Produkcija gluona u sudarima teških jezgara

Samostalni seminar iz istraživanja u fizici

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QCD i sudari teških jezgara

- QCD bez kvarkova:

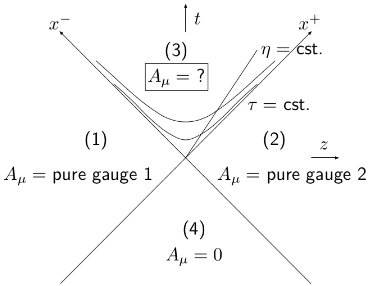
$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu}\right];\;\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\left[A_\mu, A_\nu\right]. \quad (1)$$

- Jednadžbe gibanja:

$$\left[D_\mu, F^{\mu\nu}\right] = J^\nu;\;\; D_\mu = \partial_\mu + igA_\mu. \quad (2)$$

- Pristup preko klasične teorije polja - velika gustoća faznog prostora

Kinematika sudara



$$J^\mu = \delta^{\mu +}\rho_{(1)}(\mathbf{x}_T)\delta(x^-) + \delta^{\mu -}\rho_{(2)}(\mathbf{x}_T)\delta(x^+) \tag{3}$$

$$x^\pm = \frac{1}{\sqrt{2}}(t \pm z) = \frac{1}{\sqrt{2}}\tau e^{\pm\eta} \tag{4}$$

Početni uvjeti

- Korelator gustoća naboja jezgri:

$$\langle \rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T)\rangle = g^2\mu^2\delta^{ab}\delta^2(\mathbf{x}_T - \mathbf{y}_T) \tag{5}$$

- Čisto baždarno polje u kauzalno nepovezanim zonama:

$$A^i_{(m)} = -\frac{i}{g}e^{i\Lambda_{(m)}}\partial^i e^{-i\Lambda_{(m)}}, \;\; \nabla^2_T\Lambda_{(m)}(\mathbf{x}_T) = -g\rho_{(m)}(\mathbf{x}_T) \tag{6}$$

- Početni uvjeti:

$$A^i|_{\tau=0} = A^i_{(1)} + A^i_{(2)}, \tag{7}$$

$$A^\eta|_{\tau=0} = \frac{ig}{2}\left[A^i_{(1)}, A^i_{(2)}\right]. \tag{8}$$

Jednadžbe baždarnog polja

- Boost invarijatnost modela $\rightarrow A_\eta \equiv \phi$
- CCD akcija:

$$
\begin{aligned}
S = \int d\eta d^2\mathbf{x}_T d\tau\tau \\
\mathrm{Tr}\left(\dot{A}_i\dot{A}_i - \frac{1}{2}F_{ij}F_{ij} + \frac{1}{\tau^2}\dot{\phi}^2 - \frac{1}{\tau^2}\left[D_i,\phi\right]\left[D_i,\phi\right]\right),
\end{aligned}
\tag{9}
$$

- Konjugirani impulsi $E^{ia} \equiv \frac{\delta S}{\delta \dot{A}_i^a} = \tau \dot{A}_i^a$ i $\pi^a \equiv \frac{\delta S}{\delta \dot{\phi}^a} = \frac{1}{\tau}\dot{\phi}^a$ daju Hamiltonijan

$$
\mathcal{H} = \mathrm{Tr}\left(\frac{1}{\tau}E^iE^i + \frac{\tau}{2}F_{ij}F_{ij} + \tau\pi^2 + \frac{1}{\tau}\left[D_i,\phi\right]\left[D_i,\phi\right]\right) \tag{10}
$$

- Jednadžbe evolucije:

$$\dot{A}_i = \frac{1}{\tau} E^i \tag{11}$$

$$\dot{\phi} = \tau \pi \tag{12}$$

$$\dot{E}^i = \tau \left[D_k, F_{ki} \right] - \frac{ig}{\tau} \left[\phi, \left[D_i, \phi \right] \right] \tag{13}$$

$$\dot{\pi} = \frac{1}{\tau} \left[D_i, \left[D_i, \phi \right] \right]. \tag{14}$$

- Početni uvjeti: $E^i(\tau = 0, \mathbf{x}_T) = 0$, $\phi(\tau = 0, \mathbf{x}_T) = 0$, $\pi(\tau = 0, \mathbf{x}_T) = -ig \left[A^i_{(1)}, A^i_{(2)} \right]$

