# Formiranje Schrödingerovih ''cat-stanja'' u nanoelektromehaničkim sustavima

MATIJA TEČER

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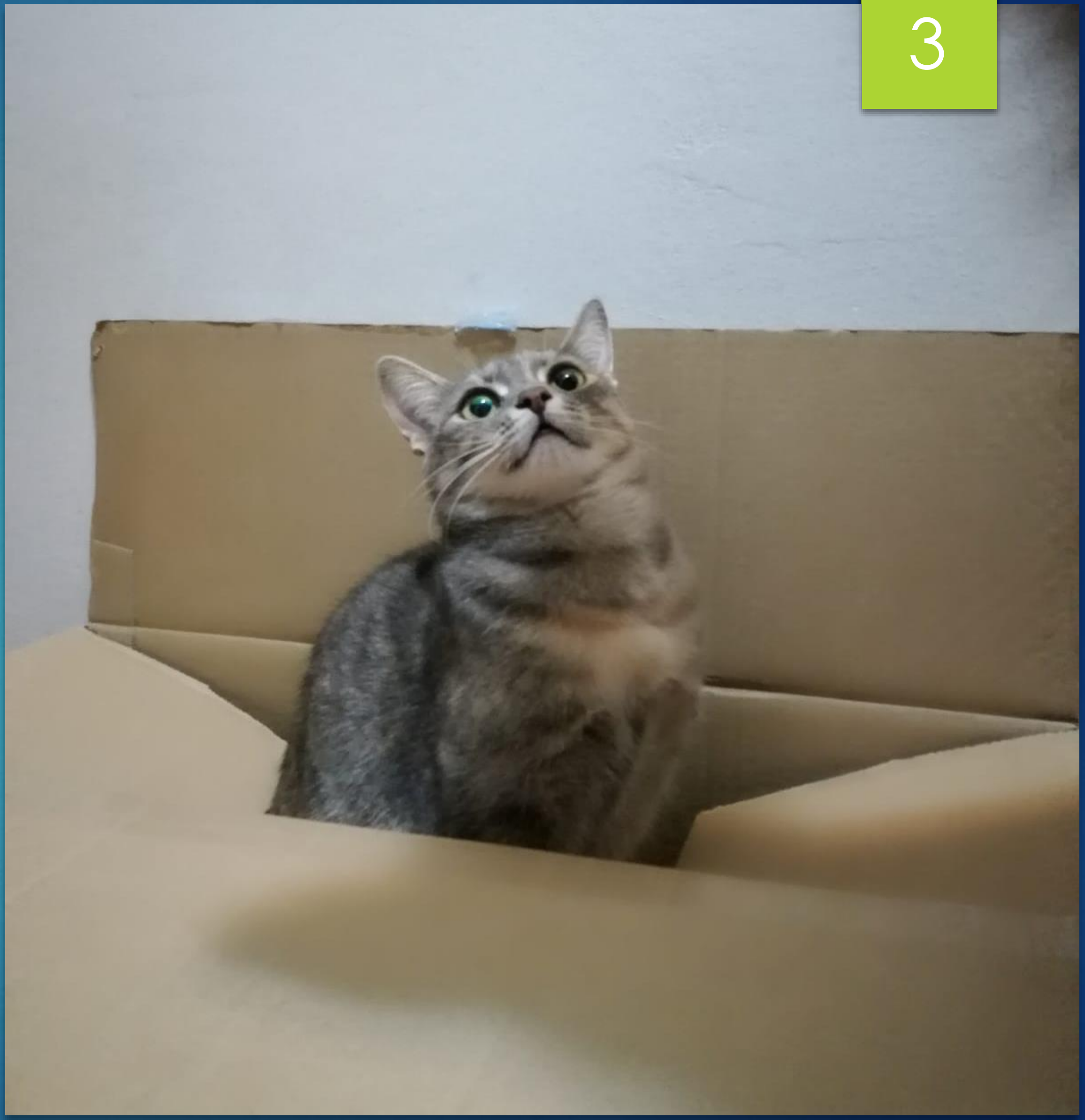
# Schrödingerova mačka

