

Gravitacijske funkcije strukture

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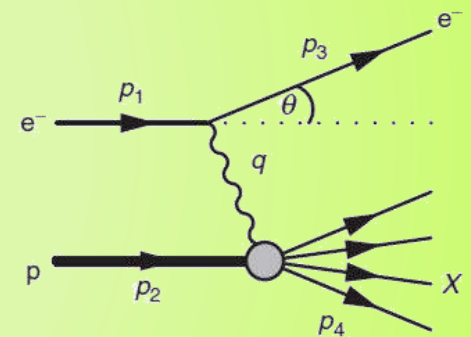
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Pregled:

- * Motivacija funkcija strukture
- * Izvod matričnog elementa tenzora energije i impulsa (TEM-a)
- * Ograničenja na funkcije strukture
- * Veza sa fizikalnim opservablama
- * D form faktor i njegova fizika
- * Veza s eksperimentom

1. Funkcije strukture

1. Eksperiment (DIS, DVCS...)
2. Udarni presjek
3. Matrični elementi
4. Rastav po Lorentzovim strukturama



Slika 1: DIS. Preuzeto iz [1]

Gravitacija se veže za tenzor energije i impulsa:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

Elektromagnetske funkcije strukture

- * Matrični element elektromagnetske struje:

$$O^\mu(x) \equiv \langle p', \sigma' | J^\mu(x) | p, \sigma \rangle$$

- * Invarijantnost na prostornovremenske translacije:

$$O^\mu(x) \equiv \langle p', \sigma' | \tau^{-1} \tau J^\mu(x) \tau^{-1} \tau | p, \sigma \rangle = e^{i(p-p')x} O^\mu(0)$$

- * Očuvanje struje:

$$\Delta_\mu O^\mu(x) = 0$$

$$\Delta^\mu = (p' - p)^\mu$$

$$P^\mu = \frac{1}{2}(p' + p)^\mu$$

Elektromagnetske funkcije strukture

- * Općeniti matrični element struje:

$$O^\mu(0) = \frac{q}{(2\pi)^3} \frac{1}{\sqrt{2p'^0} \sqrt{2p^0}} j^\mu(p', p)$$

Ovdje je
sadržana
fizika
problema!

- * Rastav 4-vektora na Lorentzove strukture:

$$j^\mu(p, p') = 2F_1(\Delta^2)P^\mu + F_2(\Delta^2)\Delta^\mu$$

Funkcije strukture (form faktori)

$$F_1(0) = 1$$

$$F_2(\Delta^2) = 0$$

2. Matrični element TEM-a

$$O^{\mu\nu}(x) \equiv \langle p', \sigma' | T^{\mu\nu}(x) | p, \sigma \rangle$$

- * Izvod se provodi u više koraka:
 1. Dijagonalni matrični element za spin 0
 2. Nedijagonalni matrični element za spin 0
 3. Dodavanje spinskog form faktora

Dijagonalni matični element spina 0

- * Zbog normalizacije imamo:

$$\langle p | T^{\mu\nu}(0) | p \rangle = 2p^\mu p^\nu$$

- * Ukupni TEM rastavljamo na doprinose po sastavnicama (kvarkovi i gluoni):

$$T^{\mu\nu} = \sum_a T_a^{\mu\nu}$$

- * Različiti doprinosi nisu očuvani (izmjena energije i impulsa) pa stoga uvodimo novu strukturu:

$$\langle p | T_a^{\mu\nu}(0) | p \rangle = 2A_a p^\mu p^\nu + 2c_a g^{\mu\nu}$$

- * Lagano se pokaže:

$$\sum_a A_a = 1 \quad \sum_a c_a = 0$$

Nedijagonalni matrični element spina 0

- * Koristi se simetričnost TEM-a (nema Levi-Civita strukture)
- * Stoga, najopćenitiji matrični element pišemo:

$$O^{\mu\nu}(0) = a_1 g^{\mu\nu} + a_2 p^\mu p^\nu + a_3 p^\mu \Delta^\nu + a_4 \Delta^\mu p^\nu + a_5 \Delta^\mu \Delta^\nu$$

- * Koristimo sačuvanje i prelazimo na varijable: P i Δ

$$O^{\mu\nu}(0) = 2a_3 P^\mu P^\nu + \frac{1}{2}(2a_5 - a_3)(\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2)$$

