ENTROPIJA CRNE RUPE I NOETHERIN NABOJ LORENTZ-DIFEOMORFIZMA

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Termodinamika crnih rupa

Law	Thermodynamics	Black Holes
Zeroth	T constant throughout body in thermal equilibrium	κ constant over horizon of stationary black hole
First	$dE = TdS +$ work terms	$dM = \frac{1}{8\pi}\kappa dA + \Omega_H dJ$
Second	$\delta S \geqq 0$ in any process	$\delta A \geqq 0$ in any process
Third	Impossible to achieve $T = 0$ by a physical process	Impossible to achieve $\kappa = 0$ by a physical process

Entropija crne rupe kao Noetherin naboj difeomorfizma

$$\delta L = E\delta\phi + \mathrm{d}\theta(\phi, \delta\phi)$$

$$\Omega(\phi, \delta_1\phi, \delta_2\phi) = \delta_1\theta(\phi, \delta_2\phi) - \delta_2\theta(\phi, \delta_1\phi)$$

$$\delta_\xi \phi = \mathcal{L}_\xi \phi$$

$$\pounds_v T^{a_1 \cdots a_k}{}_{b_1 \cdots b_l} = \lim_{t \to 0} \left\{ \frac{\phi^*_{-t} T^{a_1 \cdots a_k}{}_{b_1 \cdots b_l} - T^{a_1 \cdots a_k}{}_{b_1 \cdots b_l}}{t} \right\}$$

Entropija crne rupe kao Noetherin naboj difeomorfizma

$$\delta_\xi L = \mathcal{L}_\xi L = \mathrm{d}\, i_\xi L$$

$$j_\xi = \theta(\phi, \mathcal{L}_\xi \phi) - i_\xi L.$$

$$\mathrm{d} j_\xi = -E \mathcal{L}_\xi \phi$$

$$j_\xi = dQ_\xi$$

Integral Q po zatvorenoj površini S zove se Noetherin naboj površine S s obzirom na ξ

Entropija crne rupe kao Noetherin naboj difeomorfizma

$$
\begin{aligned}
\delta H_\xi &= \int_\Sigma \Omega(\phi, \delta\phi, \mathcal{L}_\xi \phi) \\
&= \int_\Sigma \delta\theta(\phi, \mathcal{L}_\xi \phi) - \mathcal{L}_\xi \theta(\phi, \delta\phi) \\
&= \int_\Sigma \delta j_\xi + \delta(i_\xi L) - i_\xi \mathrm{d}\theta - \mathrm{d}\, i_\xi \theta \\
&= \oint_{\partial\Sigma} \delta Q_\xi - i_\xi \theta.
\end{aligned}
$$

Entropija crne rupe kao Noetherin naboj difeomorfizma

$$\oint_{\mathcal{B}} \delta Q_\xi = \oint_{\infty} \delta Q_\xi - i_\xi \theta$$

$$\nabla_\mu \xi^\nu = \partial_\mu \xi^\nu = \kappa n_\mu^{\ \nu}$$

$$T_H \delta S = \delta \mathcal{E} - \Omega_H \delta \mathcal{J}$$

ξ Killingov vektor

$T_H = \hbar \kappa / 2\pi$ Hawkingova temperatura

$$S = \frac{2\pi}{\hbar} \oint_{\mathcal{B}} \widehat{Q}_\xi$$

Difeomorfizam Noetherinog naboja za opcu relativnost s ortonormiranom bazom

$$L(e,\omega) = \epsilon_{a\ldots bcd} e^a \wedge \ldots \wedge e^b \wedge R^{cd}$$

$$\delta L = \delta e^a \wedge \frac{\partial L}{\partial e^a} + D\delta\omega^{ab} \wedge \frac{\partial L}{\partial R^{ab}}$$

$$= \delta e^a \wedge \frac{\partial L}{\partial e^a} + \delta\omega^{ab} \wedge D\frac{\partial L}{\partial R^{ab}} + \mathrm{d}\left(\delta\omega^{ab} \wedge \frac{\partial L}{\partial R^{ab}}\right)$$