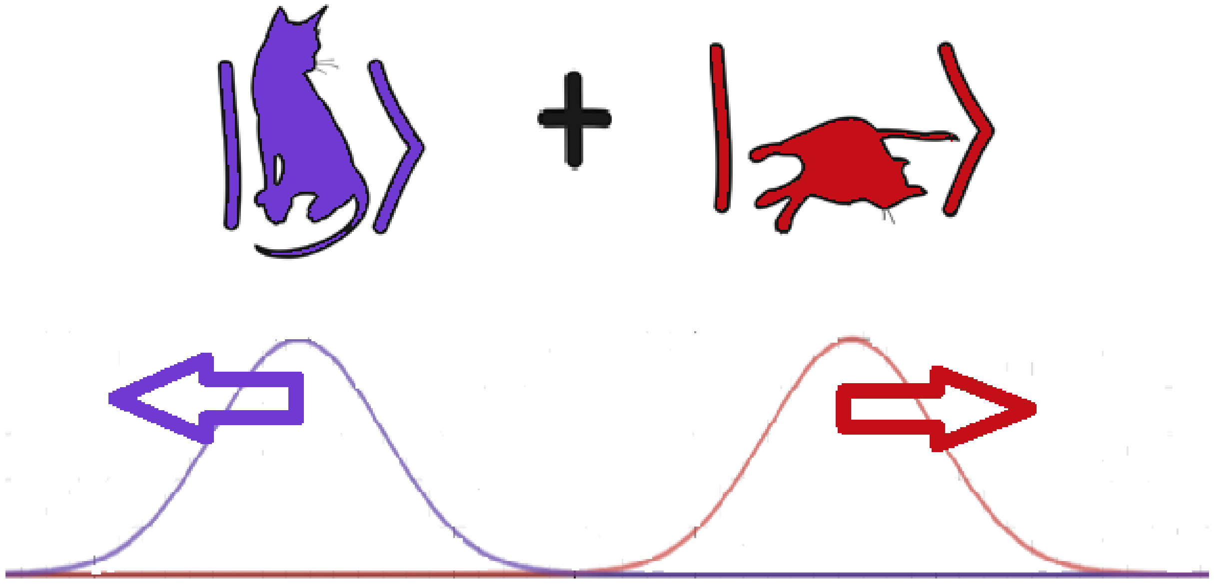
- $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle|Z\rangle + |1\rangle|M\rangle)$
- Superpozicija makroskopskih stanja?
- Dekoherencija → ansambl stanja



# ''Cat – stanja''



- Koherentna stanja:
  - Superpozicija Fockovih stanja
  - Otporna na perturbacije
  - Zadovoljavaju klasične jednadžbe gibanje
- ''Cat-stanja'' = superpozicija koherentnih stanja

