Slobodna ekspanzija anyona iz osnovnog stanja u potencijalu harmoničkog oscilatora

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Pregled seminara

1. Uvod u anyone
2. 2 anyona u potencijalu harmoničkog oscilatora
3. Eksperimentalni aspekti

1. Uvod

Uvod

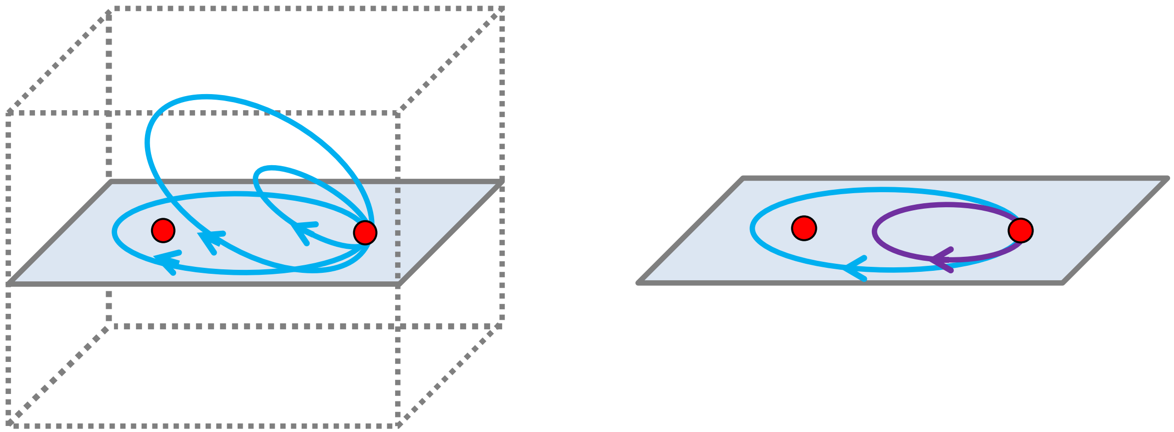
- anyoni: čestice koje nisu ni bozoni ni fermioni
    - bozoni – BE statistika, cjelobrojan spin
    - fermioni – FD statistika, polucjelobrojan spin
    - anyoni – frakcionalna statistika, spin
- mogući u dvodimenzionalnom svijetu
    - eksperimentalno: 2D sustavi, kvazičestice
- zanimljivi
    - fundamentalno (nova stanja materije)
    - tehnološki (kvantna računala)

Koja je razlika između dvije i tri dimenzije?

- 3D
  - simetrija valne funkcije s obzirom na zamjenu čestica: $$\psi(x_2, x_1) = e^{i\pi\alpha}\psi(x_1, x_2)$$
  - dvostruka zamjena
    → početna konfiguracija $$\implies \alpha = 0, 1$$
  - bozoni, fermioni, i svijet kakav poznajemo
- 2D?

Koja je razlika između dvije i tri dimenzije?

- dvostruka zamjena ≡ kruženje jedne čestice oko druge



  - 3D: putanja ekvivalentna točki $\psi \rightarrow \psi$
  - 2D: nemoguće svesti na točku $\psi \rightarrow e^{2\pi i\alpha}\psi$

Koja je razlika između dvije i tri dimenzije?

- 2D svijet ⇒ kontinuum čestica između B i F
  - proizvoljna faza kod zamjene
  - anyonska valna funkcija:

$$\psi(r, \varphi + 2\pi) = e^{2\pi i \alpha} \psi(r, \varphi)$$

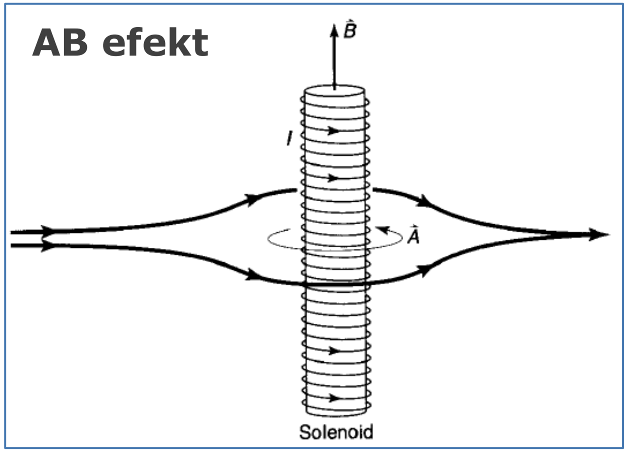
  - $\alpha \in [0,1]$ statistički parametar
  - frakcionalna statistika
  - frakcionalan spin

Aharonov-Bohm model anyona

F. Wilczek, Phys. Rev. Lett **49**, 957 (1982).

- zamjena anyona ↔ Aharonov-Bohm efekt
  - e⁻ u vektorskom potencijalu solenoida:

$$\boldsymbol{A} = \frac{\Phi}{2\pi r}\hat{\boldsymbol{\varphi}} \qquad \Rightarrow \exp\left(\frac{ie}{\hbar}\int \boldsymbol{A}\, d\boldsymbol{l}\right) = \exp\left(i\frac{e\Phi}{h}\Delta\varphi\right)$$



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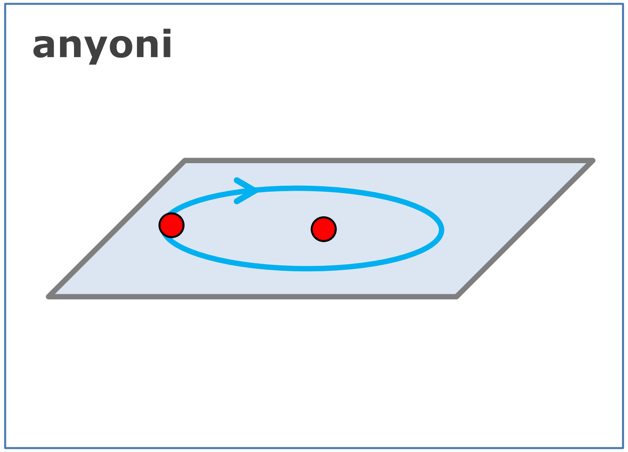


