin k_1 \frac{2\pi}{L} x$; $\phi(x) = \sin k_2 \frac{2\pi}{L} x$
  $\Rightarrow$ mreža je diskretna $(x_j = j\Delta x)$ i neka je $k_1, k_2 < k_{max}$
- sada: $\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} = 0 \Rightarrow -\frac{\partial \phi}{\partial t} = A = u \frac{\partial \phi}{\partial x}$

$$\Rightarrow A = \sin\left(k_1 \frac{2\pi}{L} x\right) k_2 \frac{2\pi}{L} \cos\left(k_2 \frac{2\pi}{L} x\right) = k_2 \frac{2\pi}{L} \frac{1}{2} \Big[\underbrace{\sin \frac{(k_1 - k_2) 2\pi}{L} x}_{1.} + \underbrace{\sin \frac{(k_1 + k_2) 2\pi}{L} x}_{2.}\Big]$$

$$\left\{ \sin\alpha \cos\beta = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right] \right\}$$

- ako je $|k_1 + k_2|, |k_1 - k_2| \leq k_{max} \Rightarrow$ OK
- ako je $|k_1 + k_2|, |k_1 - k_2| \geq k_{max} \Rightarrow$ ALIASING (to je kritično!!)
- pogledajmo kako se pogrešno predstavlja val (onaj za $k > k_{max}$!!) $\Rightarrow$ može se dobiti pomoću trigonometrijskih stavova

$$\Rightarrow \sin\left[k \frac{2\pi}{L} x\right] = \sin\left[(2k_{max} - 2k_{max} + k) \frac{2\pi}{L} x\right] = \sin \frac{2\pi}{L}\left[2k_{max} - (2k_{max} - k)\right] x =$$

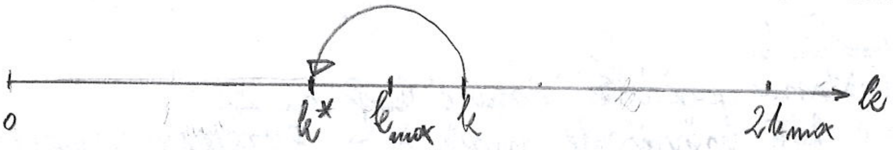
$$= \sin \frac{2\pi}{L} 2k_{max} x \cos \frac{2\pi}{L}(2k_{max} - k) x - \underbrace{\cos \frac{2\pi}{L} 2k_{max} x}_{1} \sin \frac{2\pi}{L} (2k_{max} - k) x$$

- sada prelazimo na diskretnu mrežu: $x_j = j\Delta x$
- znamo da je $k_{max} = \frac{N}{2}$; $L = N\Delta x$

$$\Rightarrow \sin \frac{2\pi}{L} 2k_{max} x = \sin \frac{2\pi}{N\Delta x} \cdot 2\frac{N}{2} j\Delta x = \sin(2\pi j) = 0 \quad \forall j$$

$$\cos \frac{2\pi}{L} 2k_{max} x = \cos \frac{2\pi}{N\Delta x} 2\frac{N}{2} j\Delta x = \cos(2\pi j) = 1 \quad \forall j$$

$$\Rightarrow \sin\left(k \frac{2\pi}{L} j\Delta x\right) = -\sin \frac{2\pi}{L}(2k_{max} - k)x \quad ; \quad \boxed{k^* = 2k_{max} - k} \rightarrow \text{BITNO!}$$



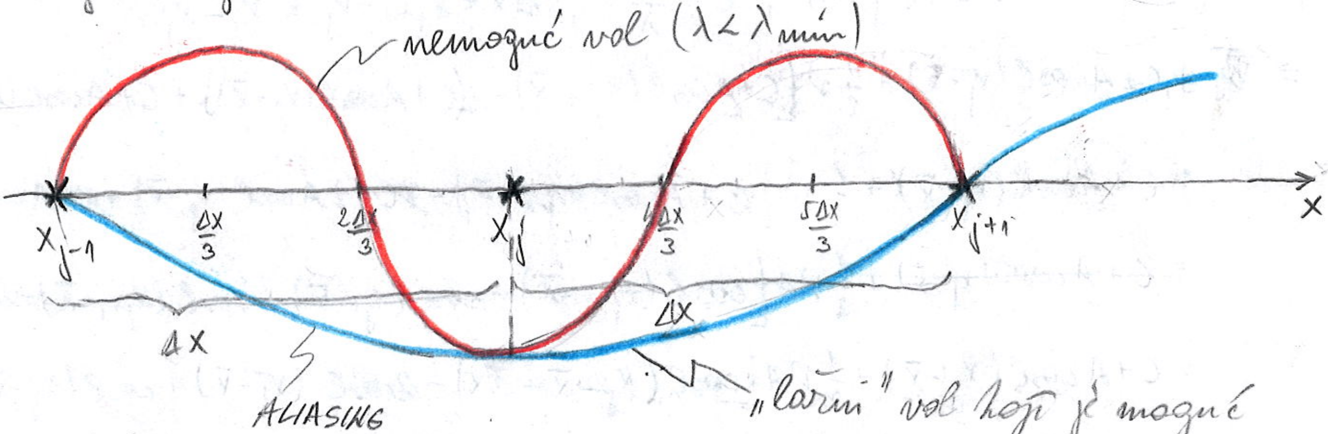
$\Rightarrow$ vidimo da se val sa $k > k_{max}$ „LAŽNO PREDSTAVIO" $\Rightarrow$ model misli da mu je $k < k_{max}$ jer on vidi k^*