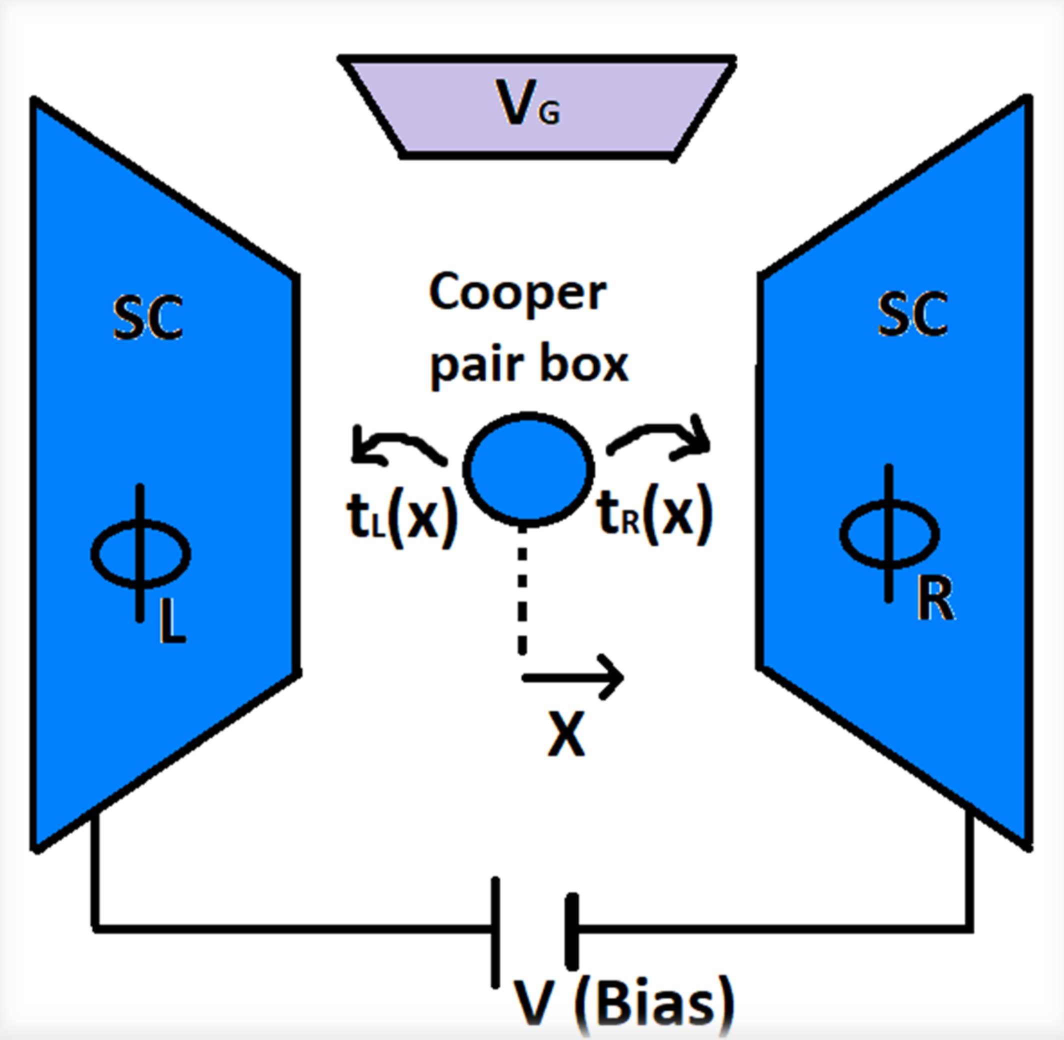


# Shema postava

- Prednapon V određuje fazu supravodljivih kontakata
  - $\phi = \phi_R - \phi_L$

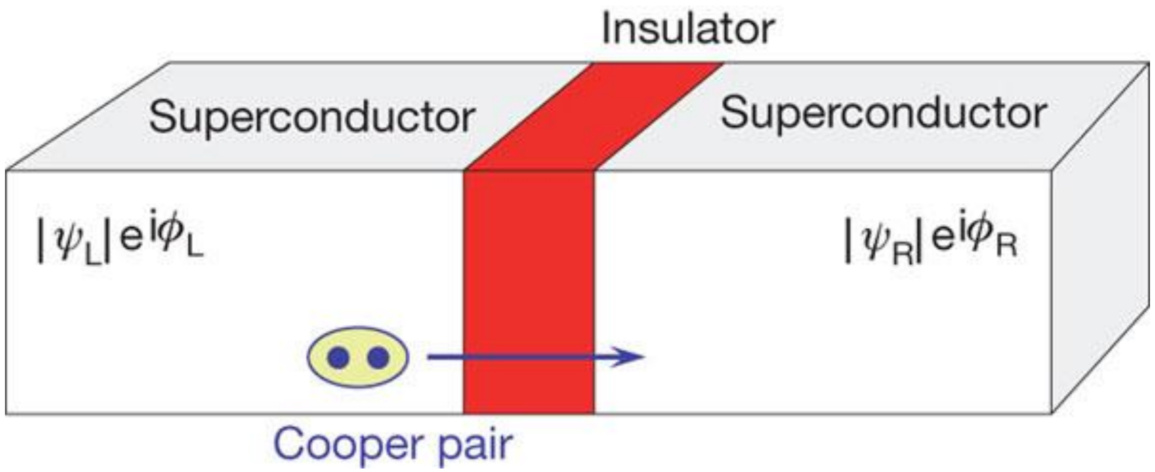
$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar}$$