Granica slabog polja

- Raspodjela energije po stupnjevima slobode:

$$
\begin{aligned}
H \approx\, & 2\int d^2\mathbf{x}_T \,\mathrm{Tr}\left[\frac{1}{\tau}E^iE^i + \tau\pi^2\right] \\
=\,& 2\int \frac{d^2\mathbf{k}_T}{(2\pi)^2}\,\mathrm{Tr}\left[\frac{1}{\tau}E^i(\mathbf{k}_T)E^i(-\mathbf{k}_T) + \tau\pi(\mathbf{k}_T)\pi(-\mathbf{k}_T)\right]
\end{aligned}
\tag{15}
$$

- Diferencijalni multiplicitet stanja:

$$
n(\mathbf{k}_T) = \frac{1}{(2\pi)^2}\frac{2}{|\mathbf{k}_T|}\left[\frac{1}{\tau}E^i(\mathbf{k}_T)E^i(-\mathbf{k}_T) + \tau\pi(\mathbf{k}_T)\pi(-\mathbf{k}_T)\right]
\tag{16}
$$

- Linearizacijom sustava (11)-(14) slijedi

$$\dot{E}^i = \partial_\tau(\tau \dot{A}_i) = \tau \nabla_T^2 A_i \tag{17}$$

$$\Longrightarrow \left(\tau^2 \partial_\tau^2 + \tau \partial \tau + \tau^2 \mathbf{k}_T^2\right) A_i(\tau, \mathbf{k}_T) = 0, \tag{18}$$

$$\dot{\pi} = \partial_\tau \left(\frac{1}{\tau}\dot{\phi}\right) = \frac{1}{\tau}\nabla_T^2 \phi \tag{19}$$

$$\Longrightarrow \left(\tau^2 \partial_\tau^2 - \tau \partial_\tau + \tau^2 \mathbf{k}_T^2\right) \phi(\tau, \mathbf{k}_T) = 0. \tag{20}$$

- Rješenja su Besselove funkcije

$$A_i(\tau, \mathbf{k}_T) = A_i(0, \mathbf{k}_T) J_o(|\mathbf{k}_T|\tau), \tag{21}$$

$$\phi(\tau, \mathbf{k}_T) = \frac{\tau}{|\mathbf{k}_T|}\pi(0, \mathbf{k}_T) J_1(|\mathbf{k}_T|\tau). \tag{22}$$

- Uvrštavanje i asimptotski razvoj Besselovih funkcija:

$$\langle n(\tau, \mathbf{k}_T)\rangle = \frac{1}{(2\pi)^2}\frac{2}{\pi \mathbf{k}_T^2}\Big(\mathbf{k}_T^2 \sin^2(|\mathbf{k}_T|\tau - \pi/4)\times \\ \times \langle A_i^a(0,\mathbf{k}_T)A_i^a(0,-\mathbf{k}_T)\rangle + \sin^2(|\mathbf{k}_T|\tau - 3\pi/4)\times \\ \times \langle \pi^a(0,\mathbf{k}_T)\pi^a(0,-\mathbf{k}_T)\rangle\Big) \tag{23}$$

- Usrednjeno po vremenu:

$$\langle n(\mathbf{k}_T)\rangle = \frac{\pi R_A^2}{(2\pi)^2}\frac{N_c(N_c^2-1)g^6\mu^4}{\mathbf{k}_T^2}\frac{1}{\pi}\int \frac{d^2\mathbf{p}_T}{(2\pi)^2}\frac{1}{\mathbf{p}_T^2\,(\mathbf{k}_T-\mathbf{p}_T)^2}. \tag{24}$$

- PV regularizacija integrala s $m \propto g^2\mu$

$$\langle n(\mathbf{k}_T)\rangle = \frac{\pi R_A^2}{(2\pi)^3}\frac{1}{\pi}\frac{N_c(N_c^2-1)g^6\mu^4}{\mathbf{k}_T^4}\ln\frac{\mathbf{k}_T^2}{m^2}, \tag{25}$$

- Dimenzijski:

$$\langle n(\mathbf{k}_T)\rangle = \frac{\pi R_A^2}{g^2} f\left(\mathbf{k}_T/g^2\mu\right) \tag{26}$$

