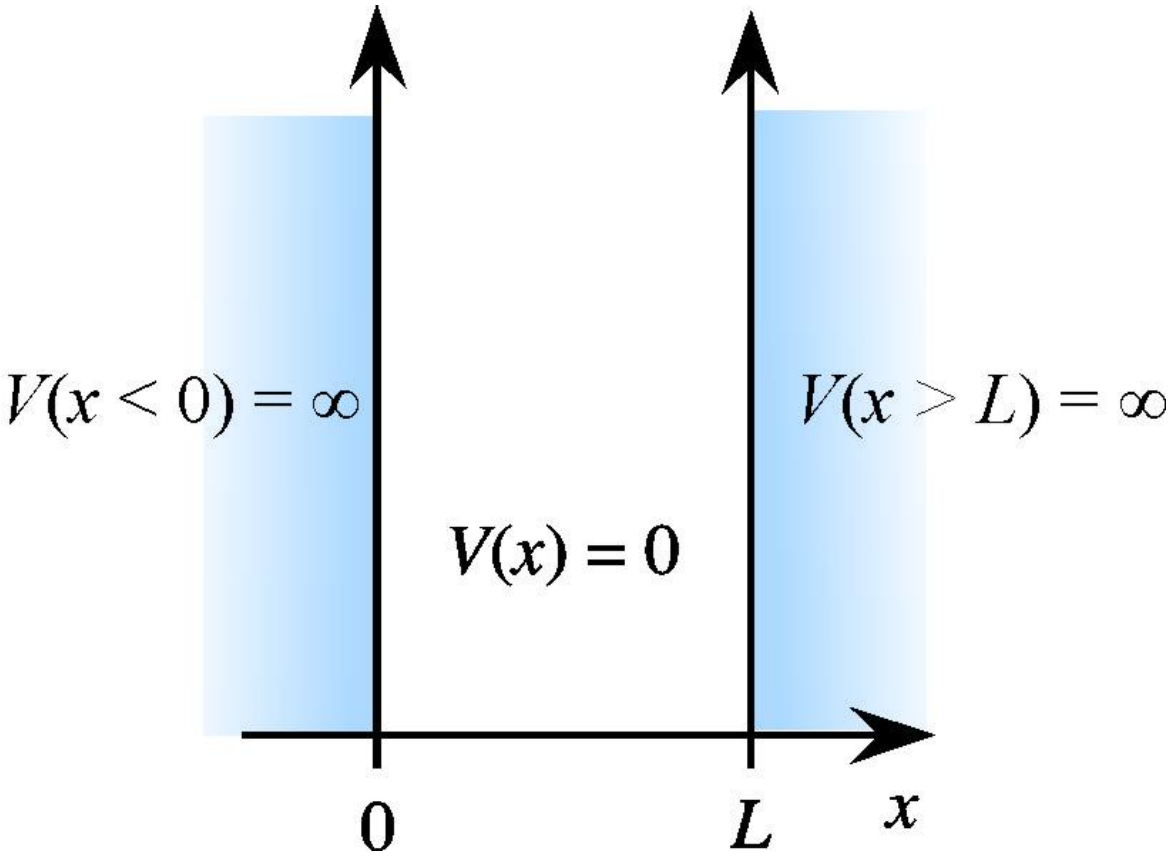
Čestica u kutiji



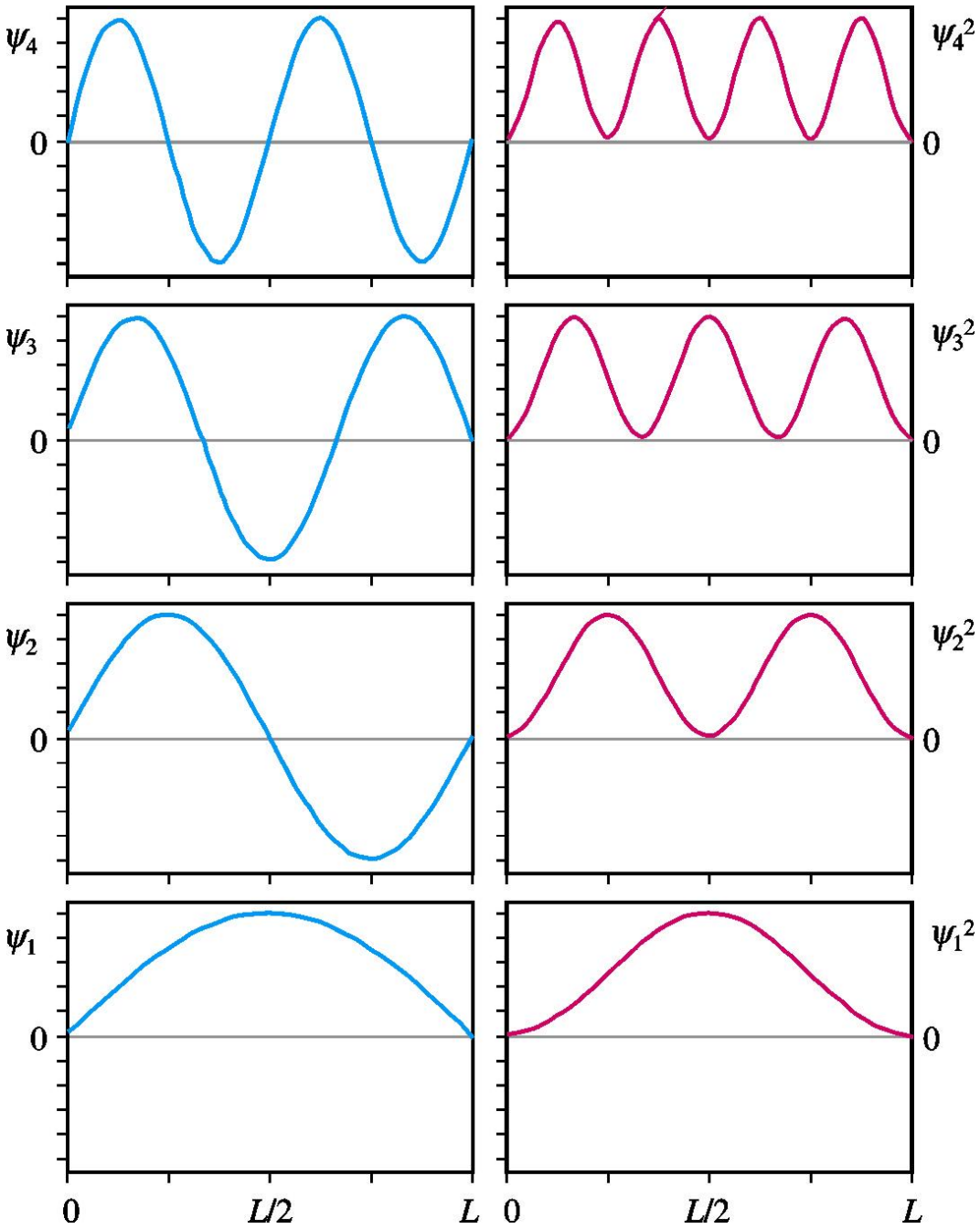
1. Napisati klasični hamiltonijan
2. Pretvoriti klasični hamiltonijan u kvantnomehanički operator
3. Postaviti Schrödingerovu jednadžbu
4. Riješiti Schrödingerovu jednadžbu

