Određivanje totalnog udarnog presjeka procesa $e^-e^+ \to$ hadroni do prvog reda u QCD konstanti vezanja

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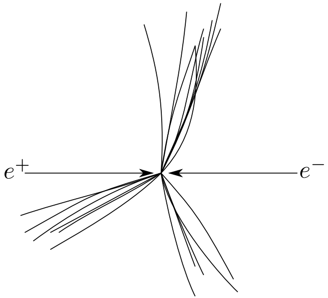
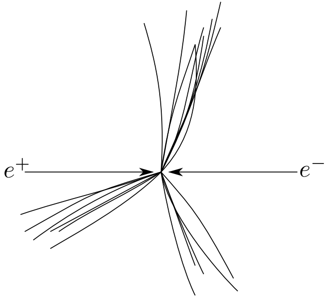
IRB, Zagreb

Proces $e^- e^+ \to hadroni$



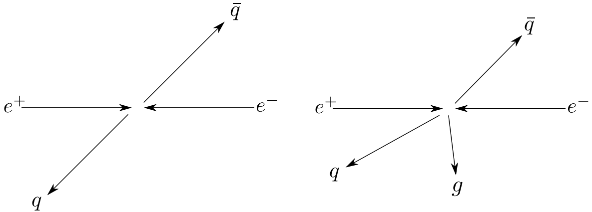
- Sudaramo elektron i pozitron, zanima nas totalni udarni presjek σ za nastajanje hadrona, neovisno o vrstama, brzinama ili kvantnim brojevima.
- Do $\mathcal{O}(\alpha_s)$, uključujući, u URL limesu (mase zanemarujemo).

Proces $e^-e^+ \rightarrow hadroni$



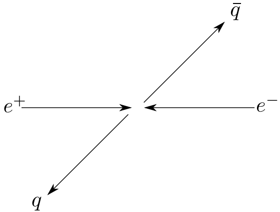
- Hadroni se u eksperimentu pojavljuju u *jet*-ovima $\rightarrow$ pokazatelj postojanja realnih kvarkova i gluona (partona).
- Dovoljno je promatrati proces $e^-e^+ \rightarrow partoni$, jer je vjerojatnost hadronizacije vrlo velika.

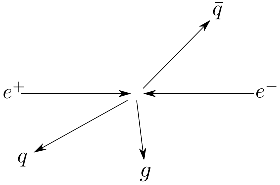
Proces $e^-e^+ \to hadroni$



- Proces opisujemo kvantnom teorijom polja.
- Radimo **račun smetnje** - baza su slobodne čestice, smetnja je interakcija.
- Rezultat **razvijamo po QCD konstanti vezanja** α_s.

Proces $e^-e^+ \rightarrow hadroni$





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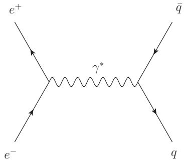
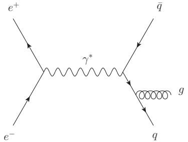
- U redu do vodećeg, $\mathcal{O}(\alpha_s)$, pojavljuju se dva procesa: $e^-e^+ \rightarrow q\bar{q}$ i $e^-e^+ \rightarrow q\bar{q}g$, za koje **odvojeno** računamo totalni udarni presjek.

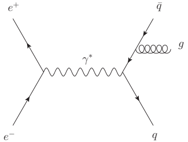
Feynmanovi dijagrami za $e^-e^+ \to$ *partoni* do $\mathcal{O}(\alpha_s)$



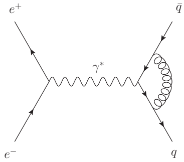
(a) $\tilde{A}_0$



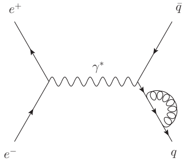
(b) $\tilde{A}_{\mathrm{R}}$



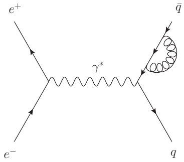
(c) $\tilde{B}_{\mathrm{R}}$



(d) $\tilde{A}_{\mathrm{V}}$



(e) $\tilde{B}_{\mathrm{V}}$



(f) $\tilde{C}_{\mathrm{V}}$

$$\sigma_{q\bar{q}} \propto \left|\mathcal{M}_{e^-e^+\to q\bar{q}}\right|^2 = \left|\tilde{A}_0 + \tilde{A}_{\mathrm{V}} + \tilde{B}_{\mathrm{V}} + \tilde{C}_{\mathrm{V}}\right|^2 + \mathcal{O}(\alpha_s^2)$$