- Ukupni multiplicitet po jedinici rapiditeta:

$$\frac{dN}{d\eta} = \int d^2\mathbf{k}_T \langle n(\mathbf{k}_T)\rangle = \frac{\pi R_A^2}{g^2}(g^2\mu)^2 f_N, \tag{27}$$

uz

$$f_N \equiv \int d^2\left(\frac{\mathbf{k}_T}{g^2\mu}\right) f\left(\frac{\mathbf{k}_T}{g^2\mu}\right), \tag{28}$$

- Analogno za energiju:

$$\frac{dE_T}{d\eta} = \int d^2\mathbf{k}_T \langle n\left(\mathbf{k}_T\right)\rangle |\mathbf{k}_T| = \frac{\pi R_A^2}{g^2}(g^2\mu)^3 f_E. \tag{29}$$

Dinamika na rešetci

- Matrica prijelaza: $U_\mu(x) = e^{igaA_\mu(x)}$
- Hamiltonijan na rešetci:

$$aH = \sum_{x_T} \left(\frac{g^2 a}{\tau} \operatorname{Tr} E^i E^i + \frac{2N_c \tau}{g^2 a} \left(1 - \frac{1}{N_c} \operatorname{Re} \operatorname{Tr} U_\perp \right) + \frac{\tau}{a} \operatorname{Tr} \pi^2 + \frac{a}{\tau} \sum_i \operatorname{Tr} \left(\phi - \tilde{\phi}_i \right)^2 \right), \tag{30}$$

$$U_\perp(x_T) = U_{x,y} = U_x(x_T) U_y(x_T + e_x) U_x^\dagger(x_T + e_y) U_y^\dagger(x_T) \tag{31}$$

$$\tilde{\phi}_i(x_T) \equiv U_i(x_T) \phi(x_T + e_i) U_i^\dagger(x_T). \tag{32}$$

Jednadžbe gibanja

$$\dot{U}_i = \frac{ig^2}{\tau} E^i U_i \text{ (nema sume po i)}, \tag{33}$$

$$\dot{\phi} = \tau\pi, \tag{34}$$

$$\dot{E}^x = \frac{i\tau}{2g^2}(U_{x,y} + U_{x,-y} - h.c.) - \text{trag} + \frac{i}{\tau}\left[\tilde{\phi}_x, \phi\right], \tag{35}$$

$$\dot{E}^y = \frac{i\tau}{2g^2}(U_{y,x} + U_{y,-x} - h.c.) - \text{trag} + \frac{i}{\tau}\left[\tilde{\phi}_y, \phi\right], \tag{36}$$

$$\dot{\pi} = \frac{1}{\tau}\sum_i(\tilde{\phi}_i + \tilde{\phi}_{-i} - 2\phi). \tag{37}$$

Početni uvjeti na rešetci

$$0 = \mathrm{Tr}\left[t_a\left(\left(U_i^{(1)} + U_i^{(2)}\right)\left(1 + U_i^{\dagger}\right) - h.c.\right)\right], \tag{38}$$

$$E^i = 0, \tag{39}$$

$$\phi = 0, \tag{40}$$

$$\pi(x_T) = \frac{-i}{4g}\sum_i \left[\left(U_i(x_T) - 1\right)\left(U_i^{\dagger(2)}(x_T) - U_i^{\dagger(1)}(x_T)\right)\right. \tag{41}$$

$$\left. + \left(U_i^{\dagger}(x_T - e_i) - 1\right)\left(U_i^{(2)}(x_T - e_i) - U_i^{(1)}(x_T - e_i)\right) - \mathrm{h.c.}\right] \tag{42}$$

Multiplicitet u lQCD

- Na primjeru slobodnog skalarnog polja:

$$H = \int d^D x \left[\frac{1}{2}\pi^2(x) + \frac{1}{2}(\nabla\phi)^2(x) + m^2\phi^2 \right] \tag{43}$$

$$= \int \frac{d^D k}{(2\pi)^D} \left[\frac{1}{2}|\pi(k)|^2 + \frac{1}{2}\omega^2(k)|\phi(k)|^2 \right] \tag{44}$$

$$= \int d^D k \omega(k) n(k), \tag{45}$$

$$\Rightarrow \overline{|\pi(k)|^2} = \omega(k)n(k), \ \overline{|\phi(k)|^2} = \frac{n(k)}{\omega(k)}, \tag{46}$$