- Napon vrata $V_G$ određuje elektrostatsku energiju supravodljive kvantne točke
  - Kvantna točaka se ponaša kao qubit (CPB)



# Josephsonov spoj

- Kvantni opis: $[\hat{\phi}, \hat{n}] = i$
  - $|\phi\rangle = \sum_n e^{in\phi}|n\rangle$
- Operator faze:
  - $\widehat{e^{i\phi}}|\phi\rangle = e^{i\phi}|\phi\rangle$
  - $\widehat{e^{i\phi}} = \sum_n |n-1\rangle\langle n|$

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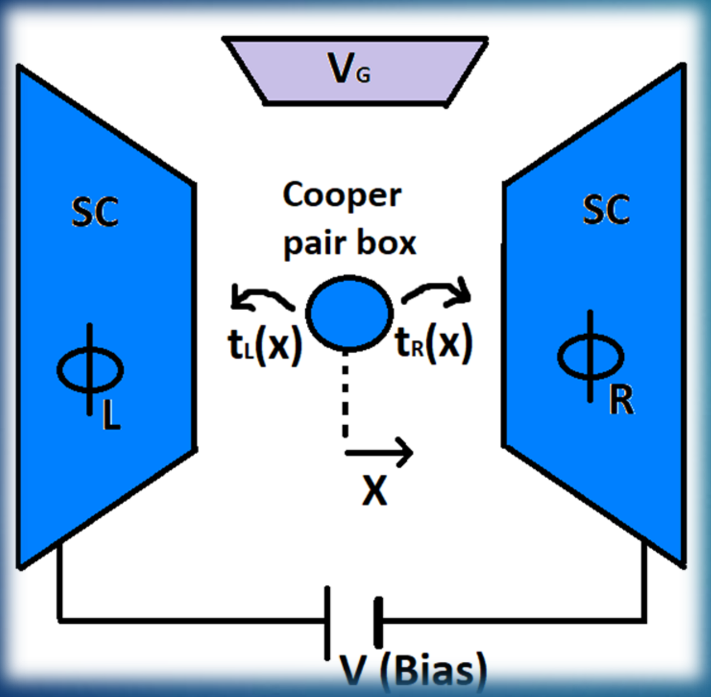
# Hamiltonijan sustava




$$H = H_C + H_{TUN} + H_{HO} + H_{SC}$$

$$H_C = \left(-2eV_G(t) + \frac{4e^2}{C}\right)|1\rangle\langle 1|$$

$$H_{HO} = \hbar\omega\left(\frac{\hat{P}^2}{2} + \frac{\hat{X}^2}{2}\right) = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$H_{SC} = |\phi_L\rangle\langle\phi_L| + |\phi_R\rangle\langle\phi_R|$$

$$H_{TUN} = t_L(\hat{x})\sum_{n_L}(|n_L+1\rangle\langle n_L| \otimes |0\rangle\langle 1| + |n_L\rangle\langle n_L+1| \otimes |1\rangle\langle 0|) +$$
$$t_R(\hat{x})\sum_{n_R}(|n_R+1\rangle\langle n_R| \otimes |0\rangle\langle 1| + |n_R\rangle\langle n_R+1| \otimes |1\rangle\langle 0|))$$

$$\hat{P} = \sqrt{\frac{1}{\hbar m\omega}}\hat{p} \qquad \hat{X} = \sqrt{\frac{\hbar}{m\omega}}\hat{x}$$