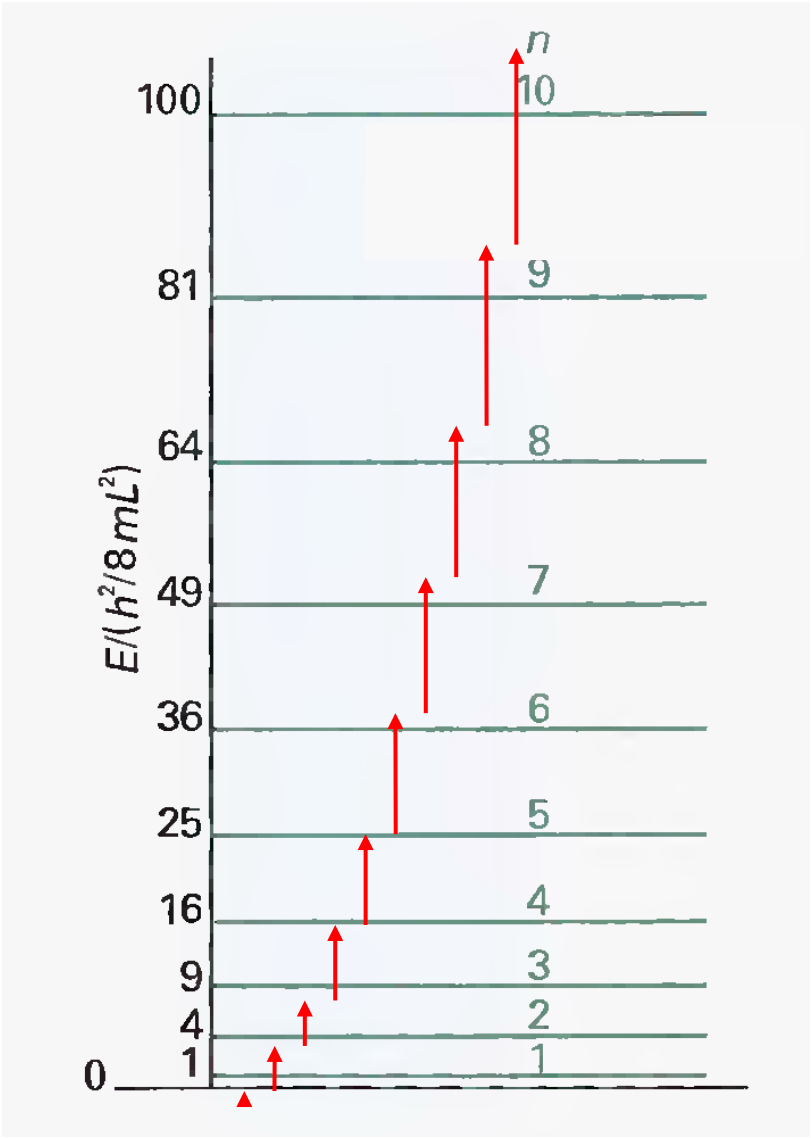
1. $H = T$
2. $\hat{H} = \hat{T}$
3. $\hat{H}\Psi = E\Psi$
4. $\Psi(x) = N\sin(ax + \delta)$

Općenito rješenje za česticu u 1-D kutiji:

$$E_n = \frac{h^2}{8mL^2} n^2 \qquad n = 1, 2, 3, \ldots$$

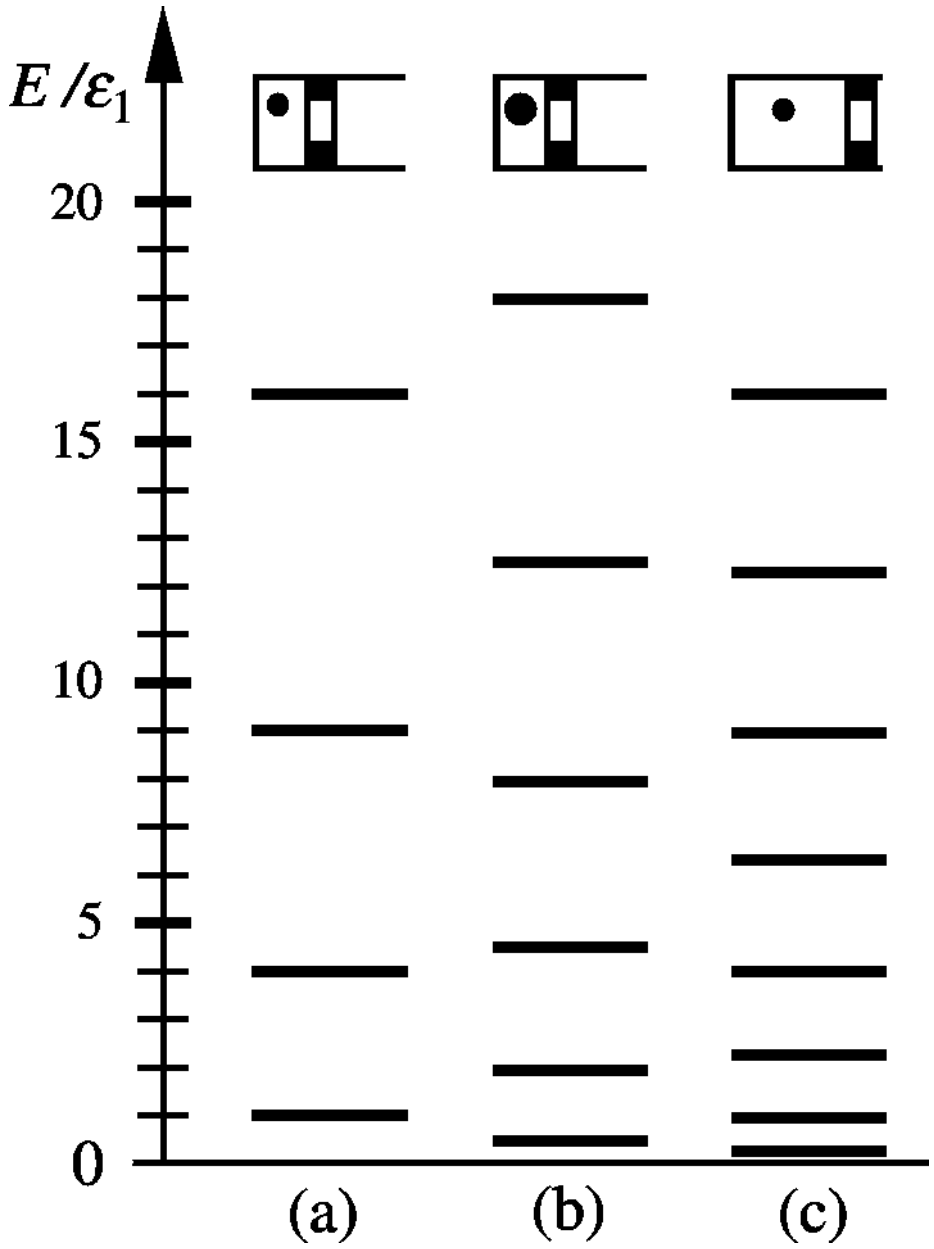
$$\Psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$