- Iz toga se definira:

$$\omega(k) = \sqrt{\frac{\overline{|\pi(k)|^2}}{\overline{|\phi(k)|^2}}}. \tag{47}$$

- Multiplicitet na rešetci:

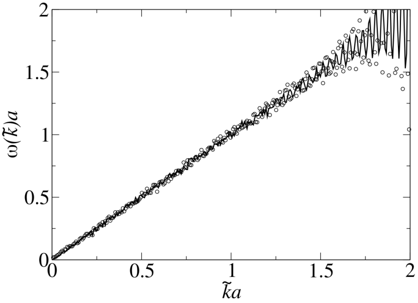
$$n(k_T) = \frac{2}{N^2}\frac{1}{\tilde{k}}\left[\frac{g^2}{2\tau}E_i^a(k_T)E_i^a(-k_T) + \frac{\tau}{2}\pi^a(k_T)\pi^a(-k_T)\right], \tag{48}$$

$$\tilde{k}^2 = \frac{4}{a^2}\left[\sin^2\frac{ak_x}{2} + \sin^2\frac{ak_y}{2}\right]. \tag{49}$$

Disperzija gluona

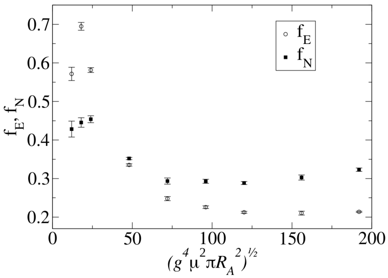
$$\frac{1}{\tau}\sqrt{\frac{\langle E_i^a(k_T)E_i^a(-k_T)\rangle}{\langle A_i^a(k_T)A_i^a(-k_T)\rangle}} \quad (50)$$

$$\tau\sqrt{\frac{\langle \pi^a(k_T)\pi^a(-k_T)\rangle}{\langle \phi^a(k_T)\phi^a(-k_T)\rangle}} \quad (51)$$

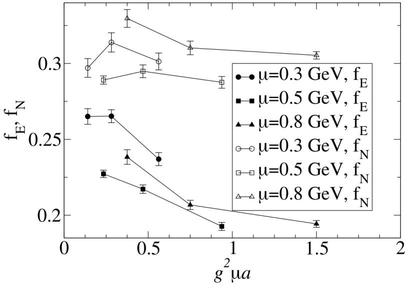


- Ovisnost (50) i (51) o transverzalnom impulsu $\mathbf{k}_T$. Kružići su disperzija iz E^i i A_i, a puna linija iz ϕ i π.

Ispitivanje ponašanja f_E i f_N

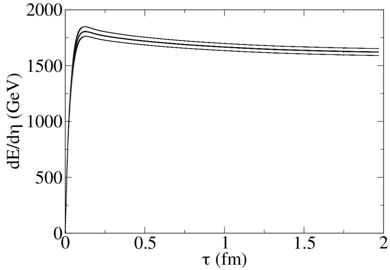


Ovisnost konstanti f_E i f_N o bezdimenzionalnom parametru $\sqrt{g^4\mu^2\pi R_A^2} \equiv C$ na 256^2 rešetci.

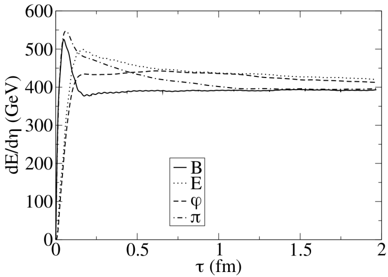


Ovisnost f_E i f_N o veličini ćelije uz konstantan C.

Saturacija

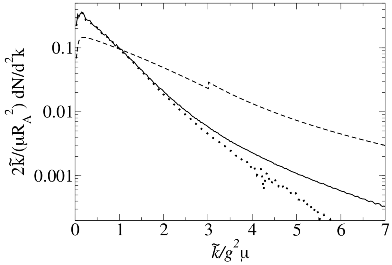


Ukupna energija po jedinici rapiditeta kao funkcija vremena za $\mu = 0.5$ GeV na 512^2 rešetci.