D. J. Griffiths, *introduction to Quantum Mechanics* (Prentice-Hall, 1995).

Aharonov-Bohm model anyona

- baždarna transformacija:

$$\boxed{\boldsymbol{A}' = \boldsymbol{A} - \nabla\lambda = 0} \quad \longrightarrow \quad \lambda = \frac{\Phi\varphi}{2\pi}$$

$$\psi'(r,\varphi) = e^{iq\lambda}\psi(r,\varphi)$$

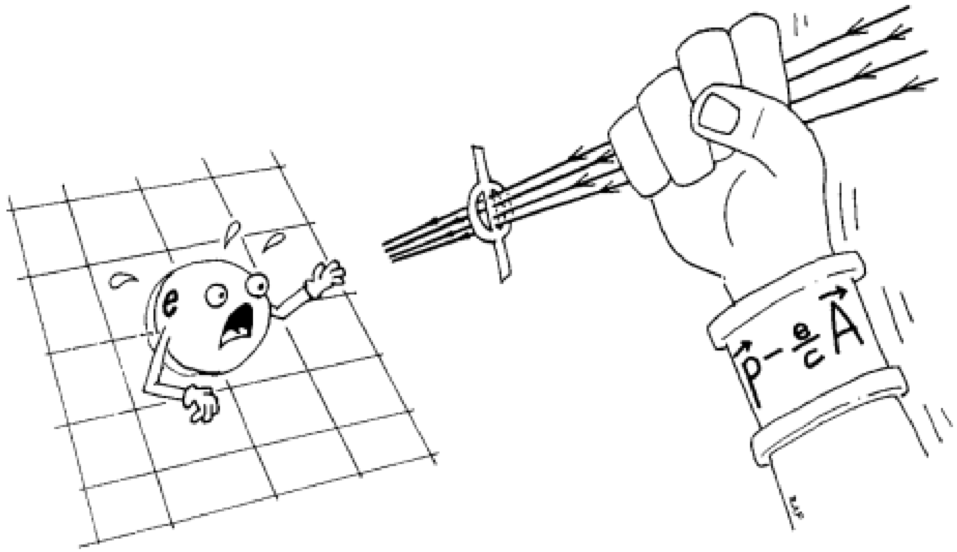
$$\boxed{\Longrightarrow \psi'(r,\varphi + 2\pi) = e^{iq\Phi}\psi'(r,\varphi)}$$

→ anyonska valna funkcija

- višeznačna $\psi \leftrightarrow$ vektorska interakcija

Aharonov-Bohm model anyona

- anyoni: naboj + magnetski tok
- anyonsko baždarenje
  - ψ višeznačna
  - $H_{kin.} = \frac{\boldsymbol{p}^2}{2m}$
- bozonsko baždarenje
  - $\psi' = e^{-i\alpha\varphi}\psi$ jednoznačna
  - $H'_{kin.} = \frac{(\boldsymbol{p} - q\boldsymbol{A})^2}{2m}$



A. Shapere, F. Wilczek, *Geometrical Phases in Physics* (World Scientific, 1989).

2. Problem 2 anyona u potencijalu harmoničkog oscilatora i slobodna ekspanzija

Anyoni u harmoničkom potencijalu

- Hamiltonijan sustava:

$$H = \frac{\boldsymbol{p}_1{}^2}{2m} + \frac{\boldsymbol{p}_2{}^2}{2m} + \frac{1}{2}m\omega^2\boldsymbol{r}_1{}^2 + \frac{1}{2}m\omega^2\boldsymbol{r}_2{}^2$$

- Transformacija u koordinate centra mase i relativnog pomaka ($\boldsymbol{R} = (R,\theta), \boldsymbol{r} = (r,\varphi)$)

$$H = \left(-\frac{\hbar^2}{2M}\nabla_{CM}{}^2 + \frac{1}{2}M\omega^2R^2\right) + \left(-\frac{\hbar^2}{2\mu}\nabla_{\mathrm{rel}}{}^2 + \frac{1}{2}\mu\omega^2r^2\right)$$

- Ansatz za valnu funkciju:

$$\psi(R,\theta,r,\varphi) = \Theta(R,\theta)\phi(r,\varphi)$$

Anyoni u harmoničkom potencijalu

- $\Theta(R,\theta)$ ne ovisi o statistici

$$\left(-\frac{\hbar^2}{2M}{\nabla_{CM}}^2 + \frac{1}{2}M\omega^2R^2\right)\Theta(R,\theta) = E_{CM}\Theta(R,\theta)$$

- Rješenje 2D harmoničkog oscilatora u polarnim koordinatama:

$$\boxed{\Theta_{N_cL_c}(R,\theta) = \mathcal{N} e^{-iL_c\theta} R^{|L_c|} e^{-\frac{M\omega}{2\hbar}R^2} L_{N_c}^{|L_c|}\left(\frac{M\omega R^2}{2\hbar}\right)}$$

$N_c = 0, 1, 2, \ldots$
$L_c = 0, \pm 1, \pm 2, \ldots$
$\mathcal{N}$ – normalizacija
$L_{N_c}^{L_c}(x)$ – generalizirani Laguerrov polinom

$$E_{CM} = \left(\frac{1}{2} + N_c + |L_c|\right)\hbar\omega$$

F. Wilczek, Phys. Rev. Lett **49**, 957 (1982).

Anyoni u harmoničkom potencijalu

- relativna koordinata: $\phi'(r,\varphi) = e^{-i\alpha\varphi}\phi(r,\varphi)$

$$-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\left(\frac{\partial}{\partial\varphi}+i\alpha\right)^2\right)\phi'+\frac{1}{2}\mu\omega^2r^2\phi' = E_{\mathrm{rel}}\phi'$$

- ansatz: $\phi' = \rho(r)e^{iL_c\varphi}$

$$-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}(L_c-\alpha)^2\right)\rho+\frac{1}{2}\mu\omega^2r^2\rho = E_\rho\rho$$

- 2D h.o. $(L_r \longrightarrow \mathrm{L_r} - \alpha)$

$$\boxed{\phi_{N_rL_r}(r,\varphi) = \mathcal{N}e^{-i(L_r-\alpha)\varphi}r^{|L_r-\alpha|}e^{-\frac{\mu\omega}{2\hbar}r^2}L_{N_r}^{|L_r-\alpha|}\left(\frac{\mu\omega r^2}{2\hbar}\right)}$$