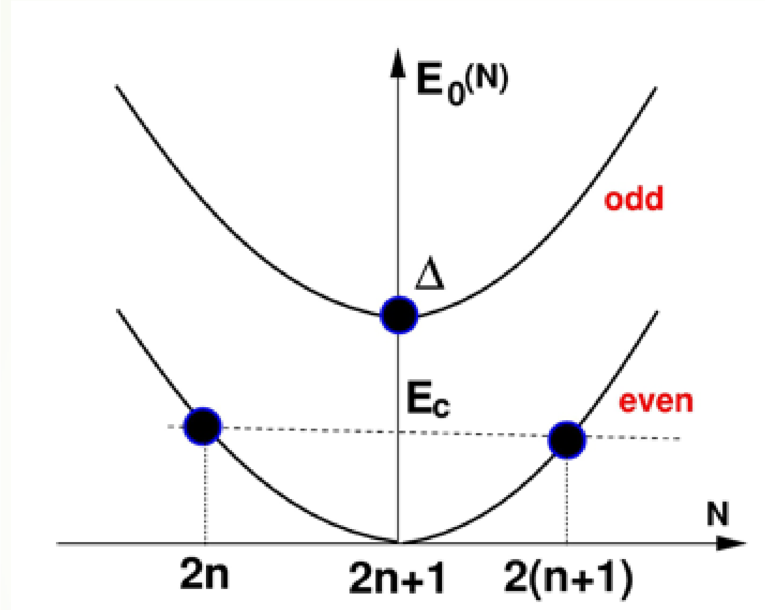
# Elektrostatska energija

- Najopćenitiji izraz za elektrostatsku energiju kvantne točke:

$$E_C = \frac{e^2}{2C}(N - \alpha V_G)^2 + \Delta$$

$$\Delta_N = \begin{cases} 0, & N = 2n \\ \Delta, & N = 2n + 1 \end{cases}$$

- Postavimo $\alpha V_G = 2n + 1$ → degenerirano osnovno stanje
- $\Delta \gg \frac{e^2}{2C}$ i $E_J < \frac{9e^2}{2C}$ → dvorazinski sustav (qubit):
  - $|0\rangle = |2n\rangle,\ |2(n+1)\rangle = |1\rangle$



# Hamiltonijan tuneliranja

$$H_{TUN} = t_L(\hat{x}) \sum_{n_L} (|n_L+1\rangle \langle n_L| \otimes |0\rangle \langle 1| + |n_L\rangle \langle n_L+1| \otimes |1\rangle \langle 0|) +$$
$$t_R(\hat{x}) \sum_{n_R} (|n_R+1\rangle \langle n_R| \otimes |0\rangle \langle 1| + |n_R\rangle \langle n_R+1| \otimes |1\rangle \langle 0|))$$

- Amplitude tuneliranja: $t_L(x) = -\frac{E_J}{2} e^{\frac{-\hat{x}}{\lambda}}$, $t_R(x) = -\frac{E_J}{2} e^{\frac{\hat{x}}{\lambda}}$, $E_J$ – Josephsonova energija
- Definiramo mali parameter: $\varepsilon = \frac{x_0}{\lambda} \ll 1,\quad x_0 = \sqrt{\frac{\hbar}{m\omega}}$
- Prepoznajući operator faze i razvojem po malom parametru dobivamo:

$$H_{TUN} = -E_J \cos(\phi)\hat{\sigma}_x + \epsilon E_J \sin(\phi)\hat{X}\hat{\sigma}_y, \qquad \phi = \frac{\phi_R - \phi_L}{2}$$

# Slika interakcije



- $H = H_0 + H_I,$
  - $H_0 = -E_J \cos(\phi)\,\sigma_x + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$
  - $H_I = \varepsilon E_J \sin(\phi)\,\sigma_y \hat{X}, \qquad \hat{X} = \frac{1}{\sqrt{2}}(a^\dagger + a)$
- Valna funkcija u slici interakcije: $|\tilde{\psi}\rangle = U_0^\dagger\,|\psi\rangle$
- $U_0$ zadovoljava jednadžbu: $i\hbar\frac{\partial U_0(t,t')}{\partial t} = H_0 U_0(t,t')$
- $|\tilde{\psi}\rangle$ evoluira operatorom $U_I$ : $i\hbar\frac{\partial U_I(t,t')}{\partial t} = \widetilde{H_I} U_I(t,t')$
- $H_I$ u slici interakcije: $\widetilde{H_I} = U_0^\dagger H_I U_0$
- $|\psi(t)\rangle$ evolouira kao: $|\psi(t)\rangle = U_0(t) U_I(t,t_0) U_0^\dagger(t_0)|\psi(t_0)\rangle$