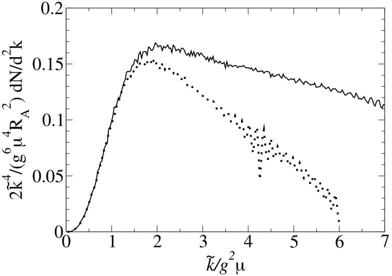


Raspodjela energije iz procesa s lijeve slike po komponentama polja.

Diskretni efekti

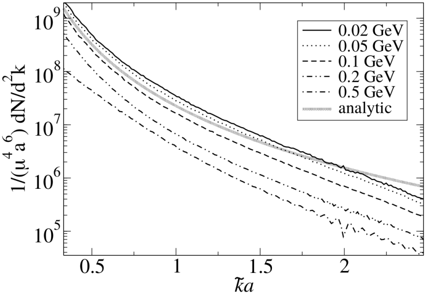


Odnos $\frac{2\tilde{k}}{\mu R_A^2}\frac{dN}{d^2k_T}$ naspram $\tilde{k}/g^2\mu$ za $C=120$. Puna crta je rezultat za 512^2 rešetku, točke za 256^2 i crtkana za rad ranijeg autora.

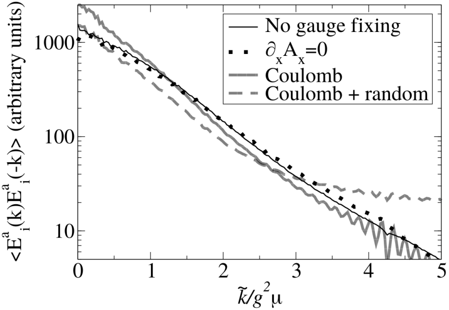


Ovisnost $\frac{2\tilde{k}^4}{g^6\mu^4R_A^2}\frac{dN}{d^2k_T}$ o $\tilde{k}/g^2\mu$ iz istih simulacija kao lijeva slika. Točkasta crta je rezultat 256^2, a puna 512^2 rešetke.

Odstupanja

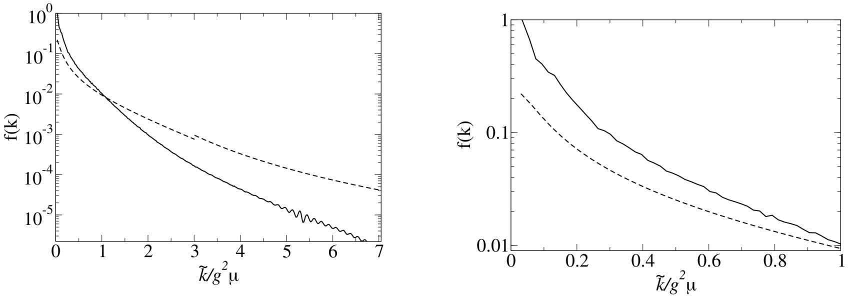


Ovisnost $\frac{1}{\mu^4 a^6}\frac{dN}{d^2 k_T}$ o $\tilde{k}a$ za $C = 240\mu/\mathrm{GeV}$, s različitim vrijednostima μ, i usporedba s analitičkim rezultatom (25)



Korelator $\langle E_i^a(k_T)E_i^a(-k_T)\rangle$ u različitim baždarenjima, uz uvjet $\int d^2k_T E_i^a(k_T)E_i^a(-k_T) = \mathrm{inv.}$

Gustoća faznog prostora



- Gustoća dvodimenzionalnog faznog prostora $f(k_T) = \frac{1}{2(N_C^2-1)} \frac{(2\pi)^2}{\pi R_A^2} \frac{dN}{d^2k_T}$ u ovisnosti o $\tilde{k}/g^2\mu$ za $C = 120$.

Zaključak

- Iz kinematike modela određeni početni uvjeti i relacije među poljima,
- Izveden multiplicitet gluona u limesu slabog polja, uz poznati korelator gustoća,
- Objašnjene lQCD relacije i dobiveni numerički rezultati: disperzijska relacija, saturacija energije kroz vrijeme te multiplicitet gluona.

Literatura

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