Primjer: npr da imamo $\lambda = \frac{4}{3}\Delta x$ $(< 2\Delta x)$

$$\Rightarrow k = \frac{L}{\lambda} = \frac{N\Delta x}{\frac{4}{3}\Delta x} = \frac{3}{4}N > k_{max} = \frac{N}{2} \Rightarrow k^* = 2k_{max} - k = 2\frac{N}{2} - \frac{3}{4}N = \frac{N}{4} < k_{max} = \frac{N}{2}$$

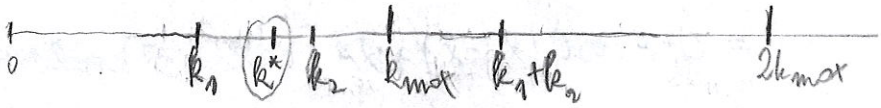
$$\Rightarrow \lambda^* = \frac{L}{k^*} = \frac{N\Delta x}{\frac{N}{4}} = 4\Delta x > 2\Delta x$$

$\rightarrow$ to je novi „lažni" val (koji je moguć) pomoću kojeg se prikazuje onaj nemogući



- pogledajmo gdje se grupiraju ovi valovi (lažni)?
  - gledamo $k_1 + k_2 > k_{max}$ $(k_1, k_2 < k_{max})$

$$\Rightarrow k^* = 2k_{max} - (k_1 + k_2)$$



- neka se postavi takav odnos k-ova da ispadne $k^* = k_1 \Rightarrow k_1 = 2k_{max} - k_1 - k_2 \Rightarrow 2k_1 = 2k_{max} - k_2 \Rightarrow k_1 = k_{max} - \frac{k_2}{2}$
- sada mijenjamo raspon od $k_2 \in [0, k_{max}] \Rightarrow k_1 \in [\frac{1}{2}k_{max}, k_{max}]$

$$\Rightarrow k^* \in \left[\frac{k_{max}}{2}, k_{max}\right]$$

- granice u λ^*: $\lambda = \frac{L}{k} \Rightarrow \frac{L}{\frac{k_{max}}{2}} = \frac{N\Delta x}{\frac{1}{2}\cdot\frac{N}{2}} = 4\Delta x$; $\frac{L}{k_{max}} = \frac{N\Delta x}{\frac{N}{2}} = 2\Delta x = \lambda_{min}$

$$\Rightarrow \boxed{\lambda^* \in [2\Delta x, 4\Delta x]}$$

PRIMJER:

ZAŠTO SE 17-h PERIOD ALIASIRA U 2.4-d PERIOD?

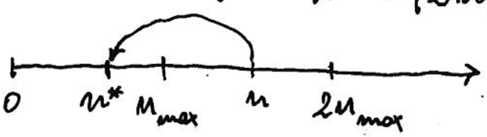
- Interval uzorkovanja niza dnevnih srednjaka iznosi:

$$\Delta t = 1\,d = 24\,h$$

- Fourierov razvoj funkcija može ići maksimalno do člana $n_{max} = \frac{N}{2}$ gdje je $N = \frac{T}{\Delta t}$; T... duljina niza jer:

$$n_{max} = \frac{T}{\tau_{min}} = \frac{N\cancel{\Delta t}}{2\cancel{\Delta t}} = \frac{N}{2}$$

- F. razvoj: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{N/2} \left(a_n \cos \frac{2\pi}{\tau_n} t + b_n \sin \frac{2\pi}{\tau_n} t\right)$
- Članovi F. razvoja čiji je $n > n_{max}$ se ALIASIRAJU na način da njihov n postaje: $n^* = |2n_{max} - n|$



- Dakle, što je s periodom $\tau = 17\,h$?
- U nizu dnevnih srednjaka se ne vide periodi kraći od $\tau_{min} = 2\Delta t = 2d = 48\,h$ pa se tako neće vidjeti ni 17-satni

$$\tau = 17h = \frac{17}{24}\Delta t$$

$$n = \frac{T}{\tau} = \frac{N\Delta t}{\frac{17}{24}\Delta t} = \frac{24}{17}N > n_{max} = \frac{N}{2}$$

$$\Rightarrow n^* = |2n_{max} - n| = \left|2\frac{N}{2} - \frac{24}{17}N\right| = \frac{7}{17}N < n_{max} \; //$$

$$\Rightarrow \tau^* = \frac{T}{n^*} = \frac{\cancel{N}\Delta t}{\frac{7}{17}\cancel{N}} = \frac{17}{7}\Delta t = 58{,}2h \approx 2.4\,d \; //_{OK}$$