Sintetički vektorski potencijal zavojnice za ultrahladne atome

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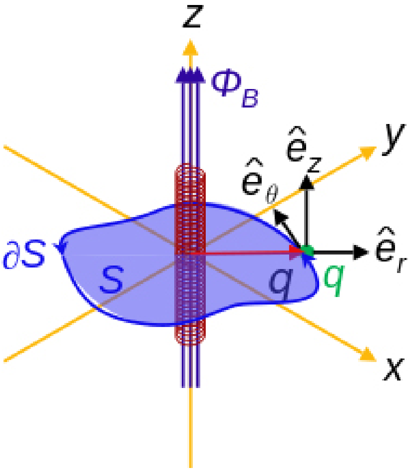
$$\boldsymbol{A} = \frac{\Phi}{2\pi}\frac{\hat{\boldsymbol{\varphi}}}{r}$$

$$\boldsymbol{B} = \frac{\Phi}{2\pi}\frac{\delta(r)}{r}\hat{\boldsymbol{z}}$$

$$\Delta\phi = \frac{1}{\hbar}\oint_{\mathcal{C}} q\boldsymbol{A}\cdot \mathrm{d}\boldsymbol{r}$$

Ideja: U sustav ultrahladnih atomskih plinova u xy ravnini upisati sintetički vektorski potencijal

$$\boldsymbol{\mathcal{A}} \sim \frac{\hat{\boldsymbol{\varphi}}}{r}.$$



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- Geometrijske faze
- Adijabatska evolucija obučenih stanja
- Sustav s dva nivoa
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- Rabijeva frekvencija i detuning
- Sintetički vektorski potencijal zavojnice za ultrahladne atome

Geometrijske faze

Sustav koji ovisi o vanjskim parametrima $\boldsymbol{\lambda}$ koji evoluiraju **adijabatski**

$$\boldsymbol{\lambda}(0) \to \boldsymbol{\lambda}(t) \to \boldsymbol{\lambda}(T) = \boldsymbol{\lambda}(0)$$

Schrödingerova jednadžba

$$\hat{H}(\boldsymbol{\lambda}) \left|\psi_n(\boldsymbol{\lambda})\right\rangle = E_n(\boldsymbol{\lambda}) \left|\psi_n(\boldsymbol{\lambda})\right\rangle$$

$\{\left|\psi_n(\boldsymbol{\lambda})\right\rangle\}$ je ortogonalna baza

$$\left|\psi(t)\right\rangle = \sum_n c_n(t) \left|\psi_n[\boldsymbol{\lambda}(t)]\right\rangle$$

Geometrijske faze

Sustav je početno u stanju $|\psi_l\rangle$

$$c_l(0) = 1, \;\; c_n(0) = 0, \;\; \forall n \neq l$$

Evolucija koeficijenata

$$\begin{aligned} i\hbar \dot{c}_l &= [E_l(t) - i\hbar \dot{\boldsymbol{\lambda}} \cdot \langle \psi_l | \boldsymbol{\nabla} \psi_l \rangle] c_l \\ &= [E_l - \dot{\boldsymbol{\lambda}} \cdot \boldsymbol{\mathcal{A}}_l(\boldsymbol{\lambda})] c_l \end{aligned}$$

Berryjeva veza

$$\boldsymbol{\mathcal{A}}_l(\boldsymbol{\lambda}) = i\hbar \, \langle \psi_l | \boldsymbol{\nabla} \psi_l \rangle$$

Geometrijske faze

Pretp. da $\boldsymbol{\lambda}$ evoluira po zatvorenoj krivulji t. da $\boldsymbol{\lambda}(T) = \boldsymbol{\lambda}(0)$.

Koeficijenti

$$c_l(T) = e^{i\Phi^{dyn.}(T)} e^{i\Phi^{geom.}(T)} c_l(0)$$

Dinamička faza

$$\Phi^{dyn.}(T) = -\frac{1}{\hbar} \int_0^T E_l(t)\,\mathrm{d}t$$

Geometrijska faza ili **Berryjeva faza**

$$\Phi^{geom.} = \frac{1}{\hbar} \int_0^T \dot{\boldsymbol{\lambda}} \cdot \boldsymbol{\mathcal{A}}_l(\boldsymbol{\lambda})\,\mathrm{d}t = \frac{1}{\hbar} \oint \boldsymbol{\mathcal{A}}_l(\boldsymbol{\lambda}) \cdot \mathrm{d}\boldsymbol{\lambda}$$

Geometrijske faze

$$\Phi^{geom.} = \frac{1}{\hbar} \oint \boldsymbol{\mathcal{A}}_l(\boldsymbol{\lambda}) \cdot \mathrm{d}\boldsymbol{\lambda}$$