- * Definiranjem form faktora:

$$\langle p' | T_a^{\mu\nu}(x) | p \rangle = \left[2P^\mu P^\nu A_a(\Delta^2) + \frac{1}{2}(\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2) D_a(\Delta^2) + 2M^2 c_a(\Delta^2) g^{\mu\nu} \right] e^{i(p' - p)x}$$

Nedijagonalni matrični element spina 1/2

- * Očuvanje, kontrakcije i Diracova jednažba:

$$O^{\mu\nu}(x) = \bar{u}' \Gamma^{\mu\nu} u e^{i\Delta \cdot x}$$

- * Spinske strukture koje možemo koristiti:

$$S_1^{\mu\nu} = \bar{u}' F(\Delta^2) P^{\{\mu} \gamma^{\nu\}} u$$

$$S_2^{\mu\nu} = \bar{u}' J(\Delta^2) \frac{i P^{\{\mu} \sigma^{\nu\}} \rho \Delta_\rho}{2M} u$$

- * Ove strukture donose jednaku količinu informacija

Nedijagonalni matrični element spina 1/2

$$\langle p' \sigma' | T^{\mu\nu}(x) | p, \sigma \rangle = \bar{u}' \left[A(\Delta^2) \frac{P^\mu P^\nu}{M} + J(\Delta^2) \frac{iP^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{2M} + \right. \\ \left. + D(\Delta^2) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} \right] u e^{i(p'-p)x}$$



Korištenje Gordonovog identiteta

$$2J = A + B$$



$$\langle p' \sigma' | T_a^{\mu\nu}(x) | p, \sigma \rangle = \bar{u}' \left[A_a(\Delta^2) \frac{\gamma^{\{\mu} \gamma^{\nu\}}}{2} + B_a(\Delta^2) \frac{iP^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{4M} + \right. \\ \left. + D_a(\Delta^2) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + M c_a(\Delta^2) g^{\mu\nu} \right] u e^{i(p'-p)x}$$

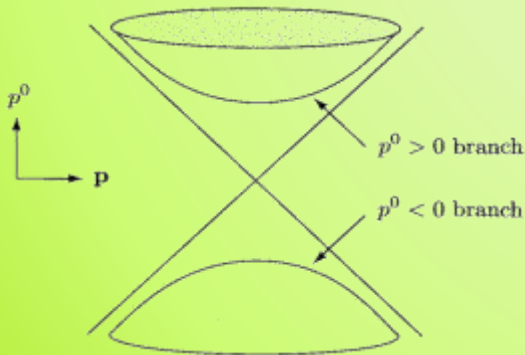
2. Ograničenja na form faktore

- * Kvantizirano polje je distribucija
- * Želimo stanja na gornjem hiperboloidu:

$$|p, \sigma, M\rangle \equiv \delta_M^{(+)}(p) |p, \sigma\rangle \quad \delta_M^{(+)}(p) \equiv 2\pi\theta(p^0)\delta(p^2 - M^2)$$

- * Matrični element: $\langle p', \sigma', M | T^{\mu\nu}(0) | p, \sigma, M \rangle =$

$$= \bar{u}' \left[\frac{1}{2} \gamma^{\{\mu} P^{\nu\}} A(\Delta^2) + \frac{i}{4M} P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho B(\Delta^2) + \frac{1}{4M} (\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) D(\Delta^2) \right] u \delta_M^{(+)}(p) \delta_M^{(+)}(p')$$



Slika 2: Gornji hiperboloid. Preuzeto iz [8]

Matrični element impulsa

- * Generator translacije definiramo pomoću TEM-a:

$$P^\mu \equiv \lim_{\substack{d \rightarrow 0 \\ R \rightarrow \infty}} \int d^4x f_{d,R}(x) T^{0\mu}(x) \longrightarrow \text{Test funkcija je odabrana tako da osigura konvergenciju}$$

- * Matrični element generatora:

$$\langle p', \sigma', M | P^\mu | p, \sigma, M \rangle = \lim_{\substack{d \rightarrow 0 \\ R \rightarrow \infty}} \tilde{f}_{d,R}(\Delta) \langle p', \sigma', M | T^{0\mu}(0) | p, \sigma, M \rangle$$