Difeomorfizam Noetherinog naboja za opcu relativnost s ortonormiranom bazom

$$\epsilon_{abc..df} e^c \wedge \ldots \wedge e^d \wedge De^f = 0$$

$$\epsilon_{ab...cde} e^b \wedge \ldots \wedge e^c \wedge R^{de} = 0$$

$$\mathcal{L}_\xi \omega = i_\xi \mathrm{d}\omega + \mathrm{d}(i_\xi \omega) = i_\xi R + D(i_\xi \omega)$$

$$\theta = \delta\omega^{ab} \wedge \frac{\partial L}{\partial R^{ab}}$$

Difeomorfizam Noetherinog naboja za opcu relativnost s ortonormiranom bazom

$$j_\xi = \mathrm{d}\left(i_\xi \omega^{ab} \wedge \frac{\partial L}{\partial R^{ab}} \right) - (i_\xi \omega^{ab}) \wedge D \frac{\partial L}{\partial R^{ab}} + (i_\xi R^{ab}) \wedge \frac{\partial L}{\partial R^{ab}} - i_\xi L$$

$$i_\xi L = (i_\xi e^a) \wedge \frac{\partial L}{\partial e^a} + (i_\xi R^{ab}) \wedge \frac{\partial L}{\partial R^{ab}}$$

$$Q_\xi = i_\xi \omega^{ab} \wedge \frac{\partial L}{\partial R^{ab}}$$

$$\mathcal{L}_\xi e^a = i_\xi \mathrm{d} e^a + \mathrm{d} i_\xi e^a = i_\xi D e^a + D i_\xi e^a - i_\xi \omega^a{}_b \wedge e^b$$

Difeomorfizam Noetherinog naboja za opcu relativnost s ortonormiranom bazom

$$i_\xi(\omega^e)^a{}_b = e^\mu_b D_\mu(i_\xi e^a)$$

$$i_\xi(\omega^e)^a{}_b = e^\mu_b e^a_\nu \nabla_\mu \xi^\nu$$

$$\lim_{\to\mathcal{B}} i_\xi(\omega^e)^{ab} = -\kappa n^{ab}$$

$$\lim_{\to\mathcal{B}}(Q_\xi) = -\kappa n^{ab}\epsilon_{abc\ldots d}e^c \wedge \ldots \wedge e^d$$

Difeomorfizam Noetherinog naboja za opcu relativnost s ortonormiranom bazom

$$\oint_{\mathcal{B}} Q_\xi = 2\kappa A/16\pi G$$

$$S_{BH} = A/4\hbar G$$

Bekenstein-Hawkingova etropija

No imamo problem: Na bifurkacijskoj površini entropija iščezava što je nefizikalno, a i implicira da konekcija spina divergira !

Lorentz-Lieva derivacija

Lieva derivacija ortonormirane baze općenito nije 0 jer osjeća Lorentzovu transformaciju.

Lorentz-Lieva derivacija zato dobija kompenzirajući član kojim ostvarujemo isčezavanje.

$$\mathcal{K}^e_\xi e^a = \mathcal{L}_\xi e^a + (\lambda^e_\xi)^a{}_b e^b$$

$$e_{a\mu}\mathcal{K}^e_\xi e^a_\nu = e_{a(\mu}\mathcal{K}^e_\xi e^a_{\nu)} + e_{a[\mu}\mathcal{K}^e_\xi e^a_{\nu]}$$

Lorentz-Lieva derivacija

$$e_{a(\mu}\mathcal{K}^{e}_{\xi}e^{a}_{\nu)} = \frac{1}{2}\mathcal{L}_{\xi}g_{\mu\nu}$$

$$e_{a[\mu}\mathcal{K}^{e}_{\xi}e^{a}_{\nu]} = e_{a[\mu}\mathcal{L}_{\xi}e^{a}_{\nu]} + e_{a\mu}e_{b\nu}(\lambda^{e}_{\xi})^{ab}$$

$$(\lambda^{e}_{\xi})^{ab} = e^{\sigma[a}\mathcal{L}_{\xi}e^{b]}_{\sigma}$$

$$\mathcal{K}^{e}_{\xi}e^{a}_{\mu} = \frac{1}{2}e^{a\nu}\mathcal{L}_{\xi}g_{\mu\nu}$$