$$E_{\mathrm{rel}} = \left(\frac{1}{2}+N_r+|L_r-\alpha|\right)\hbar\omega$$

Osnovno stanje

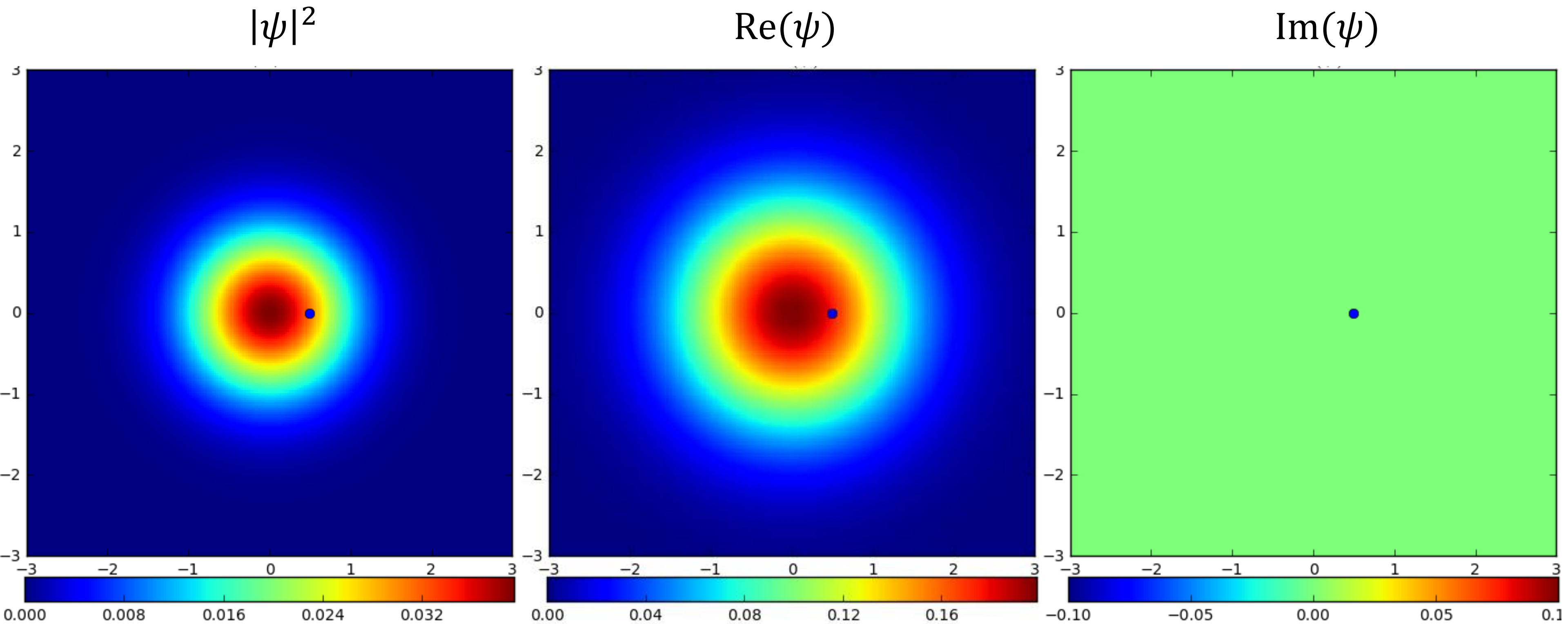
$$\psi_0(z_1, z_2) = \mathcal{N}(z_1 - z_2)^\alpha e^{-\frac{m\omega}{2\hbar}(|z_1|^2 + |z_2|^2)}$$

$$\boldsymbol{r}_1 \to z_1 = x_1 + iy_1$$

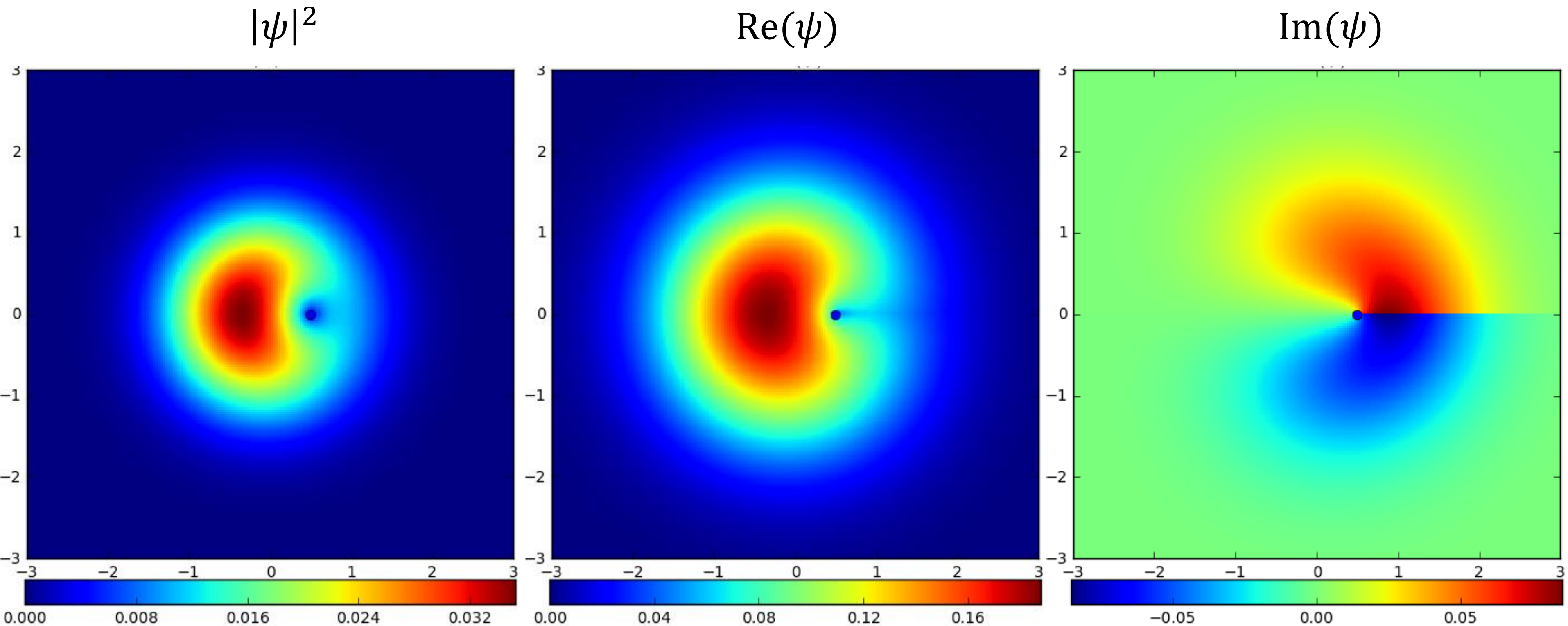
$$\boldsymbol{r}_2 \to z_2 = x_2 + iy_2$$

- nije umnožak jednočestičnih
- generalizirani Paulijev princip

Osnovno stanje, $\alpha = 0$ (bozoni)