# Konstantni prednapon

- Postavimo: $V(t) = V_0\Theta(t)$ → $\phi = \nu t, \quad \upsilon = \frac{2eV_0}{\hbar}$ (Josephsonova frekvencija)
- Također pretpostavljamo: $\omega = k\nu, \quad k \in \mathbb{N}$ (promatrati ćemo slučaj k=1)
- $[H_0(t), H_0(t')] = 0$ → $U_0 = \exp\left(-i\omega a^\dagger a t + i\frac{E_j}{\hbar\nu}\sin(\nu t)\right)$
- $\widetilde{H}_I(t) = \varepsilon E_J\left(\left(f_1(t)\hat{X} + f_2(t)\hat{P}\right)\sigma_y + \left(f_3(t)\hat{X} + f_4(t)\hat{P}\right)\sigma_z\right)$

$$f_1(t) = \cos\left(\frac{2E_J}{\hbar\nu}\sin(\nu t)\right)\sin(\nu t)\cos(k\nu t)$$

$$f_2(t) = \cos\left(\frac{2E_J}{\hbar\nu}\sin(\nu t)\right)\sin(\nu t)\sin(k\nu t)$$

$$f_3(t) = \sin\left(\frac{2E_J}{\hbar\nu}\sin(\nu t)\right)\sin(\nu t)\cos(k\nu t)$$

$$f_4(t) = \sin\left(\frac{2E_J}{\hbar\nu}\sin(\nu t)\right)\sin(\nu t)\sin(k\nu t)$$

# Aproksimacija stacionarnih faza

- Zanemarivanje svih oscilatornih doprinosa Hamiltonijanu
- Hamiltonijan se razvija u Fourierov red i zadržava se samo konstantni doprinos:

$$f_i(x) = \frac{a_{0i}}{2} + \sum_{n=1}^{\infty} a_{ni}\cos(nx) + \sum_{n=1}^{\infty} b_{ni}\sin(nx), \quad x = \nu t$$

- Dobiva se **vremenski neovisan** $\widetilde{H_I}$ → možemo izračunati $U_I$: $\tilde{U}_I = \exp\left(-i\epsilon\alpha t\hat{P}\sigma_y\right)$
- $\alpha = \frac{E_J}{2\hbar} a_{02} = \frac{E_J}{2\hbar}\left( J_{k-1}\left(\frac{2E_J}{\hbar\nu}\right) - J_{k+1}\left(\frac{2E_J}{\hbar\nu}\right)\right)$
- Operator prostorne translacije: $e^{-i\bar{X}\hat{P}}|0\rangle = |\bar{X}\rangle$ (koherentno stanje s <X>= $\bar{X}$ )