$$\sigma_{q\bar{q}g} \propto \left|\mathcal{M}_{e^-e^+\to q\bar{q}g}\right|^2 = \left|\tilde{A}_{\mathrm{R}} + \tilde{B}_{\mathrm{R}}\right|^2 + \mathcal{O}(\alpha_s^2)$$

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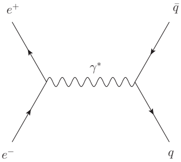

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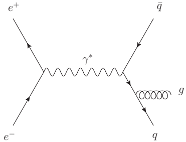
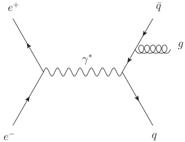
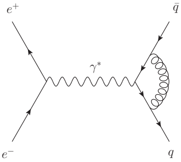


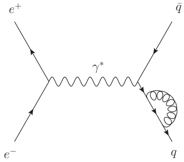
(b) $\tilde{A}_{\mathrm{R}}$



(c) $\tilde{B}_{\mathrm{R}}$



(d) $\tilde{A}_{\mathrm{V}}$



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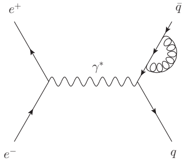

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$$\sigma_{q\bar{q}} \propto \left|\tilde{A}_0\right|^2 + 2\,\mathrm{Re}(A_0\tilde{A}_{\mathrm{V}}^*) + 2\,\mathrm{Re}(\tilde{A}_0(\tilde{B}_{\mathrm{V}}^* + \tilde{C}_{\mathrm{V}}^*)) + \mathcal{O}(\alpha_s^2)$$

$$\sigma_{q\bar{q}g} \propto \left|\tilde{A}_{\mathrm{R}}^2\right|^2 + 2\,\mathrm{Re}(\tilde{A}_{\mathrm{R}}\tilde{B}_{\mathrm{R}}^*) + \left|\tilde{B}_{\mathrm{R}}\right|^2 + \mathcal{O}(\alpha_s^2)$$

Problem divergirajućih integrala - primjer

- Promotrimo sljedeće integrale:

$$\int_0^\infty \frac{1}{x^2} \mathrm{d}x$$

$$\int_0^\infty \frac{-\cos x}{x^2} \mathrm{d}x$$

- Oba **divergiraju** u $x = 0$.

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- No, **zajedno konvergiraju**:

$$\int_0^\infty \frac{1-\cos x}{x^2} = \frac{2\sin\frac{x}{2}}{x^2} = \frac{\pi}{2}$$

Problem divergirajućih integrala

- Znači li to da je dovoljno samo pozbrajati sve integrale za neki proces prije integriranja?

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- Taj udarni presjek divergira kada jedan od kvarkova uzme pola ukupne energije.
  $\Longrightarrow$ drugi kvark i gluon su usmjereni u istom smjeru
  $\Longrightarrow$ ne razlikujemo taj slučaj od slučaja $e^-e^+ \to q\bar{q}$!

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- Odnosno, **sve zajedno moramo pozbrajati**.
  $\Longrightarrow$ **Regularizacija**

Regularizacija

- Modificiramo integrale u ovisnosti o vanjskom parametru:

$$A(a) \stackrel{\text{def}}{=} \int_0^\infty \frac{1}{x^2 + a^2} \mathrm{d}x = \frac{\pi}{2a}$$

$$B(a) \stackrel{\text{def}}{=} \int_0^\infty \frac{-\cos x}{x^2 + a^2} \mathrm{d}x = \frac{-e^{-a}\pi}{2a}$$

- Za $a = 0$ se reproduciraju originalni integrali, ali za $a \neq 0$ integrali više **ne divergiraju**.