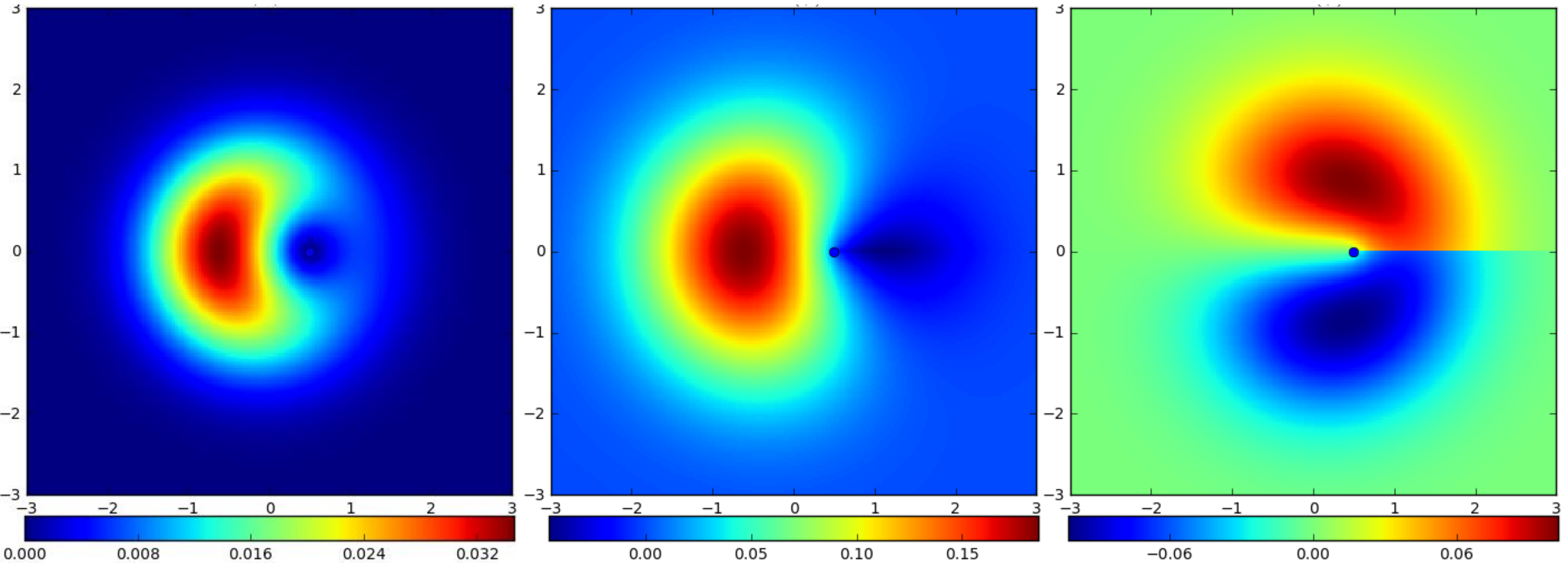


Osnovno stanje, $\alpha = 0.3$

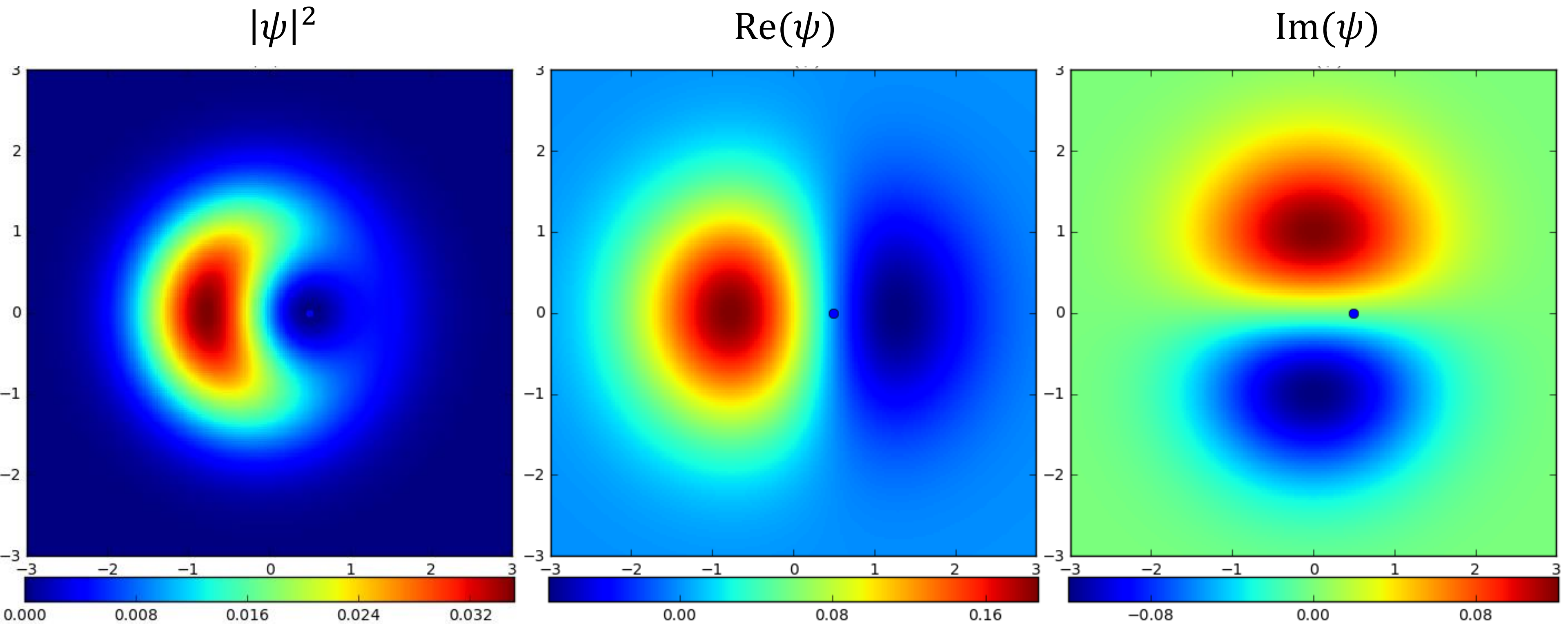


Osnovno stanje, $\alpha = 0.7$

$|\psi|^2$ $\mathrm{Re}(\psi)$ $\mathrm{Im}(\psi)$



Osnovno stanje, $\alpha = 1$ (fermioni)