ψ4
0
ψ4²
0
ψ3
0
ψ3²
0
ψ2
0
ψ2²
0
ψ1
0
ψ1²
0
0
L/2
L
0
L/2
L

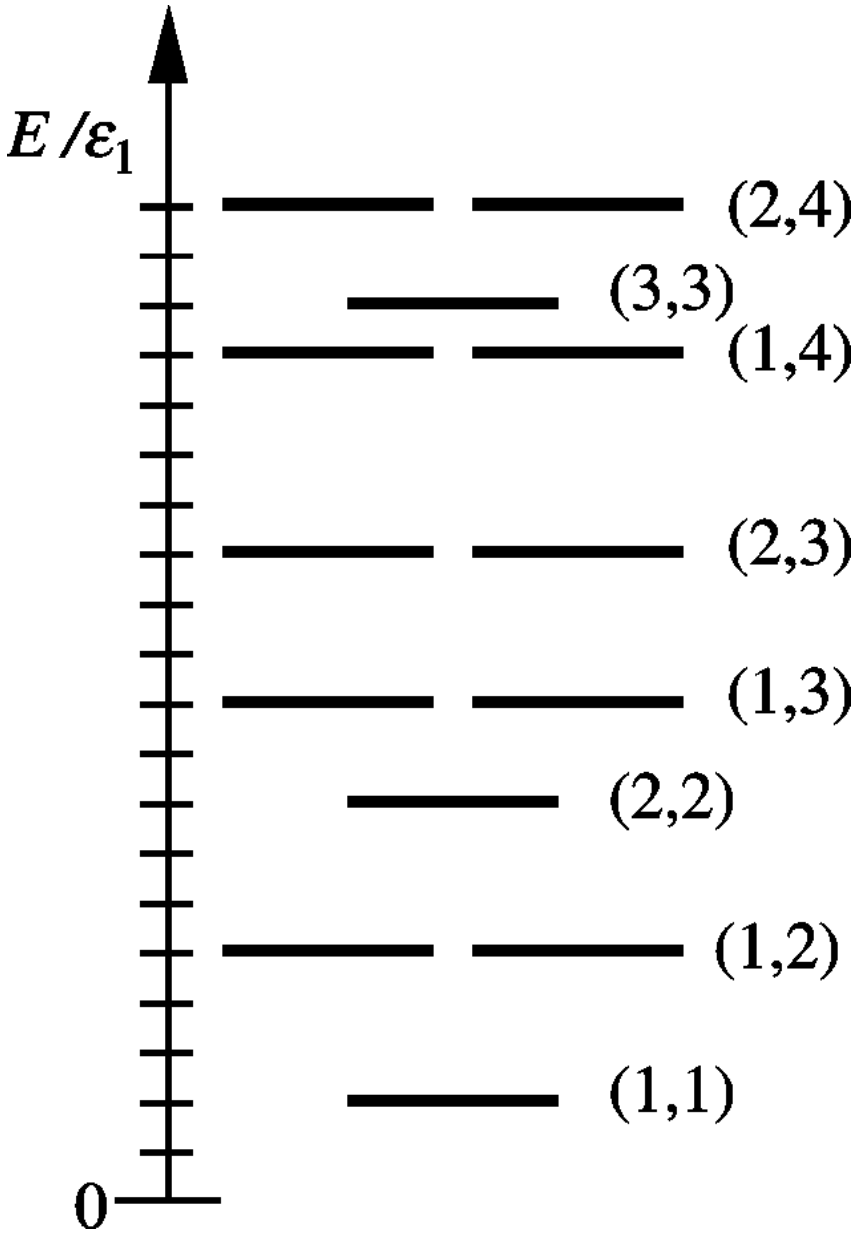


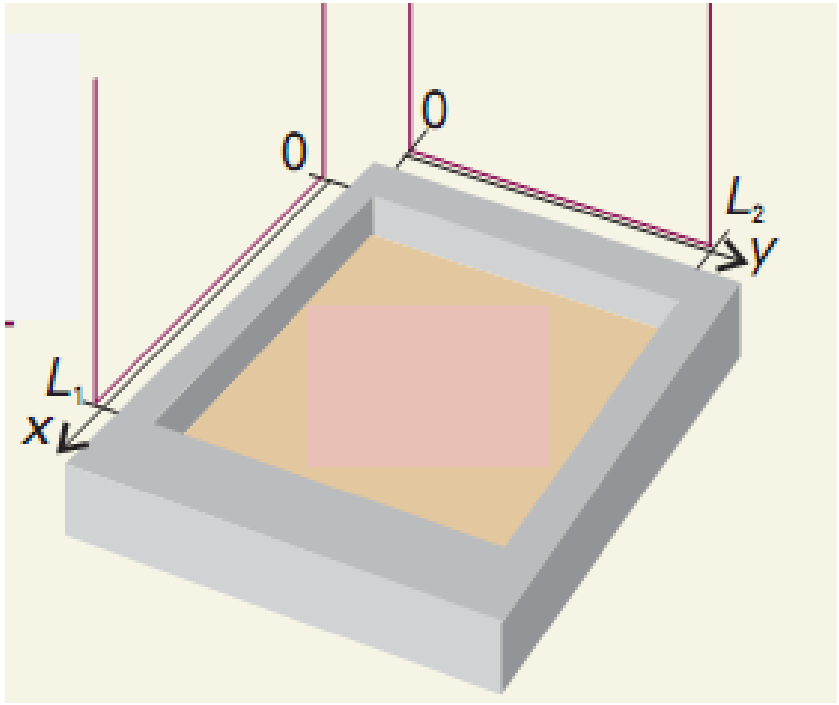
$$E = \frac{h^2}{8mL^2} n^2$$

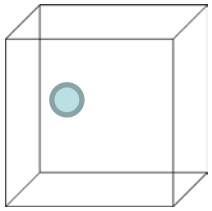
$$\Delta E = E_{n+1} - E_n = \frac{h^2}{8mL^2}(2n+1)$$