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- Za $a = 0$ se reproduciraju originalni integrali, ali za $a \neq 0$ integrali više **ne divergiraju**.
- Na kraju uzimamo limes $a \to 0$:

$$\text{rezultat} = \lim_{a\to 0} A(a) + B(a) = \frac{\pi}{2}$$

Dimenzionalna regularizacija

- Račun provodimo u $D = 4 + \epsilon$ dimenzija, $\epsilon \in \mathbb{R}$. Na kraju uzimamo $\lim_{\epsilon \to 0}$.
- Sve računamo za $D \in \mathbb{N}$, a zatim analitički proširujemo rezultate.

$$D! \to \Gamma(D+1)$$

$$\frac{\mathrm{d}^4 k}{(2\pi)^4} \to \frac{\mathrm{d}^D k}{(2\pi)^D}$$

$$\mathrm{d}^D k \to \mathrm{d}k\; k^{D-1}\; S_{D-1}, \quad S_d = \frac{2\pi^{(d+1)/2}}{\Gamma\left(\frac{d+1}{2}\right)}$$

$$g^\mu{}_\mu = D$$

$$\gamma^\mu \gamma_\mu = D\mathbb{1}$$

$$\gamma^\mu \gamma_\nu \gamma_\mu = (2-D)\gamma_\nu$$

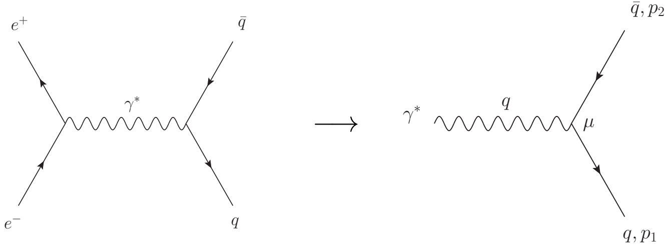
$$\gamma^\mu \gamma_\nu \gamma_\rho \gamma_\nu = 4g_{\nu\rho} - (4-D)\gamma_\nu \gamma_\rho$$

$$\vdots$$

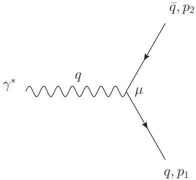
Podjela procesa na dva dijela

- Proces se može podijeliti na dva dijela:

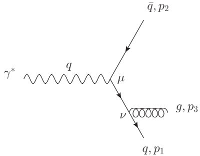
$$e^- e^+ \to \gamma^* \to kvarkovi\ i\ gluoni$$



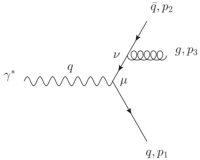
Feynmanovi dijagrami za $\gamma^* \to$ *partoni* do $\mathcal{O}(\alpha_s)$



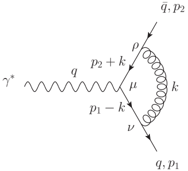
(a) A_0



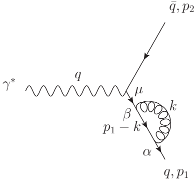
(b) A_R



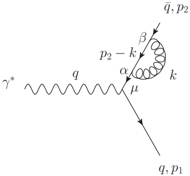
(c) B_R



(d) A_V



(e) B_V

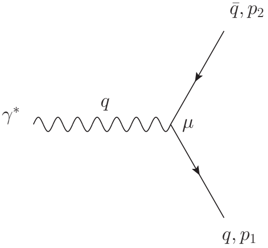


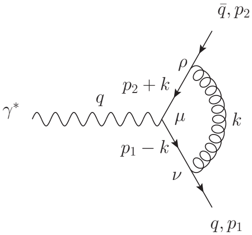
(f) C_V

$$\Gamma_{q\bar{q}} \propto |\mathcal{M}_{\gamma^* \to q\bar{q}}|^2 = |A_0 + A_\mathrm{V} + B_\mathrm{V} + C_\mathrm{V}|^2$$

$$\Gamma_{q\bar{q}g} \propto |\mathcal{M}_{\gamma^* \to q\bar{q}g}|^2 = |A_\mathrm{R} + B_\mathrm{R}|^2$$