Slobodna ekspanzija

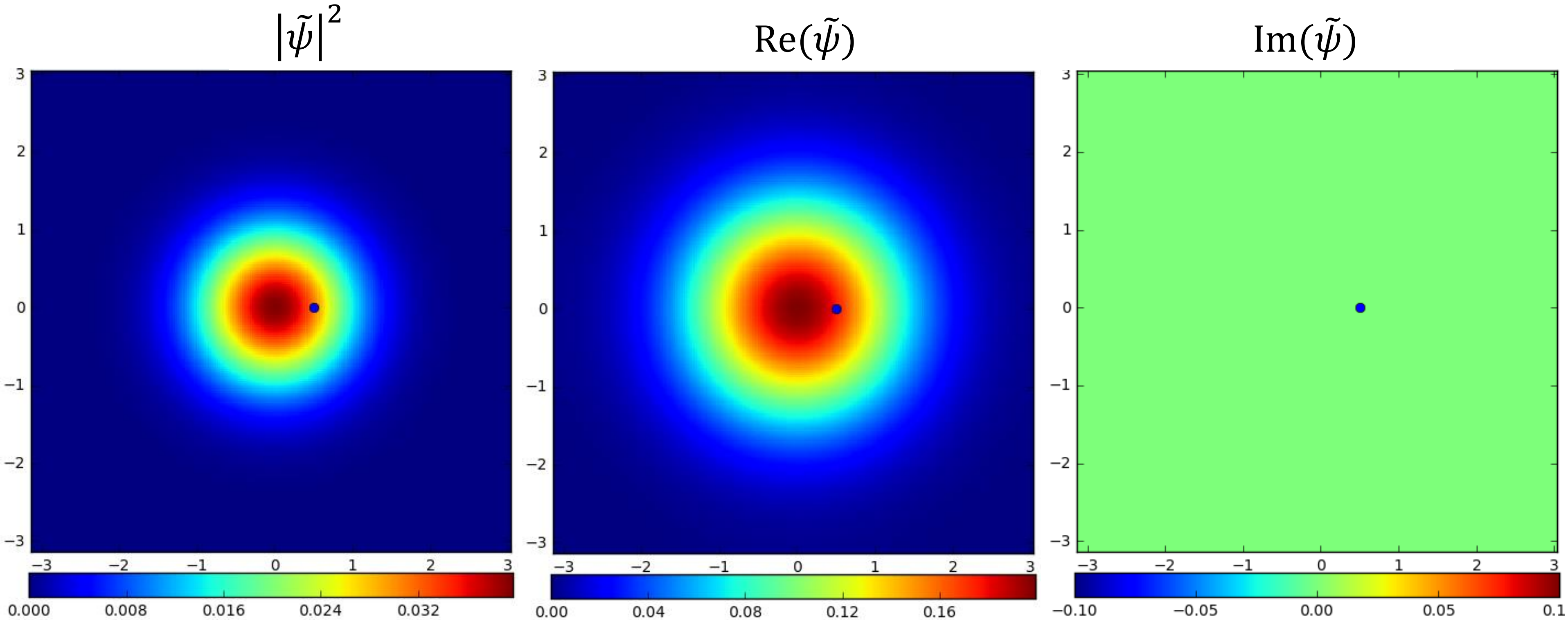
- u nekom trenutku $V_{\text{harm.}} = 0$
- motivacija:
  - eksperimentalna metoda *(time of flight)*
  - ideja: direktno mjerenje statistike α

- ponašanje valne funkcije u slobodnoj ekspanziji:

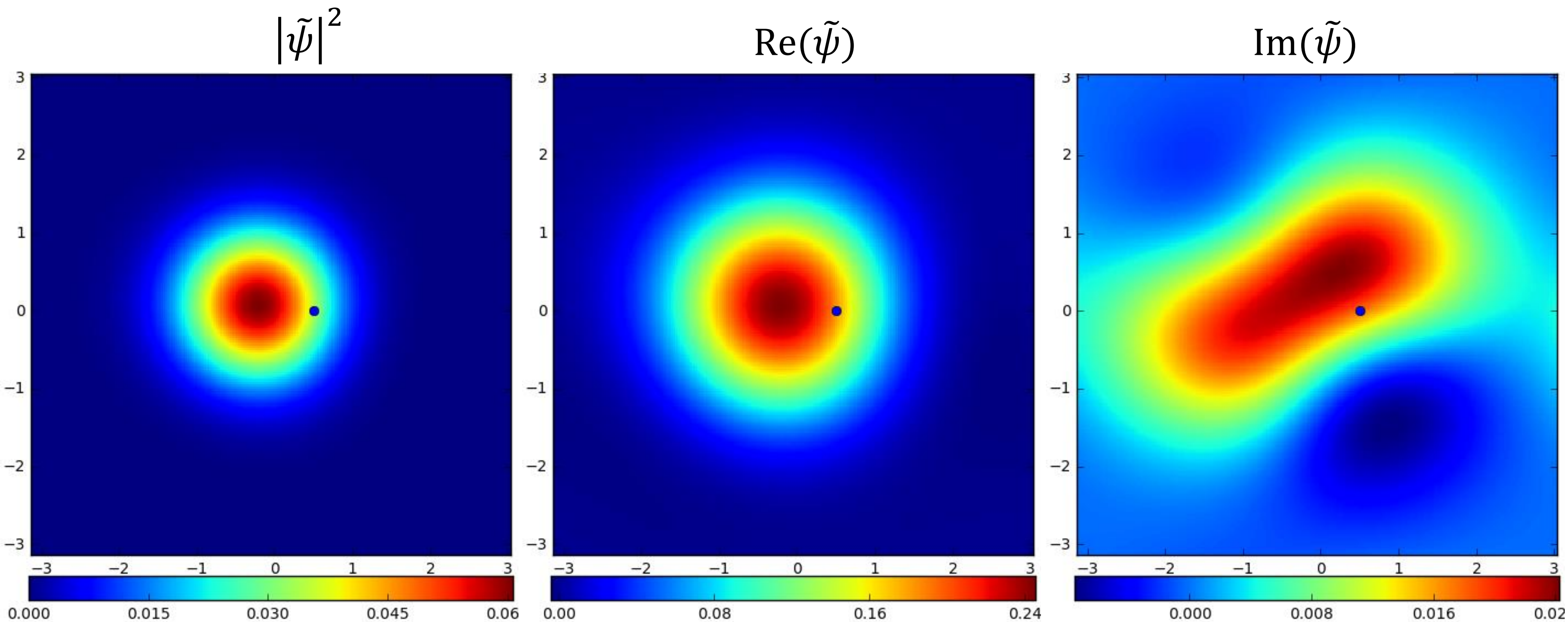
$$\boxed{\lim_{t\to\infty} |\psi(\boldsymbol{r}, t)|^2 \propto \left| \tilde{\psi}_{\boldsymbol{0}}(\boldsymbol{M}\boldsymbol{r}/\hbar t) \right|^{\boldsymbol{2}}}$$

- $\left|\tilde{\psi}_{\boldsymbol{0}}(\boldsymbol{k}_1, \boldsymbol{k}_2)\right|^{\boldsymbol{2}}$ numerički