Berryjeva zakrivljenost

$$\boldsymbol{\mathcal{B}}_l = \boldsymbol{\nabla} \times \boldsymbol{\mathcal{A}}_l$$

$$\Phi^{geom.}(T) = \frac{1}{\hbar} \iint_S \boldsymbol{\mathcal{B}}_l \cdot \mathrm{d}^2 S$$

Adijabatska evolucija obučenih stanja

Ultrahladni atomi ($T \lesssim \mu K$); dva stupnja slobode

$$\hat{H}_{tot} = \frac{\hat{\boldsymbol{p}}^2}{2M} + \hat{H}_{int}(\hat{\boldsymbol{r}})$$

Obučena stanja

$$\hat{H}_{int}(\boldsymbol{r}) \left|\psi_n(\boldsymbol{r})\right\rangle = E_n(\boldsymbol{r}) \left|\psi_n(\boldsymbol{r})\right\rangle$$

$\{|\psi_n(\boldsymbol{r})\rangle\}$ čini bazu

$$\Psi(\boldsymbol{r}, t) = \sum_n \phi_n(\boldsymbol{r}, t) \left|\psi_n(\boldsymbol{r})\right\rangle$$

Adijabatska evolucija obučenih stanja

Schrödingerova jednadžba

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}_{tot}\Psi(\boldsymbol{r},t) = \left(-\frac{\hbar^2}{2M} + \hat{H}_{int}(\boldsymbol{r})\right)\Psi(\boldsymbol{r},t)$$

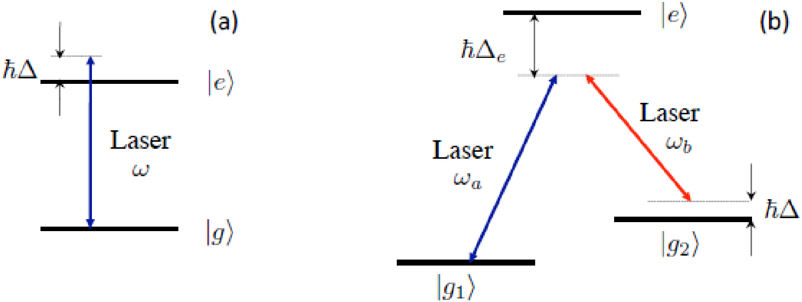
Jednadžba za amplitudu vjerojatnosti

$$i\hbar\frac{\partial\psi_l}{\partial t} = \left[\frac{(\hat{\boldsymbol{p}} - \boldsymbol{\mathcal{A}}_l(\boldsymbol{r}))^2}{2M} + E_l(\boldsymbol{r}) + \mathcal{V}_l(\boldsymbol{r})\right]\psi_l(\boldsymbol{r},t)$$

Berryjeva veza

$$\boldsymbol{\mathcal{A}}_l(\boldsymbol{r}) = i\hbar\,\langle\psi_l|\boldsymbol{\nabla}\psi_l\rangle$$

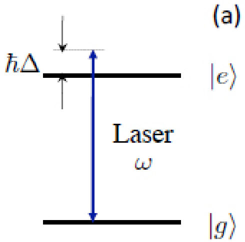
Sustav s dva nivoa

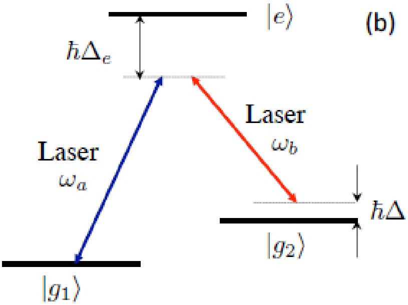


Slika: (a) Atom s dva nivoa. (b) Ramanov prijelaz između dva podnivoa osnovnog stanja atoma.

Sustav s **2 nivoa**: magnezij, stroncij, iterbij
Sustav s **3 nivoa**; svodi se na sustav s 2 nivoa: alkalijski metali, erbij, disprozij

Sustav s dva nivoa





Detuning

$$\Delta = \omega_L - \omega_0$$

Jednofotonske Rabijeve frekvencije za $g_j \leftrightarrow e$

$$\kappa_j = \frac{\boldsymbol{d} \cdot \boldsymbol{E}_j}{\hbar}$$

Rabijeva frekvencija

$$\kappa = \frac{\kappa_a \kappa_b^*}{2\Delta_e}$$

Unutarnji hamiltonijan

$$\hat{H}_{int} = \frac{\hbar}{2}\begin{pmatrix} \Delta & \kappa^* \\ \kappa & -\Delta \end{pmatrix}$$

Poopćena Rabijeva frekvencija Ω, **kut miješanja** θ i **fazni kut** ϕ

$$\Omega = (\Delta^2 + |\kappa|^2)^{1/2}, \ \cos\theta = \frac{\Delta}{\Omega}, \ \kappa = |\kappa| e^{i\phi}$$