- * Stanje određenog impulsa je svojstveno stanje:

$$\langle p', \sigma', M | P^\mu | p, \sigma, M \rangle = p^\mu (2\pi)^4 \delta^4(\Delta) \delta_M^{(+)}(p') \delta_{\sigma, \sigma'}$$

$$A(0) = 1$$

Matriční element momenta impulsa

$$\langle p', \sigma', M | J^i | p, \sigma, M \rangle = -i \epsilon^{ijk} \lim_{\substack{d \rightarrow 0 \\ R \rightarrow \infty}} \frac{\partial \tilde{f}_{d,R}(\Delta)}{\partial \Delta_j} \langle p', \sigma', M | T^{0k}(0) | p, \sigma, M \rangle$$

$$\langle p', \sigma', M | J^i | p, \sigma, M \rangle = i \frac{\partial}{\partial \beta_i} \langle p', \sigma', M | U(\alpha = R) | p, \sigma, M \rangle_{\beta=0}$$

$$U(\alpha) | p, \sigma, M \rangle = \sum_{\sigma'} D_{\sigma' \sigma}^s(\alpha) | \Lambda(\alpha) p, \sigma', M \rangle$$

$$B(0) = 0$$

$$J(0) = \frac{1}{2}$$

3. Fizika form faktora

* Breitov sustav: $P = (E, 0, 0, 0)$
 $\Delta = (0, \mathbf{\Delta})$
 $t = \Delta^2 = -\mathbf{\Delta}^2$

* Statični TEM:

$$t_a^{\mu\nu}(\mathbf{r}, \mathbf{s}) = \int \frac{d^3 \mathbf{\Delta}}{(2\pi)^3 2E} e^{-i\mathbf{r}\mathbf{\Delta}} \langle p' | T_a^{\mu\nu}(0) | p \rangle$$

00-komponenta statičnog TEM-a

- * Dobro definiran samo za ukupni TEM

$$t^{00}(r) = M \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\mathbf{r}\Delta} \left[A(t) - \frac{t}{4M^2} [A(t) - 2J(t) + D(t)] \right]$$

- * Integriranjem lagano vidimo fizikalnu pozadinu form faktora A:

$$\int d^3 r t^{00}(r) = M$$

- * Dakle, form faktor A je povezan s energijom u hadronu

Veza spina i form faktora J

- * ok-komponenta statičnog TEM-a
- * Ne moramo promatrati cijeli TEM

$$J_a^i(\mathbf{r}, \mathbf{s}) = \epsilon^{ijk} r^j t_a^{0k}(\mathbf{r}, \mathbf{s})$$

$$J_a^i(\mathbf{r}, \mathbf{s}) = s^j \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\mathbf{r}\Delta} \left[\left(J_a(t) + \frac{2}{3} t \frac{dJ_a(t)}{dt} \right) \delta^{ij} + \left(\Delta^i \Delta^j - \frac{1}{3} \Delta^2 \delta^{ij} \right) \frac{dJ_a(t)}{dt} \right].$$

$$\sum_a \int d^3 r J_a^i(\mathbf{r}, \mathbf{s}) = s^i J(0)$$

Tenzor naprezanja

- * ij-komponente statičnog TEM-a

$$t^{ij}(\mathbf{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3} r^2 \delta^{ij} \right) s(r) + \delta^{ij} p(r)$$

- * Posljedice očuvanja TEM-a:

1. Veza tlaka i smicanja:

$$\frac{2}{3} s'(r) + \frac{2}{r} s(r) + p'(r) = 0$$

2. Von Laueov uvjet stabilnosti:

$$\int_0^\infty dr r^2 p(r) = 0$$

Tenzor naprezanja

- * D-član je definiran za slučaj kada nema prijenosa impulsa:

$$\mathcal{D}_a = D_a(0)$$

- * Njega možemo povezati s tlakom i smicanjem:

$$\mathcal{D} = -\frac{4}{15}M \int d^3r r^2 s(r) = M \int d^3r r^2 p(r)$$

4. Posljednja globalna nepoznanica hadrona

- * Tlak i smicanje možemo dobiti iz D form faktora:

$$s(r) = -\frac{1}{4M} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}(r)$$