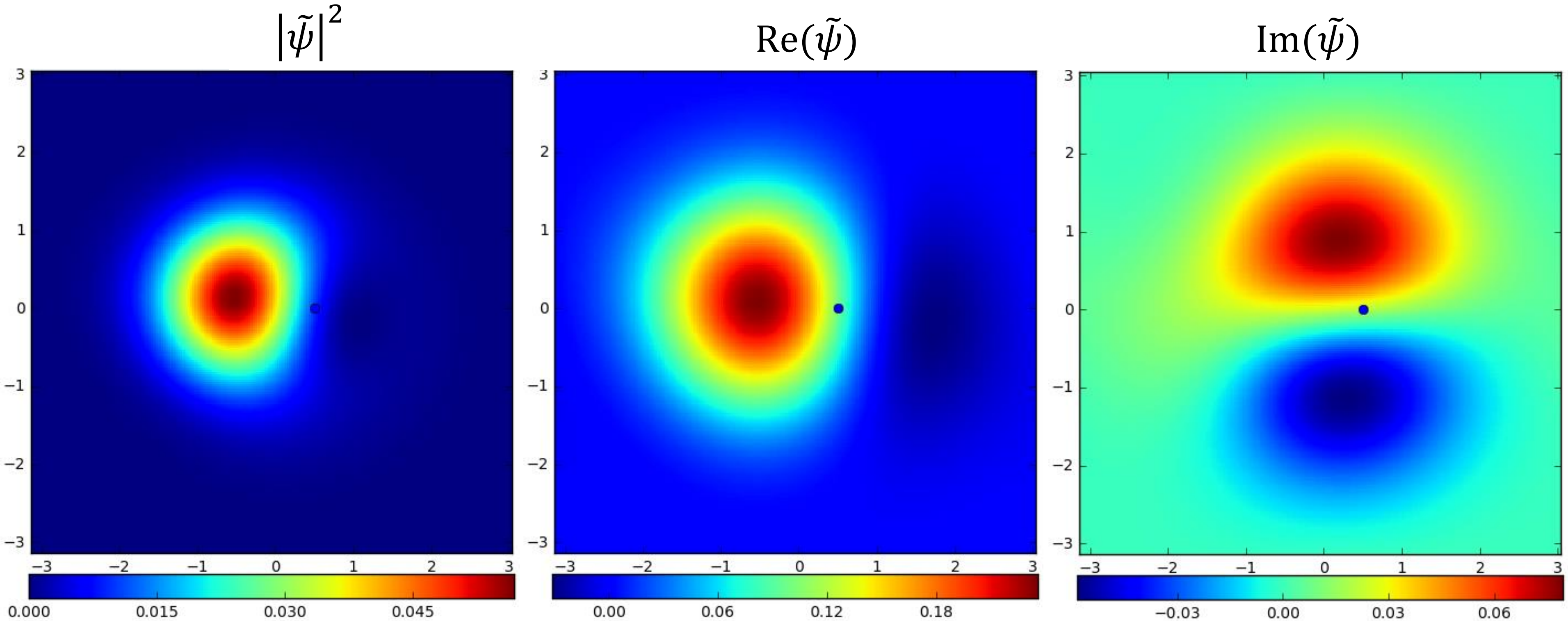
Slobodna ekspanzija, $\alpha = 0$ (bozoni)



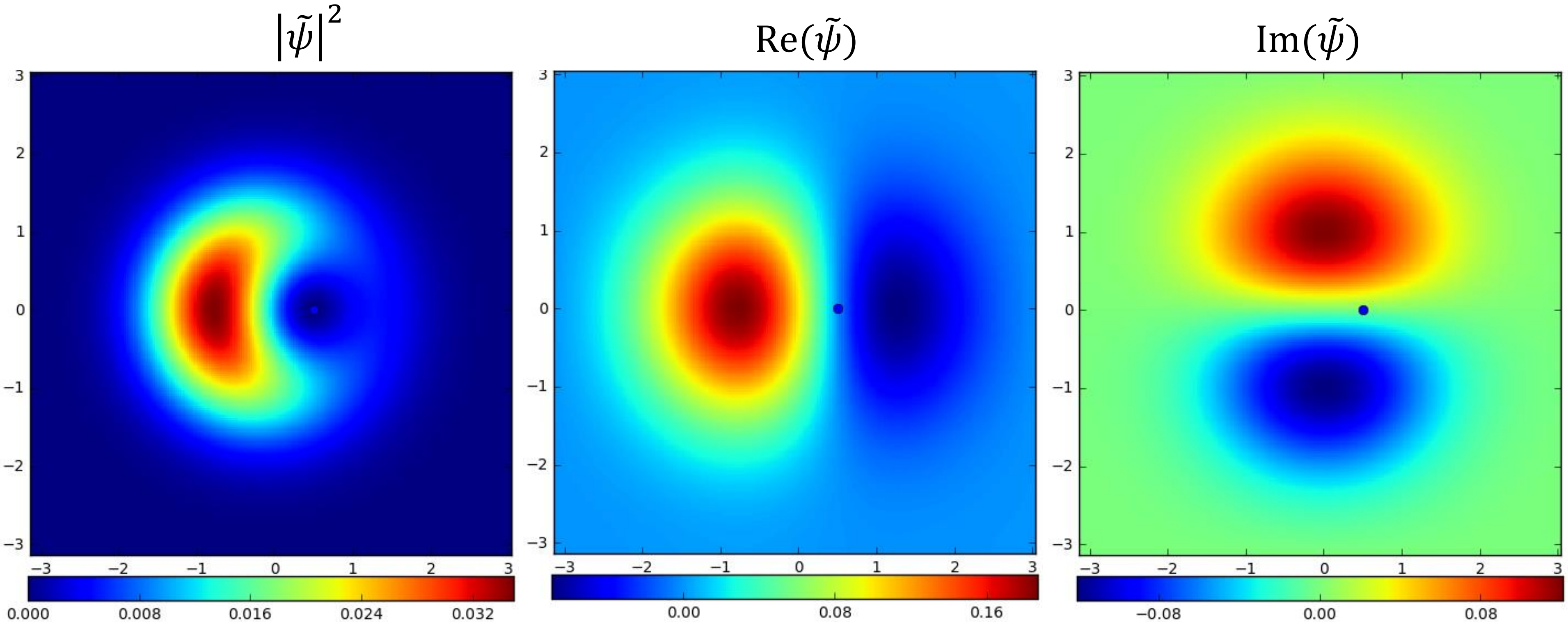
Slobodna ekspanzija, $\alpha = 0.3$



Slobodna ekspanzija, $\alpha = 0.7$

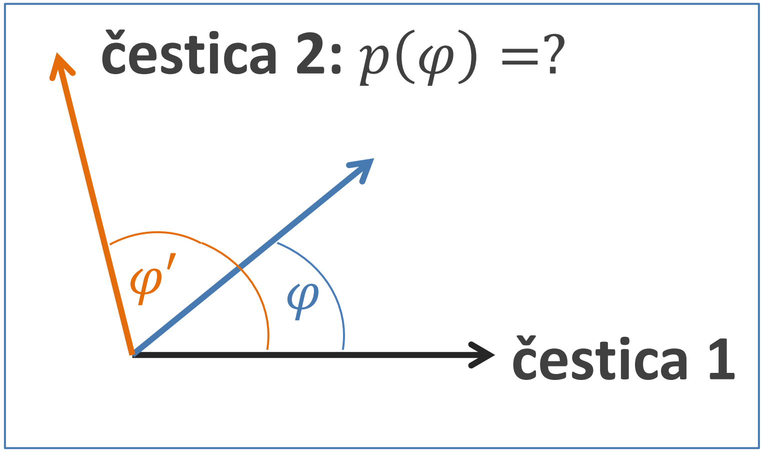


Slobodna ekspanzija, $\alpha = 1$ (fermioni)