# Evolucija sustava

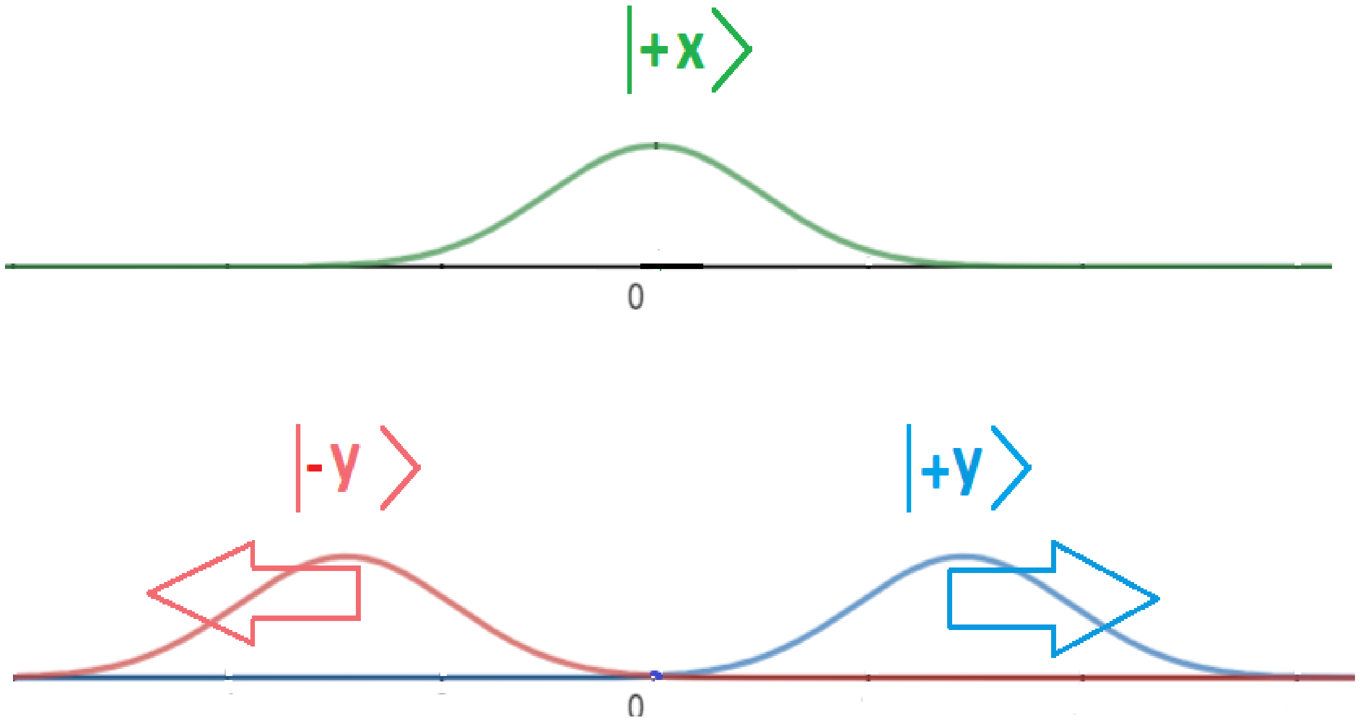
$$\tilde{U}_I = \exp\left(-i\epsilon\alpha t\hat{P}\sigma_y\right)$$

- Početno stanje:

$$|\psi(t=0)\rangle = |+x\rangle\,|0\rangle$$

- Evoluira (slika interakcije) u:

$$|\psi(t)\rangle = \frac{1+i}{2}|+y\rangle\,|\alpha\varepsilon t\rangle + \frac{1-i}{2}|-y\rangle\,|-\alpha\varepsilon t\rangle$$



# Protokol za dobivanje ''cat-stanja''



- Razlika faza prije uključivanja prednapona:

$$\phi(t=0) = -\phi_0$$

- U trenutku $t_1$ okrećemo predznak napona:

$$V(t) = \begin{cases} 0, & t < 0 \\ V_0, & t \in [0, t_1] \\ -V_0, & t > t_1 \end{cases}$$

- Zasebno tražimo $U_0$ za dva intervala:

$$U_0(t) = \begin{cases} \exp\left(-i\omega a^\dagger a t + \frac{iE_J}{\hbar\nu} h(t, \phi_0)\sigma_x\right), & t \in [0, t_1] \\ \exp\left(-i\omega a^\dagger a t + \frac{iE_J}{\hbar\nu} g(t, t_1, \phi_0)\sigma_x\right), & t > t_1 \end{cases}$$