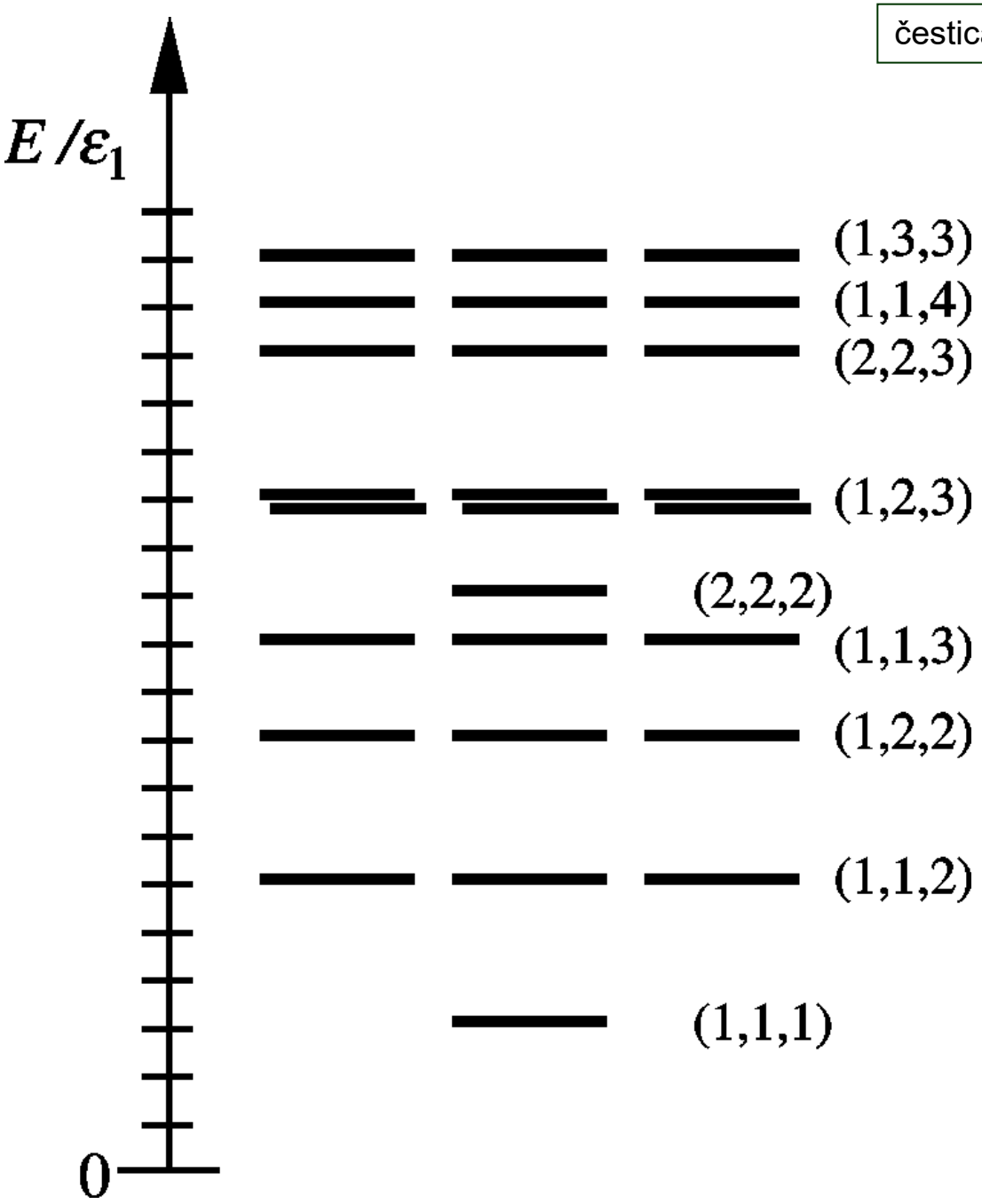


$$E = \frac{h^2}{8mL^2} n^2$$


E /ε1
(2,4)
(3,3)
(1,4)
(2,3)
(1,3)
(2,2)
(1,2)
(1,1)
0


0
0
L2
y
L1
x

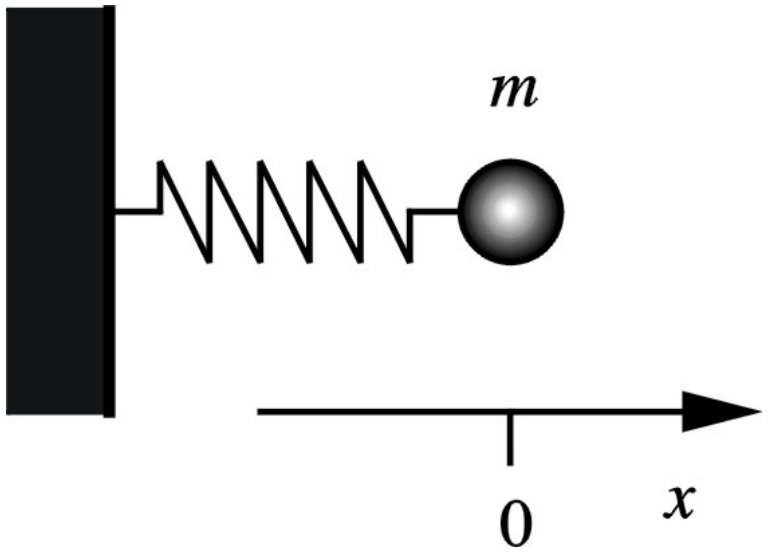



E/ε_1
0
(1,3,3)
(1,1,4)
(2,2,3)
(1,2,3)
(2,2,2)
(1,1,3)
(1,2,2)
(1,1,2)
(1,1,1)

Čestica u kutiji – pitanja za ponavljanje

1. O čemu ovise energije čestice u kutiji?
2. Kako energija ovisi o masi čestice?
3. Kako energija ovisi o dimenziji kutije?
5. Što je degeneracija energetske razine?
6. Kako se mijenja gustoća stanja s energijom?
7. Na kakve je sustave primjenjiv model čestice u kutiji?
8. Postoji li energija 0 za translaciju čestice u kutiji? Zašto?
9. Nacrtajte prve tri valne funkcije za česticu u kutiji?
10. Gdje je najveća vjerojatnost nalaženja čestice u $n = 1, 2$ i 3

Harmonički oscilator



Hooke:

$$F = -k\,x$$

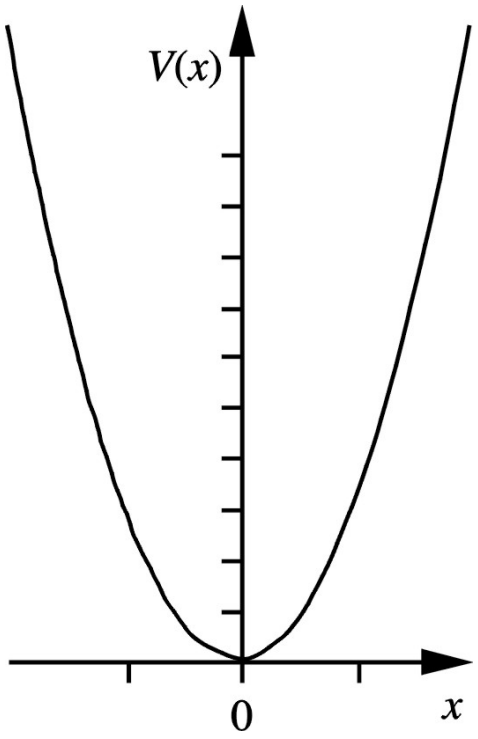
$$F = ma = m\ddot{x}$$

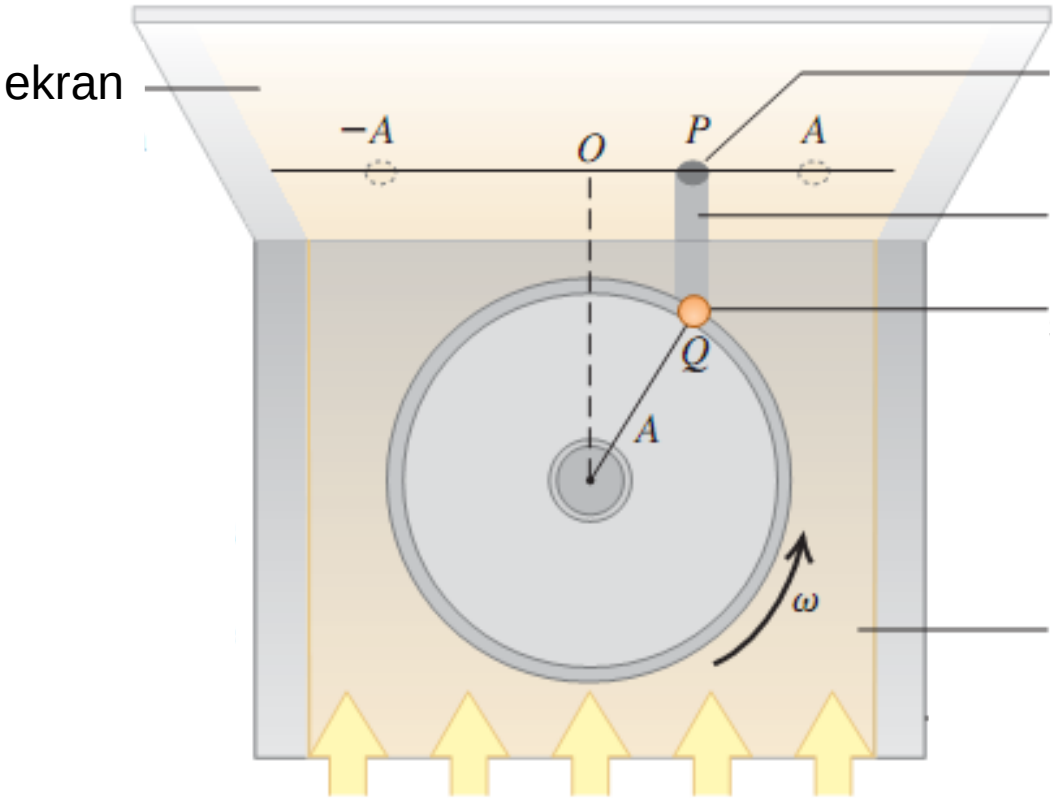
$$V = -\int F\mathrm{d}x = \int k\,x\mathrm{d}x$$

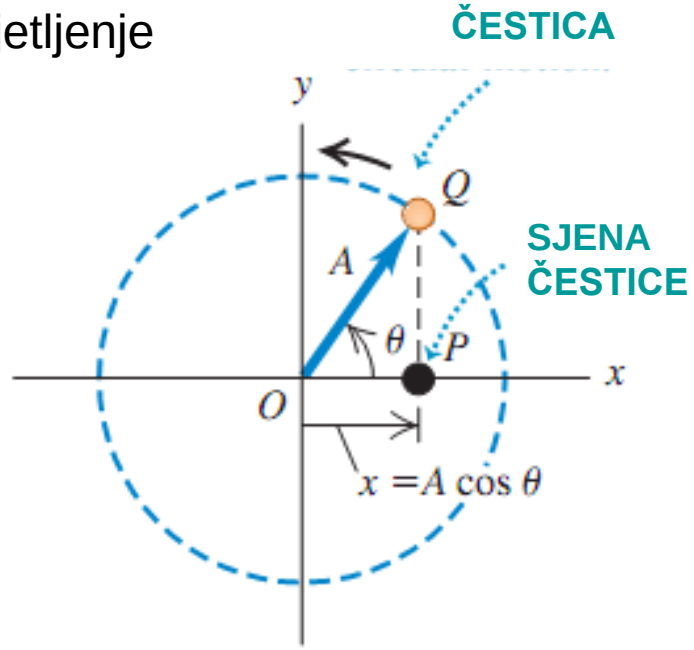
$$V = \tfrac{1}{2}kx^2$$

$$T = \frac{1}{2}mv_x^2 = \frac{1}{2}m\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = \frac{1}{2}m\dot{x}^2$$

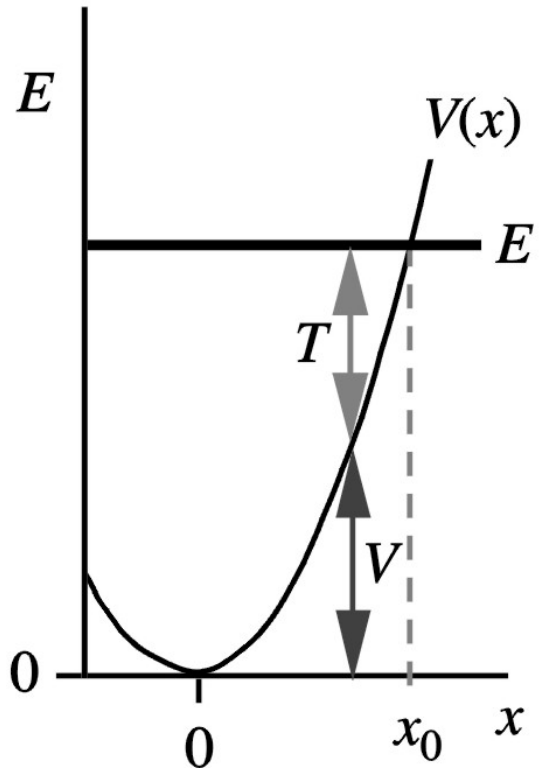
$$H = T + V$$

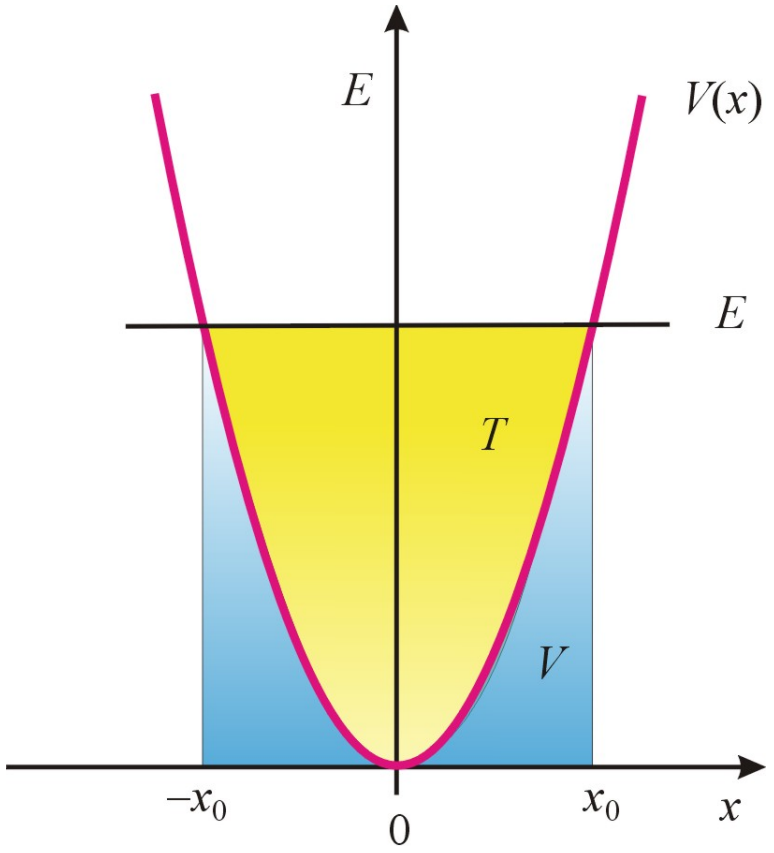



ekran
slika sjene čestice na ekranu
sjena čestice
čestica
osvjetljenje
izvor svjetla
−A
O
P
A
Q
A
ω