Korelacije položaja

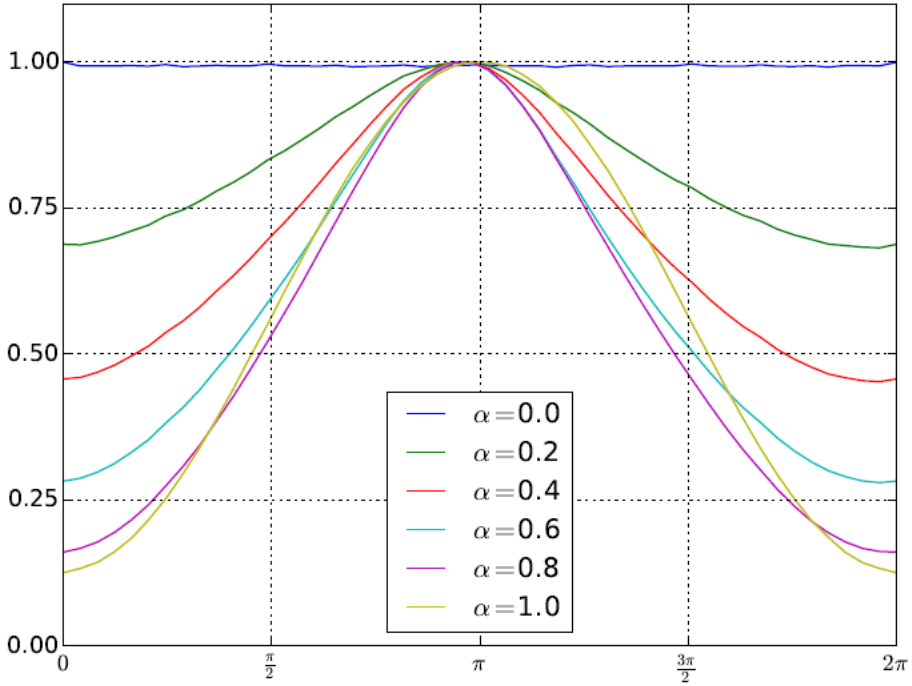
- položaj jedne čestice poznat
  - vjerojatnost nalaženja druge čestice
  - u $\boldsymbol{k}$-prostoru: ako se jedna čestica giba u jednom smjeru, kamo će se gibati druga čestica



- usrednjavanje $\left|\tilde{\psi}_{\mathbf{0}}(\boldsymbol{k}_1, \boldsymbol{k}_2)\right|^2$ po k_2:

$$p(\varphi) = \int_0^\infty \left|\tilde{\psi}_{\mathbf{0}}\left(\boldsymbol{k}_1 = (0.5,0), \boldsymbol{k}_2 = k_2 e^{i\varphi}\right)\right|^2 dk_2$$

Korelacije položaja



- kontinuirani prijelaz B – F
- anyoni: devijacija od B/F krivulja ⟷ statistički parametar
- mogućnost detekcije anyona

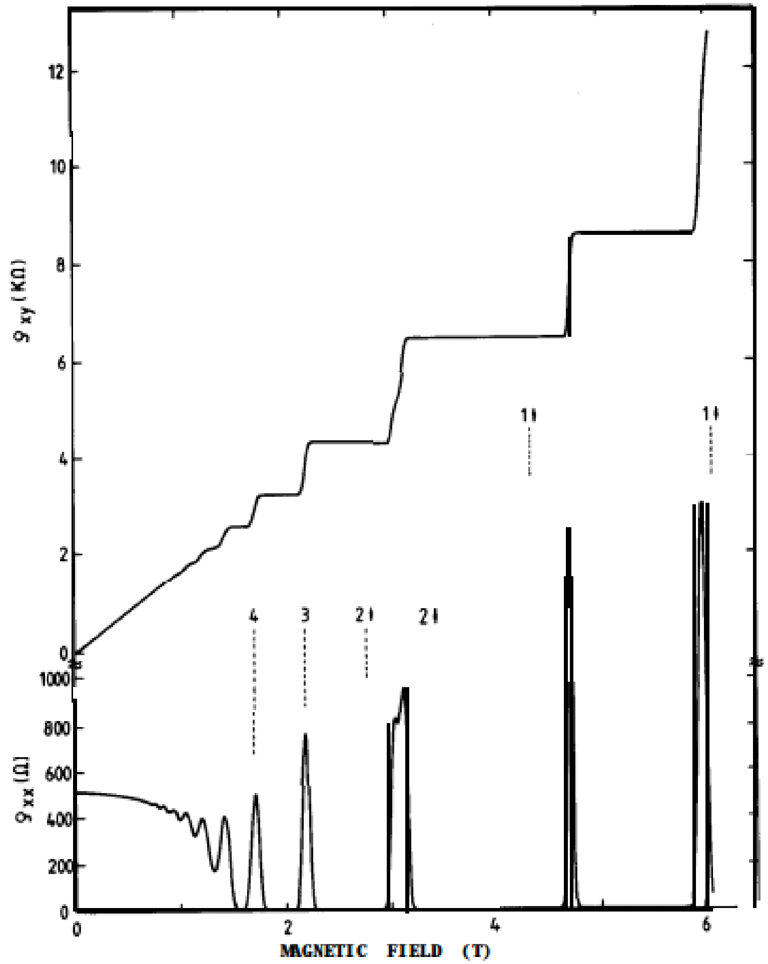
3. Eksperimentalne mogućnosti

racionalni kvantni Hallov efekt

ultrahladni atomi

Kvantni Hallov efekt (QHE)