Unutarnji hamiltonijan nakon promjene varijabli

$$\hat{H}_{int} = \frac{\hbar\Omega}{2}\begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix} = \frac{\hbar\Omega}{2}\boldsymbol{n}\cdot\hat{\boldsymbol{\sigma}}$$

Sustav s dva nivoa

Svojstvene vrijednosti $\pm\hbar\Omega/2$

Svojstvena stanja

$$|\psi_+\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix}, \quad |\psi_-\rangle = \begin{pmatrix} -e^{-i\phi}\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix}$$

Berryjeva veza i zakrivljenost

$$\boldsymbol{\mathcal{A}}_\pm = \pm\frac{\hbar}{2}(\cos\theta - 1)\boldsymbol{\nabla}\phi$$

$$\boldsymbol{\mathcal{B}}_\pm = \pm\frac{\hbar}{2}\boldsymbol{\nabla}(\cos\theta) \times \boldsymbol{\nabla}\phi$$

Možemo pogoditi rješenje

$$\cos\theta = \text{const.},\ \phi = \varphi \implies \boldsymbol{\mathcal{A}} \sim \frac{\hat{\boldsymbol{\varphi}}}{r}$$

Valna jednadžba

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)u(\boldsymbol{r}, z, t) = 0$$

Separacija varijabli

$$u(\boldsymbol{r}, z, t) = R(\boldsymbol{r}, z)e^{-i\omega t}$$

$$(\nabla^2 - k^2)R(\boldsymbol{r}, z) = 0$$

$$k = \omega/c$$

Još malo separacije

$$R(\boldsymbol{r}, z) = U(\boldsymbol{r}, z)e^{ikz}$$

Paraksijalna aproksimacija

$$\partial^2 U/\partial z^2 \ll k\partial U/\partial z$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik\frac{\partial}{\partial z}\right)U(\boldsymbol{r}, z) = 0$$

I još malo separacije

$$U(\boldsymbol{r}, z) = Z(z)F(u, v)\mathsf{GB}(r, q)$$

$$\boldsymbol{\rho} \equiv (u, v) = (x, y)/\chi(z)$$

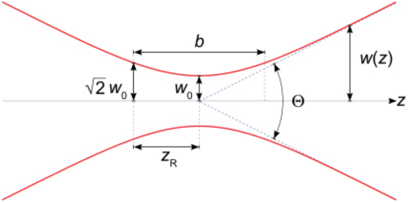
Laserske zrake s vrtlozima

GB(r, q) je gausijan

$$\mathrm{GB}(r,q) = \frac{q_0}{q_0 + z} \exp\left(\frac{ikr^2}{2q(z)}\right)$$

$$q = q_0 + z$$

Struk zrake $w_0 = w(0)$,
$q_0 = i\pi w_0^2/\lambda$



$$[\tilde{\nabla}^2 - i\boldsymbol{\rho} \cdot \tilde{\boldsymbol{\nabla}} - \gamma - i]F = 0$$

$$\frac{\partial}{\partial z}\left(\frac{\chi^2}{q^2}\right) = \frac{1}{kq^2}$$

$$\frac{\partial Z}{\partial z} = \frac{i\gamma - 1}{2k\chi^2}$$

Rješenja

$$\frac{1}{\chi^2(z)} = k\left(\frac{1}{\tilde{q}} - \frac{1}{q}\right)$$

$$Z(z) = \left(\frac{\tilde{q}q_0}{q\tilde{q}_0}\right)^{i\gamma/2 - 1/2}$$

$F(\rho) = F(u, v)$ napišemo u drugom obliku

$$F(\rho) = G(\rho) e^{im\varphi} e^{i\rho^2/4}/\rho$$

Promjenom varijable $\varrho = i\rho^2/2$, dobivamo kanonsku formu Whittakerove diferencijalne jednadžbe

$$\left(\frac{\partial^2}{\partial \varrho^2} - \frac{1}{4} + \frac{i\gamma/2}{\varrho} + \frac{1/4 - (m/2)^2}{\varrho^2} \right) G(\varrho) = 0$$

Rješenje

$$U(\boldsymbol{r}, z) = \mathsf{CiB}_\gamma^m(\boldsymbol{r}; q, \tilde{q}) = \left(\frac{\tilde{q} q_0}{q \tilde{q}_0} \right)^{i\gamma/2} \left(\frac{ir^2}{2\chi^2} \right)^{-1/2}$$
$$\times M_{i\gamma/2, m/2} \left(\frac{ir^2}{2\chi^2} \right) e^{im\varphi}$$
$$\times \left[\mathsf{GB}(r, q) \, GB(r, \tilde{q}) \right]^{1/2}$$