ČESTICA
SJENA ČESTICE
y
x
Q
A
θ
P
O
x = A cos θ

$$E = T + V = \frac{1}{2} k x_0^2$$





Harmonički oscilator –kvantna fizika

1. Napisati klasični hamiltonijan

$$H = T + V = \frac{1}{2} m v^2 + k x^2$$

2. Pretvoriti klasični hamiltonijan u kvantnomehanički operator

$$\hat{H} = \hat{T} + \hat{V} = \frac{-\hbar^2}{2m} \frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2} k x^2$$

3. Postaviti Schrödingerovu jednadžbu

$$\hat{H}\psi = \hat{T}\psi + \hat{V}\psi = \frac{-\hbar^2}{2m} \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{1}{2} k x^2 \psi = E\psi$$

4. Riješiti Schrödingerovu jednadžbu

Rješenje za osnovno stanje harmoničkog oscilatora:

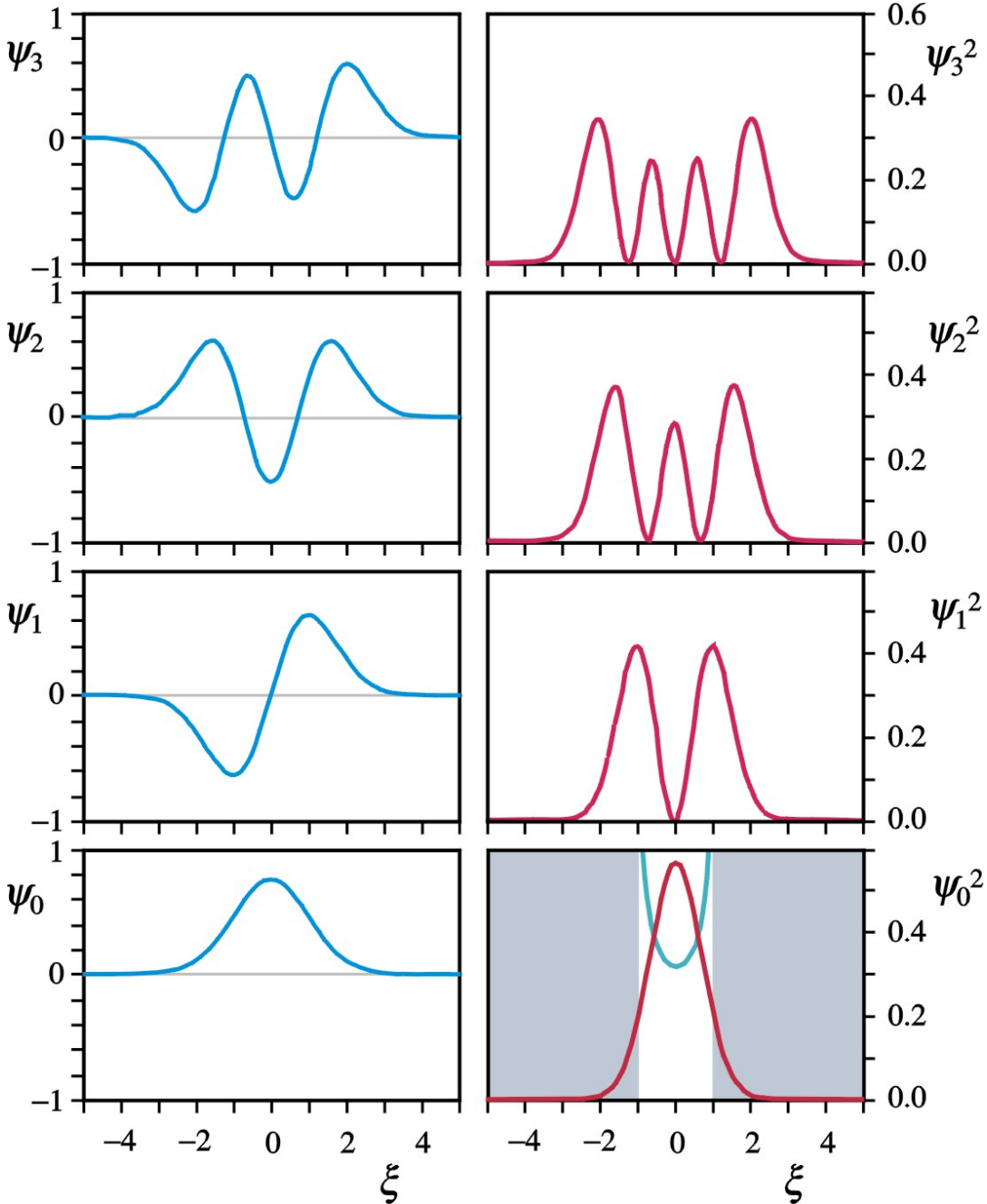
$$\Psi_0 = \exp\left\{-ax^2\right\} \qquad \Psi_0 = \exp\left\{-\frac{1}{2\hbar}\sqrt{km}\,x^2\right\} \qquad E_0 = \frac{h}{4\pi}\sqrt{\frac{k}{m}} = \frac{1}{2}h\nu$$

Općenito rješenje za harmonički oscilator:

$$\Psi_n = N_n H_n(x)\exp\left\{-ax^2\right\}$$

$$E_n = \left(n + \frac{1}{2}\right)h\nu; \qquad n = 0,1,2,3,\ldots$$

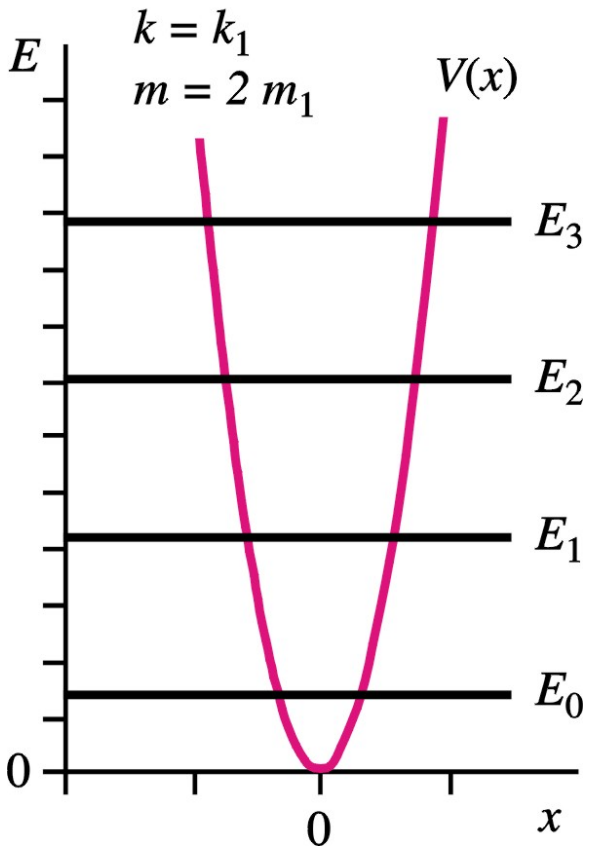
$$\Delta E = h\nu$$

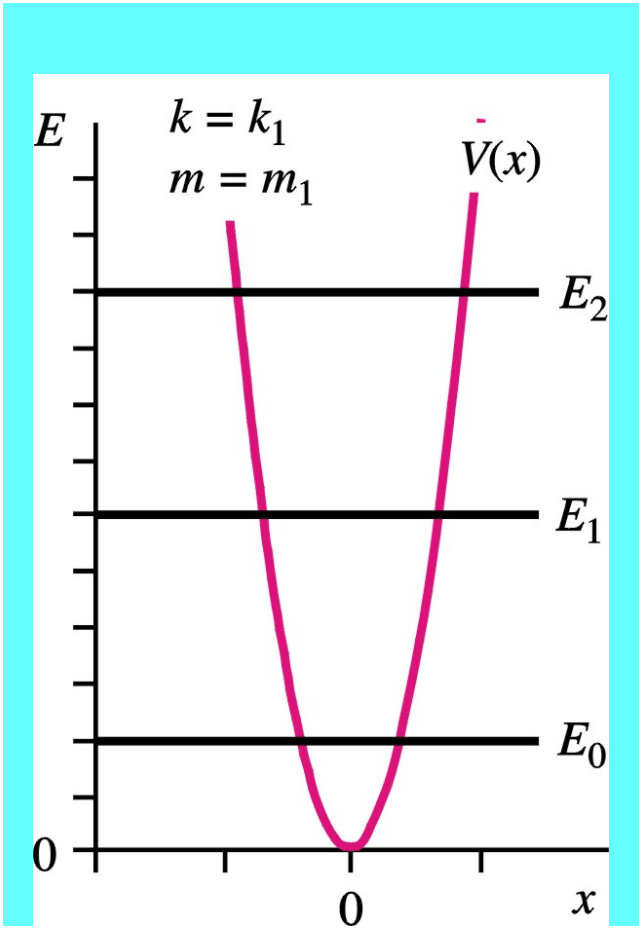


$$\Psi_n = N_n H_n(x) \exp\left\{-ax^2\right\}$$

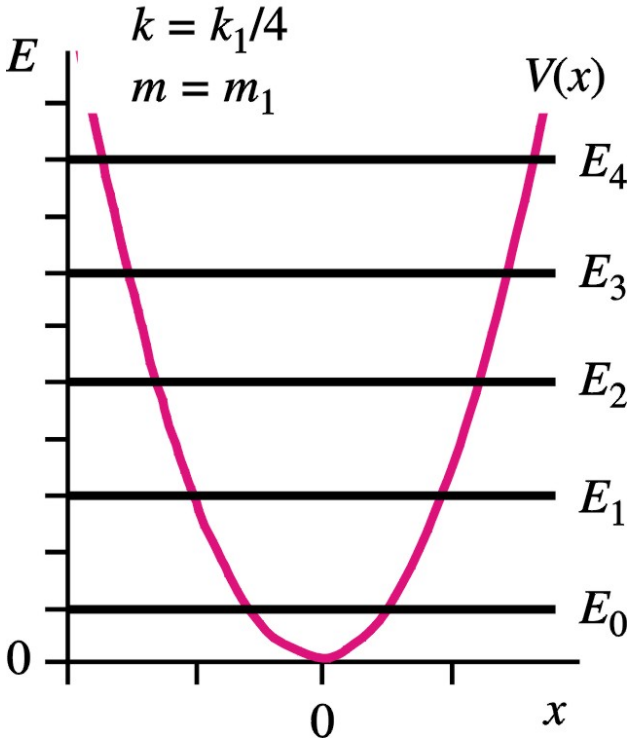
$$E_n = \left(n + \frac{1}{2}\right) h\nu;\ n = 0,1,2,3,\ldots$$