$$p(r) = \frac{1}{6M} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r)$$

$$\tilde{D}(r) = \int \frac{d^r \Delta}{(2\pi)^2} e^{-i\Delta \mathbf{r}} D(t)$$

Kriterij stabilnosti

- * Diferencijalna sila na komadić površine:

$$dF^i = t^{ij}(\mathbf{r})dS^j = \left(\frac{2}{3}s(r) + p(r) \right) dS^i$$

- * Stabilnost sustava:

$$\frac{2}{3}s(r) + p(r) \geq 0$$

- * Ekvivalentan uvjet:

$$\mathcal{D} \leq 0$$

Mehanički radijus

- * Prethodni uvjet nam služi kao distribucija

$$\left\langle r^2 \right\rangle_{mech} = \frac{\int d^3r r^2 \left(\frac{2}{3} s(r) + p(r) \right)}{\int d^3r \left(\frac{2}{3} s(r) + p(r) \right)}$$

- * Može se povezati sa D-članom i D form faktorom:

$$\left\langle r^2 \right\rangle_{mech} = \frac{6D}{\int_{-\infty}^0 dt D(t)}$$

Model tekuće kapljice

* Dovoljno veliki sustavi

$$p(r) = p_0 \theta(R - r) - \frac{p_0 R}{3} \delta(R - r) \longrightarrow s(r) = \frac{p_0 R}{2} \delta(R - r) = \gamma \delta(R - r)$$

Veza tlaka i smicanja

$$\mathcal{D} = -\frac{4M}{5} \frac{4\pi}{3} \gamma R^4$$

$$\langle r^2 \rangle_{mech} = \frac{3}{5} R^2$$

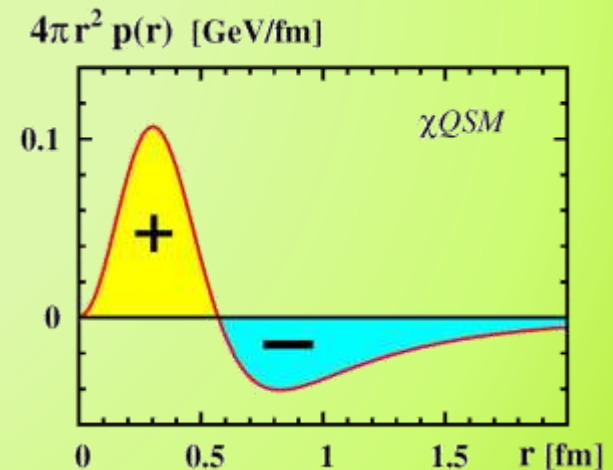
χ QSM model

- * Kiralno-kvarkovski solitonski model
- * D-član iščezava ako nema kiralnih interakcija

$$-4 \leq \mathcal{D} \leq -2$$

$$|D_u(t) + D_d(t)| \sim N_C^2 \gg |D_u(t) - D_d(t)| \sim N_C$$

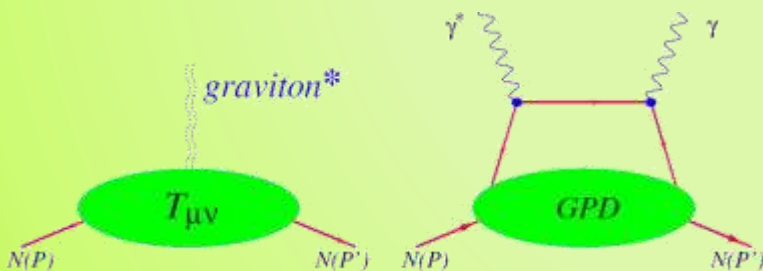
Okusna
hijerarhija D
form faktora



Slika 3: Ovisnost tlaka o udaljenosti od središta .
Preuzeto iz [13]

Eksperiment i GPD-evi

- * Duboko virtualno komptonско raspršenje (DVCS)
- * Generalizirane partonske distribucije (GPD)



$$\int_{-1}^1 dx x H_a(x, \zeta, t) = A_a(t) + \zeta^2 D_a(t)$$
$$\int_{-1}^1 dx x E_a(x, \zeta, t) = B_a(t) - \zeta^2 D_a(t)$$

Slika 4: Raspršenje gravitona lijevo i DVCS desno. Preuzeto iz [4]

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