Brzi pregled računa doprinosa - $A_0 A_{\mathrm{V}}^*$

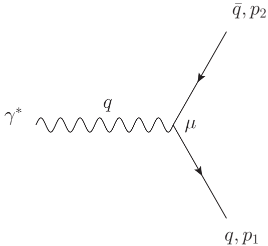
$A_0 =$ 

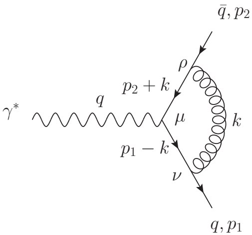
$A_{\mathrm{V}} =$ 

$$A_0 = \bar{u}_1(-ie_D\gamma^\mu)v_2 \times \varepsilon_\mu$$

$$A_{\mathrm{V}} = \int \frac{\mathrm{d}^D k}{(2\pi)^D}\bar{u}_1(-ig_D T^a_{il}\gamma^\nu)\frac{i(\not{p}_1 - \not{k})}{(p_1-k)^2}(-ie_D\gamma^\mu)$$
$$\frac{i(-\not{p}_2 - \not{k})}{(p_2+k)^2}(-ig_D T^a_{lj}\gamma^\rho)v_2\frac{-i(g_{\nu\rho} + \eta k_\nu k_\rho/k^2)}{k^2}\varepsilon_\mu$$

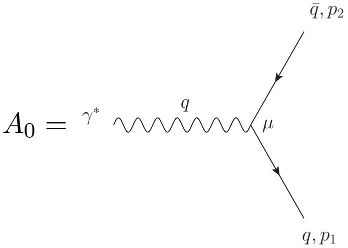
Brzi pregled računa doprinosa - $A_0 A_\mathrm{V}^*$

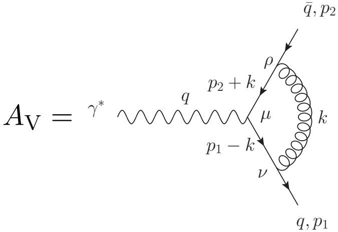
$A_0 =$ 

$A_\mathrm{V} =$ 

$$
\begin{aligned}
2 \sum_{\lambda, s_1, s_2} A_0 A_\mathrm{V}^* = -2ie_D^2 g_D^2 \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{4}{(p_1 - k)^2 (p_2 + k)^2 k^2} \times \\
\times \Big\{ \mathrm{Tr} \left[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu (\not{p}_2 + \not{k}) \gamma_\mu (\not{p}_1 - \not{k}) \gamma_\nu \right] \\
+ \frac{\eta}{k^2} \mathrm{Tr} \left[\not{p}_1 \gamma^\mu \not{p}_2 \not{k} (\not{p}_2 + \not{k}) \gamma_\mu (\not{p}_1 - \not{k}) \not{k} \right] \Big\}
\end{aligned}
$$

Brzi pregled računa doprinosa - $A_0 A_V^*$





$$2 \sum_{\lambda, s_1, s_2} A_0 A_V^* = -2ie_D^2 g_D^2 \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{4}{(p_1 - k)^2 (p_2 + k)^2 k^2} 4 \left(1 + \frac{\epsilon}{2}\right) \times$$

$$\times \Big\{ -2q^4 + 4q^2 \left(k \cdot p_2 - k \cdot p_1\right) + 8(k \cdot p_1)(k \cdot p_2) + q^2 \epsilon k^2$$

$$+ \eta q^2 k^2 + 2\eta q^2 \left(k \cdot p_2 - k \cdot p_1\right) - 4\eta q^2 \frac{(k \cdot p_1)(k \cdot p_2)}{k^2} \Big\}$$