Lorentz-Lieva derivacija

$$(\lambda^e_\xi)^{ab} = e^{\mu[a}\mathcal{L}_\xi e^{b]}_\mu = e^{\mu[a}\xi^\nu\nabla_\nu e^{b]}_\mu + e^{\mu[a}(\nabla_\mu\xi^\nu)e^{b]}_\nu$$

$$(\lambda^e_\xi)^{ab} = i_\xi(\omega^e)^{ab} + e^{\mu[a}e^{b]}_\nu\nabla_\mu\xi^\nu.$$

$$\nabla e^b = \mathcal{D}e^b - (\omega^e)^b{}_c e^c =$$

$$= -(\omega^e)^b{}_c e^c$$

$$\lambda^{Le}_\xi = L\lambda^e_\xi L^{-1} + L\mathcal{L}_\xi L^{-1}$$

$$\mathcal{K}^e_\xi\omega^{ab} = \mathcal{L}_\xi\omega^{ab} - D(\lambda^e_\xi)^{ab} = i_\xi R^{ab} + D(i_\xi\omega - \lambda^e_\xi)^{ab}$$

Entropija crne rupe kao Noetherin naboj Lorentzovog difeomorfizma

$$\delta_\xi L = \mathcal{L}_\xi L = \mathrm{d}\, i_\xi L \longrightarrow \mathcal{K}^e_\xi L = \mathcal{L}_\xi L = \mathrm{d}\, i_\xi L$$

$$j_\xi = \theta(\phi, \mathcal{L}_\xi \phi) - i_\xi L \longrightarrow j^{\mathcal{K}}_\xi = \theta(\phi, \mathcal{K}^e_\xi \phi) - i_\xi L$$

$$j_\xi = dQ_\xi \longrightarrow j^{\mathcal{K}}_\xi = \mathrm{d}Q^{\mathcal{K}}_\xi$$

$$S = \frac{2\pi}{\hbar} \oint_{\mathcal{B}} \widehat{Q}_\xi \longrightarrow S = \frac{2\pi}{\hbar} \oint_{\mathcal{B}} \widehat{Q}^{\mathcal{K}}_\xi$$

Lovelock gravitacija

$$L(e,\omega) = \epsilon_{a\ldots bcd}(c_0 e^a \wedge \ldots \wedge e^b \wedge e^c \wedge e^d +$$
$$+ c_1 e^a \wedge \ldots \wedge e^b \wedge R^{cd} + \ldots)$$

$$Q_\xi = i_\xi \omega^{ab} \wedge \frac{\partial L}{\partial R^{ab}} \xrightarrow[i_\xi \omega \;\to\; i_\xi \omega - \lambda^e_\xi]{} Q^{\mathcal{K}}_\xi = (i_\xi \omega - \lambda^e_\xi)^{ab} \wedge \frac{\partial L}{\partial R^{ab}}$$

Lovelock gravitacija

$$(\lambda_\xi^e)^{ab} = \kappa n^{ab}$$

$$\widehat{Q}_\xi^K = -\kappa n^{ab} \wedge \frac{\partial L}{\partial R^{ab}}$$

Ključna stvar je to da pošto je baza Lorentz-Lie invarijantna, a nije Lie invarijantna, može se pretpostaviti da je regularna na $\mathcal{B}$.

Stoga se može integrirati
što nam daje entropiju !

4-dimenzionalna opća relativnost

$$L(e,\omega) = \Big(*(e^a \wedge e^b) + c_H e^a \wedge e^b + c_E *R^{ab} + c_P R^{ab} \Big) \wedge R_{ab}$$

$$*R^{ab} = \tfrac{1}{2}\epsilon^{abcd} R_{cd}$$

$$S = \frac{2\pi}{\hbar} \oint_{\mathcal{B}} n^{cd} (*e_c \wedge e_d + c_H e_c \wedge e_d + 2c_E *R_{cd} + 2c_P R_{cd})$$

Raspravа

- Lorentz-Lieva derivacija ortogonalne baze isčezava --> Time smo uspjeli ostvariti da baza istovremeno bude invarijantna na difeomorfizam na bifurkacijskoj površini i da se „dobro” ponaša.

- Pokazali smo kako Lorentz-Lie Noetherin naboj daje entropiju te dali primjere.

- Ograničili smo se na Lagranžijane koji su Lorentzovi n-form skalari. Bilo bi zanimljivo proučiti situacije za drugačije Lagranžijane.

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