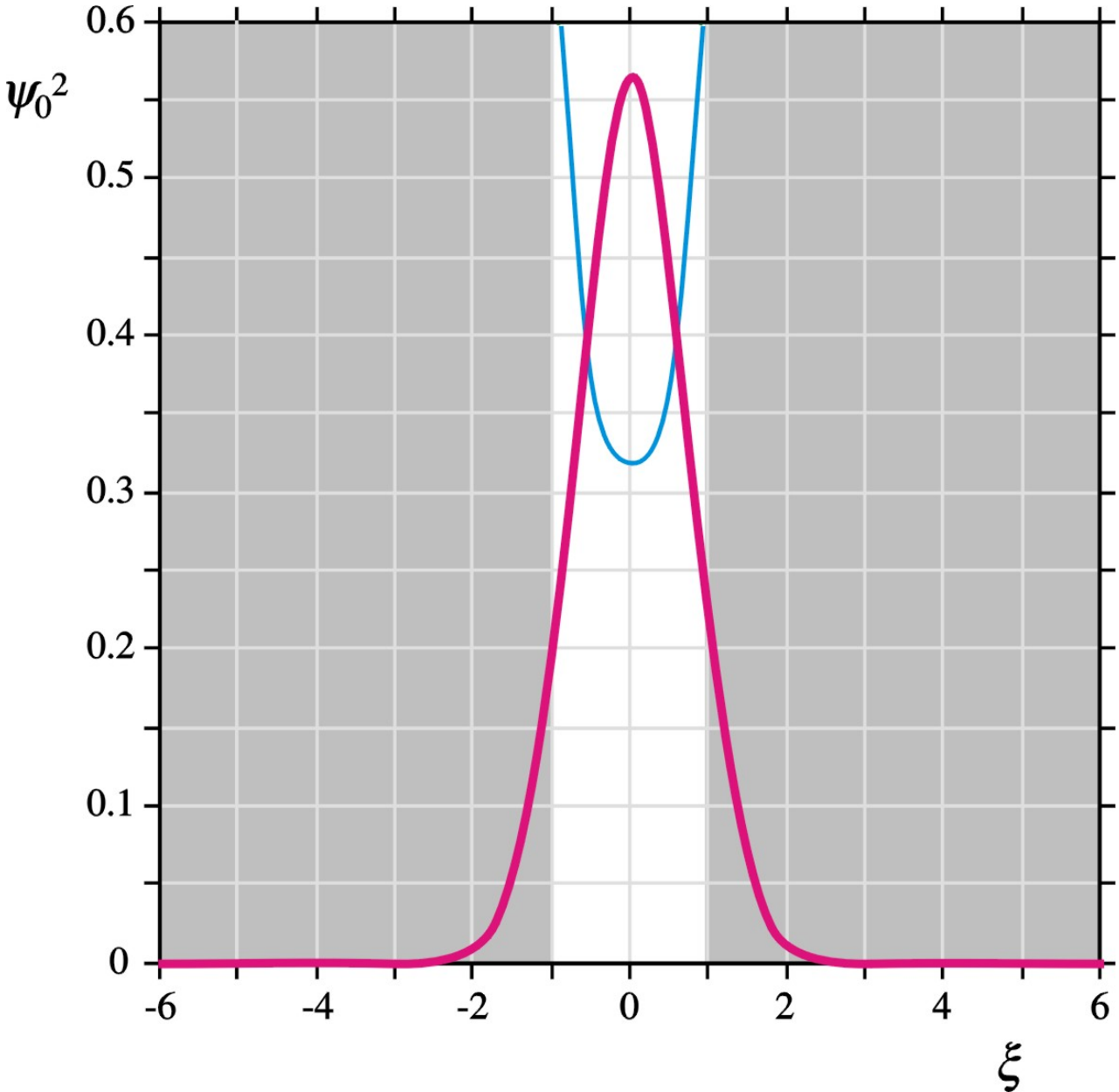
Utjecaj mase

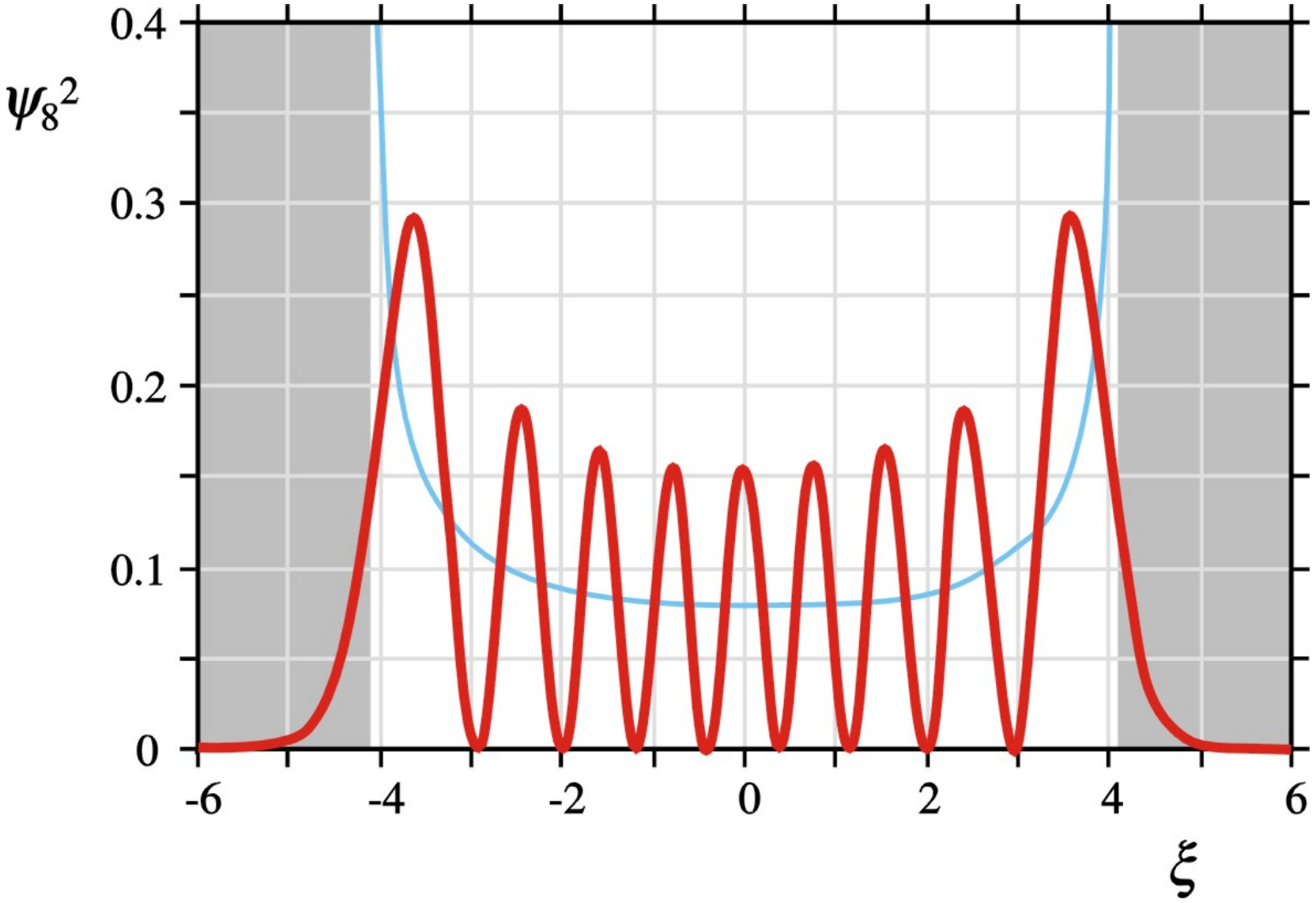


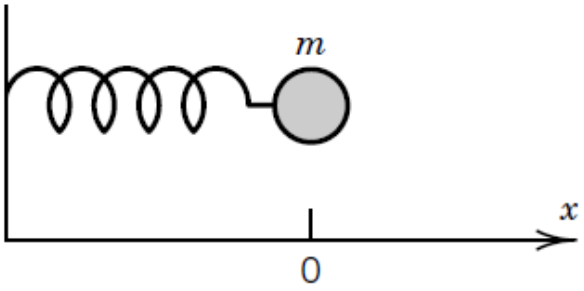


Utjecaj konstante sile

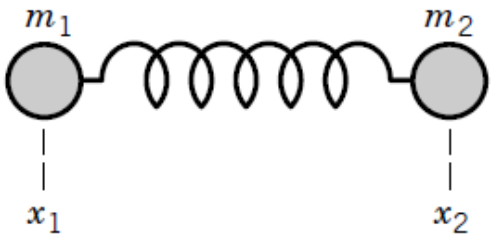



ψ_0^2
0.6
0.5
0.4
0.3
0.2
0.1
0
-6
-4
-2
0
2
4
6
ξ


ψ_8^2
0.4
0.3
0.2
0.1
0
-6
-4
-2
0
2
4
6
ξ



$$F_1 = k(x_2 - x_1 - l) = m_1 \frac{\mathrm{d}^2 x_1}{\mathrm{d}t^2}$$



$$F_2 = -k(x_2 - x_1 - l) = m_2 \frac{\mathrm{d}^2 x_2}{\mathrm{d}t^2}$$

$$(x_2 - x_1 - l) = x$$

$$-k\,x = \mu \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

Harmonički oscilator – pitanja za ponavljanje

1. O čemu ovisi frekvencija klasičnog harmoničkog oscilatora?
2. O čemu ovisi ukupna energija klasičnog harmoničkog oscilatora?
3. Kako se mijenja položaj s vremenom?
4. Kako se mijenja brzina s vremenom?
5. Kako potencijalna energija ovisi o vremenu?
6. Kako kinetička energija ovisi o vremenu?
7. Prikažite odnos kinetičke, potencijalne i ukupne energije za H.O.?
8. Kakva je raspodjela vjerojatnosti nalaženja čestice?
9. Kakve su energije kvantnog harmoničkog oscilatora?
10. Kojeg su oblika valne funkcije H.o?
11. Što je energija nulte točke?
12. Koje su glavne razlike klasičnog i kvantnog oscilatora?
13. Što je princip korespondencije?