Feynmanova parametrizacija

- Spajanje faktora u nazivniku uvođenjem dodatnih integrala:

$$\frac{1}{AB} = \int_0^1 \frac{1}{(xA + (1-x)B)^2} \mathrm{d}x$$

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- I druge varijante:

$$\frac{1}{A^2B} = \int_0^1 \frac{2x}{(xA + (1-x)B)^3} \mathrm{d}x$$

$$\frac{1}{A^2B^2} = \int_0^1 \frac{6x(1-x)}{(xA + (1-x)B)^4} \mathrm{d}x$$

$$\vdots$$

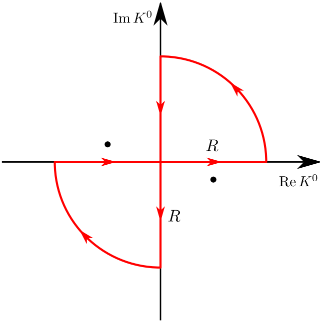
Brzi pregled računa doprinosa - $A_0 A_{\mathrm{V}}^*$

$$\Gamma_{\mathrm{V}} = const \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \, [A I_2 + B I_3 + C I_4] \tag{1}$$

$$I_M \stackrel{\mathrm{def}}{=} \int \frac{\mathrm{d}^D K}{(2\pi)^D} \frac{1}{(K^2 - C + i\epsilon)^M}$$

- Kada bismo mogli preći u sferni sustav, bitno bismo pojednostavnili integral, ali ovdje se radi o Lorentzovoj metrici:
$$K^2 = +(K^0)^2 - (K^1)^2 - \cdots - (K^{D-1})^2$$

Wickova rotacija



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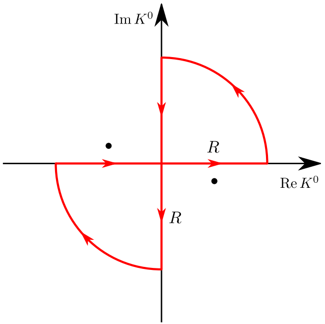

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$$K^0 = iK_E^0, \quad K^i = K_E^i, \quad K^2 = -K_E^2$$

$$I_M = i \left(\frac{C}{4\pi}\right)^{D/2} \left(\frac{-1}{C}\right)^M \frac{\Gamma(M - D/2)}{\Gamma(M)}$$

Konačni doprinos $A_0 A_V^*$

$$\Gamma_V = -\frac{\sqrt{\pi}(\epsilon^2 + \epsilon + 4)\Gamma(\frac{\epsilon}{2})\Gamma(-\frac{\epsilon}{2})}{2^{\epsilon+2}\Gamma(\frac{3+\epsilon}{2})} \left(\frac{Q^2}{4\pi\mu^2}\right)^{\epsilon/2}$$

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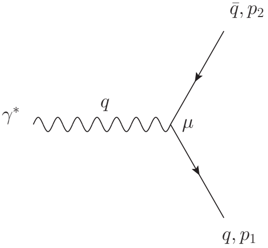
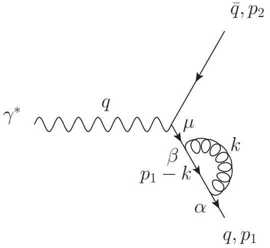
$$\begin{aligned}\Gamma_{\mathrm{V}} = \Gamma_0 \frac{2\alpha_s}{3\pi}\Bigg[&\frac{1}{\epsilon^2}(-8) \\ &+\frac{1}{\epsilon}\left(-4\ln\frac{Q^2}{4\pi\mu^2} - 4\gamma + 6\right) + \\ &+\left(-\ln^2\frac{Q^2}{4\pi\mu^2} - (2\gamma-3)\ln\frac{Q^2}{4\pi\mu^2} + 3\gamma - \gamma^2 + \frac{7}{6}\pi^2 - 8\right) \\ &+\mathcal{O}(\epsilon)\Bigg]\end{aligned} \tag{2}$$

Doprinos $A_0 B_V^*$

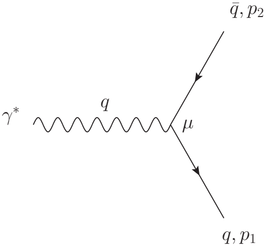
$A_0 =$ 

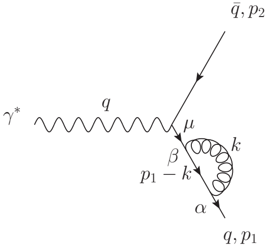
$B_\mathrm{V} =$ 

$$B_\mathrm{V} = \bar{u}_1 \Sigma(p_1) \frac{i\not{p}_1}{p_1^2} (-ie_D\gamma^\mu) v_2 \varepsilon_\mu$$

$$\Sigma(p_1) = 4 \int \frac{\mathrm{d}^D k}{(2\pi)^D} (-ig_D\gamma_\alpha T^a_{ij}) \frac{i(\not{p}_1 - \not{k})}{(p_1 - k)^2} (-ig_D\gamma_\beta T^a_{ji}) \frac{-i(g^{\alpha\beta} + \eta\frac{k^\alpha k^\beta}{k^2})}{k^2}$$