Konfluentna hipergeometrijska funkcija

$$\mathsf{CiB}_\gamma^m(\boldsymbol{r};q,\tilde{q}) = \left(\frac{\tilde{q}q_0}{q\tilde{q_0}}\right)^{i\gamma/2-1/2}\left(\frac{ir^2}{2\chi^2}\right)^{m/2} \times {}_1F_1\left(\frac{1+m-i\gamma}{2}, m+1; \frac{ir^2}{2\chi^2}\right) \times \mathsf{GB}(r,q)e^{im\varphi}$$

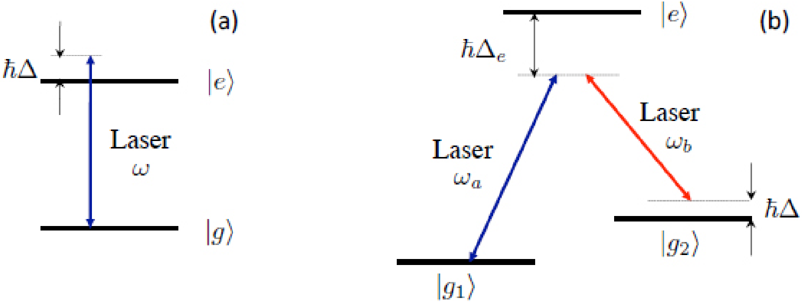
Hipergeometrijski gausovski modovi za $1/\chi^2 \to \infty$ kada $z \to 0$

$$\mathsf{HyGG}_p^m(\rho,\varphi) = \mathsf{CiB}^m_{-i(p+m+1)}(\boldsymbol{r};q_0,0) = C_{pm}r^{|m|+p}e^{-\rho^2}e^{im\varphi}$$

HyGG s $p=-|m|$= faza $e^{im\varphi}$ na TEM_{00} laserski mod = **optički vrtlozi**

$$\boldsymbol{E} = E_0 e^{-r^2/w_0^2}e^{im\varphi}e^{ikz-i\omega t}\hat{\boldsymbol{x}}$$

Rabijeva frekvencija i detuning



Laser ω_a u $+z$ smjeru

$$\boldsymbol{E}_a = E_0 e^{-r^2/w_0^2} e^{im\varphi} e^{ikz-i\omega t} \hat{\boldsymbol{x}}$$

Laser ω_b u $-z$ smjeru

$$\boldsymbol{E}_b = E_0 e^{-r^2/w_0^2} e^{-im\varphi} e^{-ikz-i\omega t} \hat{\boldsymbol{x}}$$

Rabijeva frekvencija i detuning

Rabijeva frekvencija

$$\kappa = \kappa_0 e^{2im\varphi + 2ikz} e^{-2r^2/w_0^2}$$

Faza

$$\phi = 2kz + 2m\varphi$$

Gradijent

$$\nabla\phi = 2k\hat{\boldsymbol{z}} + 2m\frac{\hat{\boldsymbol{\varphi}}}{r}$$

Kut miješanja

$$\cos\theta = \frac{\Delta}{(\Delta^2 + \kappa_0^2 e^{-4r^2/w_0^2})^{1/2}}$$

$$\cos\theta = \text{const., za } w_0 \gg r \text{ i } \Delta = \text{const.}$$

Sintetički vektorski potencijal zavojnice za ultrahladne atome

$$\boldsymbol{\mathcal{A}}' = \frac{\hbar}{2}\left[\frac{\Delta}{(\Delta^2 + \kappa_0^2 e^{-4r^2/w_0^2})^{1/2}} - 1\right](2m\frac{\hat{\boldsymbol{\varphi}}}{r} + 2k\hat{\boldsymbol{z}})$$

Napravimo baždarnu transformaciju $\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{A}}' - 2k\hat{\boldsymbol{z}}$

$$\boldsymbol{\mathcal{A}} = \frac{\hbar}{2}\left[\frac{\Delta}{(\Delta^2 + \kappa_0^2 e^{-4r^2/w_0^2})^{1/2}} - 1\right]2m\frac{\hat{\boldsymbol{\varphi}}}{r}$$

Definiramo **sintetički magnetski tok**

$$\Sigma = 2m\pi\hbar\left(\frac{\Delta}{(\Delta^2 + \kappa_0^2 e^{-4r^2/w_0^2})^{1/2}} - 1\right)$$

Sintetički vektorski potencijal zavojnice za ultrahladne atome

I dobivamo **sintetički vektorski potencijal**

$$\boldsymbol{\mathcal{A}} = \frac{\Sigma}{2\pi} \frac{\hat{\boldsymbol{\varphi}}}{r}$$

Ključna riječ: **anyon**

Kraj. Hvala na pažnji!