- klasični Hallov efekt
  - $R_H = \frac{B}{n_e e}$
- kvantni Hallov efekt
  - $T < 4\,\mathrm{K}, \;\; B \sim 5 - 10\,\mathrm{T}$
  - kvantizacija: $R_H = \frac{1}{\nu}\frac{h}{e^2}$ $\nu = 1, 2, 3, \ldots$
- objašnjenje: e^- u magnetskom polju
  - Landau nivoi
  - $E = \hbar\omega_c\left(\frac{1}{2} + n\right), \omega_c = \frac{eB}{m}$



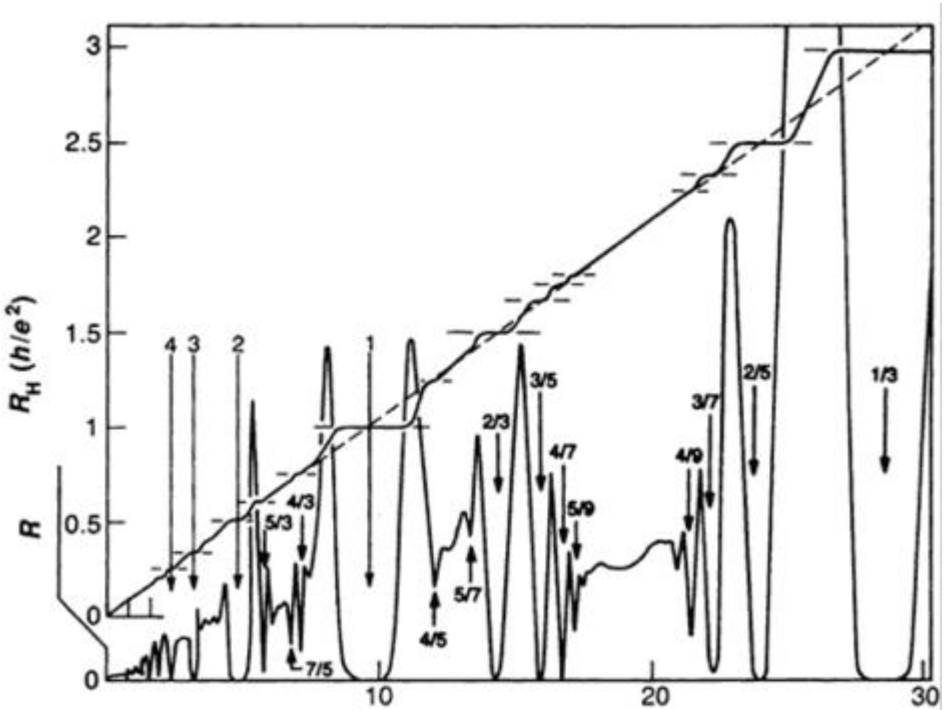
K. Klitzing, Nobel Lecture: The Quantized Hall effect (1985).

Racionalni kvantni Hallov efekt (FQHE)

- još veća polja:
  - $\nu = p/q$
  - višečestični fenomen
- Laughlinovo stanje:

$$\psi = \prod_{i>j} (\mathrm{z_i} - \mathrm{z_j})^{\mathrm{2p+1}} \exp\left(-\sum_i \frac{|z_i|^2}{4}\right)$$

  - dobar opis stanja $\nu = 1/(2p+1)$
  - pobuđenja: frakcionalni naboj i statistika $\Rightarrow$ anyoni



http://www.nobelprize.org/nobel_prizes/physics/laureates/1998/press.html

Prijedlog realizacije u ultrahladnim atomskim sustavima

B. Paredes *et al.*, Phys. Rev. Lett **87**, 010402 (2001).

- ultrahladni atomi:
  - interakcije modeliraju laseri
  - vrlo velika kontrola
  - simulator čvrstostanjskih sustava

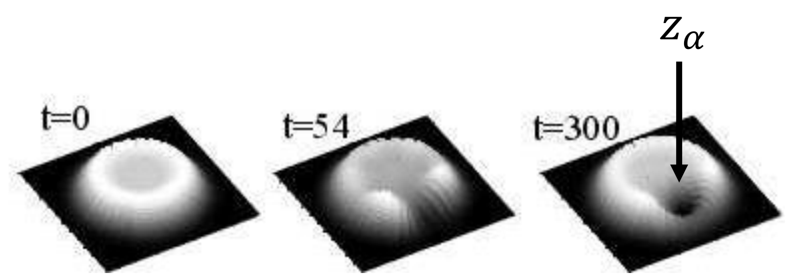
- rotirajući BEC + kontaktna interakcija:
  - Hamiltonijan analogan FQHE Hamiltonijanu
    - rotacija → magnetsko polje
    - kontaktna int. → kulonska int.

Prijedlog realizacije u ultrahladnim atomskim sustavima

- ansatz za osnovno stanje:

$$\psi_0 = \prod_{i>j} (z_i - z_j)^2 \exp\left(-\sum_i \frac{|z_i|^2}{4}\right)$$

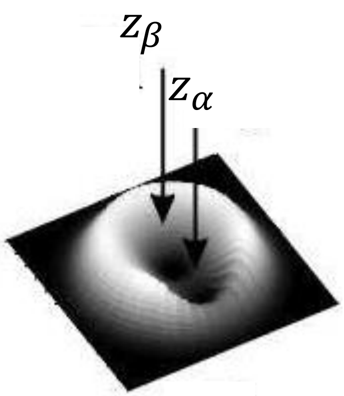
- kvazičestica 1:
  - laser $V_\alpha = V_0(t) \sum_i \delta(z_i - z_\alpha)$
  - $\psi_{z_\alpha} = \prod_i (z_i - z_\alpha) \psi_0$
  - $\psi_0 \rightarrow \psi_{z_\alpha}$
  - anyon



B. Paredes *et al.*, Phys. Rev. Lett **87**, 010402 (2001).

Prijedlog realizacije u ultrahladnim atomskim sustavima

- + kvazičestica 2
  - $\psi_{z_\alpha,z_\beta} = \prod_i (z_i - z_\beta)(z_i - z_\alpha)\psi_0$
- V_1 se naglo isključi:
  - $\Psi \rightarrow \psi_{z_\alpha} + \psi_{z_\alpha,z_\beta}$
- adijabatsko kruženje lasera V_0
  - $\Psi' \rightarrow \psi_{z_\alpha} - \psi_{z_\alpha,z_\beta}$
- V_1 se opet uključi $\Longrightarrow$ drugačija evolucija
- tehnološka ograničenja
  - eksperimentalno još neostvareno



B. Paredes *et al*., Phys. Rev. Lett **87**, 010402 (2001).

Zaključak

- proučavana su svojstva anyona
- analiziran je problem 2 anyona u potencijalu harmoničkog oscilatora
- ispitana je slobodna ekspanzija i mogućnost direktne detekcije statistike
- eksperimentalne mogućnosti

- dalje
  - proširenje analize na 3 i N čestica

Hvala na pažnji!