Doprinos $A_0 B_V^*$

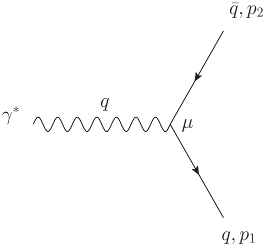
$A_0 =$ 

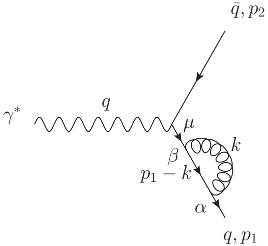
$B_{\mathrm{V}} =$ 

$$B_{\mathrm{V}} = \bar{u}_1 \Sigma(p_1) \frac{i \not{p}_1}{p_1^2} (-i e_D \gamma^\mu) v_2 \varepsilon_\mu$$

$$\Sigma(p_1) \propto \not{p}_1 (1+\eta) \left(\frac{p_1^2}{4\pi} \right)^{\epsilon/2} \frac{(\epsilon+2)\Gamma(1+\epsilon/2)\Gamma(-\epsilon/2)}{2^{\epsilon+6}\pi^{3/2}\Gamma((\epsilon+3)/2)}$$

Doprinos $A_0 B_V^*$

$A_0 =$ 

$B_{\mathrm{V}} =$ 

$$B_{\mathrm{V}} = 0$$

$$\Sigma(p_1) = 0$$

Doprinos procesa $e^-e^+ \to q\bar{q}g$

$$\Gamma_{\mathrm{R}} = \Gamma_0 \frac{2\alpha_s}{3\pi}\left[\frac{1}{\epsilon^2}(+8) \right.$$
$$+ \frac{1}{\epsilon}\left(4\ln\frac{Q^2}{4\pi\mu^2} + 4\gamma - 6\right) +$$
$$+ \left(\ln^2\frac{Q^2}{4\pi\mu^2} + (2\gamma - 3)\ln\frac{Q^2}{4\pi\mu^2} - 3\gamma + \gamma^2 - \frac{7}{6}\pi^2 + \frac{19}{2}\right)$$
$$\left. + \mathcal{O}(\epsilon)\right] \tag{3}$$

Ukupni udarni presjek

$$\Gamma = \Gamma_0 + \Gamma_{\mathrm{V}} + \Gamma_{\mathrm{R}}$$

$$\Gamma = 3\alpha Q \left(1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

$(Q =$ ukupna ulazna energija$)$

Ukupni udarni presjek

$$\sigma = \frac{4\pi\alpha}{3Q^3}\Gamma$$

$$\sigma = \sum_{e_q} \frac{4\pi\alpha^2 e_q^2}{Q^2}\left(1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

(Q = ukupna ulazna energija)

(e_q = naboj kvarka)

Sažetak postupka

- Svesti problem na Feynmanove dijagrame.
- Odrediti sve dijagrame (i procese) koji doprinose do zadanog reda.
- Ako je moguće i ako ima smisla, prepoloviti dijagrame.
- Ako integrali divergiraju, napraviti regularizaciju.
- Ako se pojavljuje više faktora u nazivnicima, napraviti Feynmanovu parametrizaciju.
- Po potrebi napraviti Wickovu rotaciju.
- Zbrojiti rezultate.
- Spojiti razdvojene dijagrame.
- Gotovo.

Hvala na pažnji!

Reference

- Richard D. Field, *Applications of Perturbative QCD*, 1989
- Michael E. Peskin, Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, 1995
- Wikipedia, N-sphere, `https://en.wikipedia.org/wiki/N-sphere`
- Wikipedia, Feynman Parametrization, `https://en.wikipedia.org/wiki/Feynman_parametrization`