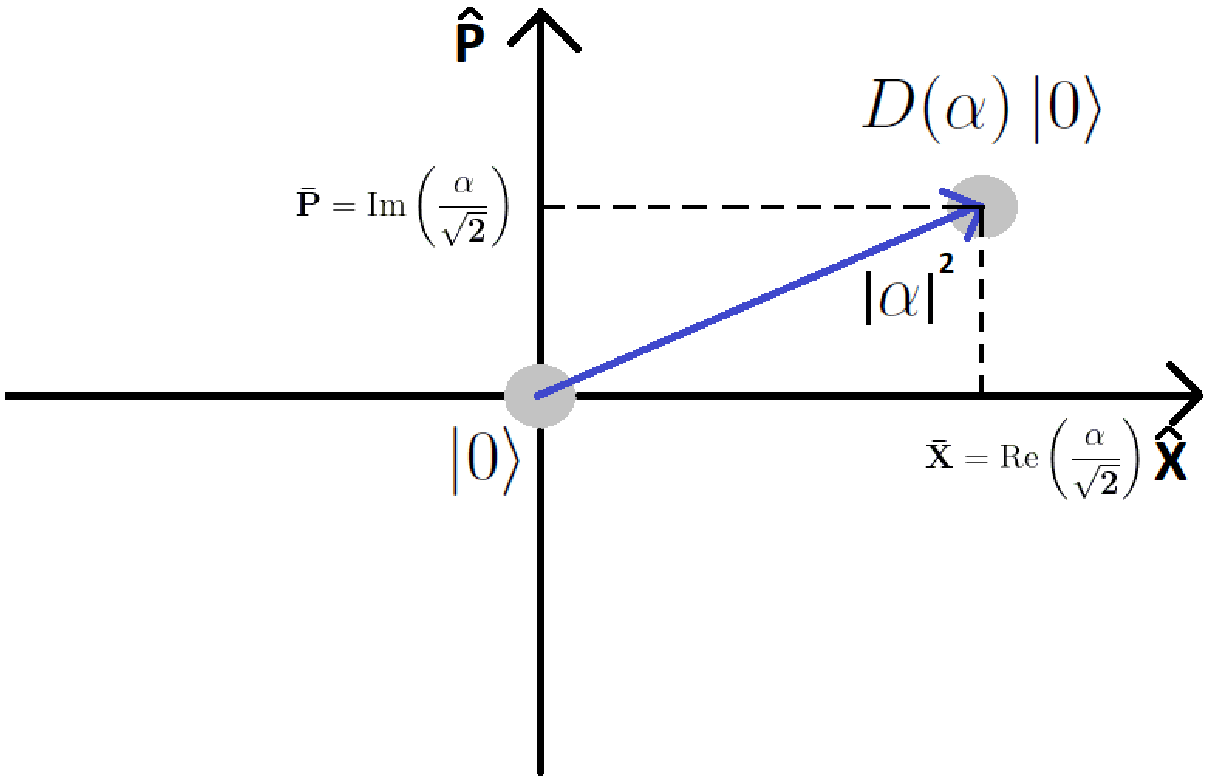
- Aproksimacija stacionarnih faza:

$$\tilde{H}_I = \frac{\epsilon E_J}{2}\left[\left(b_{01}\hat{X} + b_{02}\hat{P}\right)\sigma_y + \left(b_{03}\hat{X} + b_{04}\hat{P}\right)\sigma_z\right], \quad t \in [0, t_1]$$

$$\tilde{H}_I = \frac{\epsilon E_J}{2}\left[\left(c_{01}\hat{X} + c_{02}\hat{P}\right)\sigma_y + \left(c_{03}\hat{X} + c_{04}\hat{P}\right)\sigma_z\right], \quad t > t_1$$

# Operator pomaka mehaničkih stanja

- $D(\alpha) = |\alpha\rangle$, $|\alpha\rangle$ koherentno stanje
- $D(\alpha) = e^{i(\bar{P}\hat{X} - \bar{X}\hat{P})}$
  - $\alpha = \frac{1}{\sqrt{2}}(\bar{X} + i\bar{P})$
- $D(\alpha_2)D(\alpha_1) = e^{i\,Im(\alpha_2\alpha_1^*)}D(\alpha_1 + \alpha_2)$
- $|\langle\alpha_2|\alpha_1\rangle|^2 = e^{-|\alpha_2 - \alpha_1|^2}$



# Ideja protokola

- Početno stanje:

$$|\psi(t=0)\rangle = |+x\rangle\,|0\rangle$$

- Do trenutka $t_1$ želimo koherentna stanja vezana na svojstena stanja $\sigma_z$:

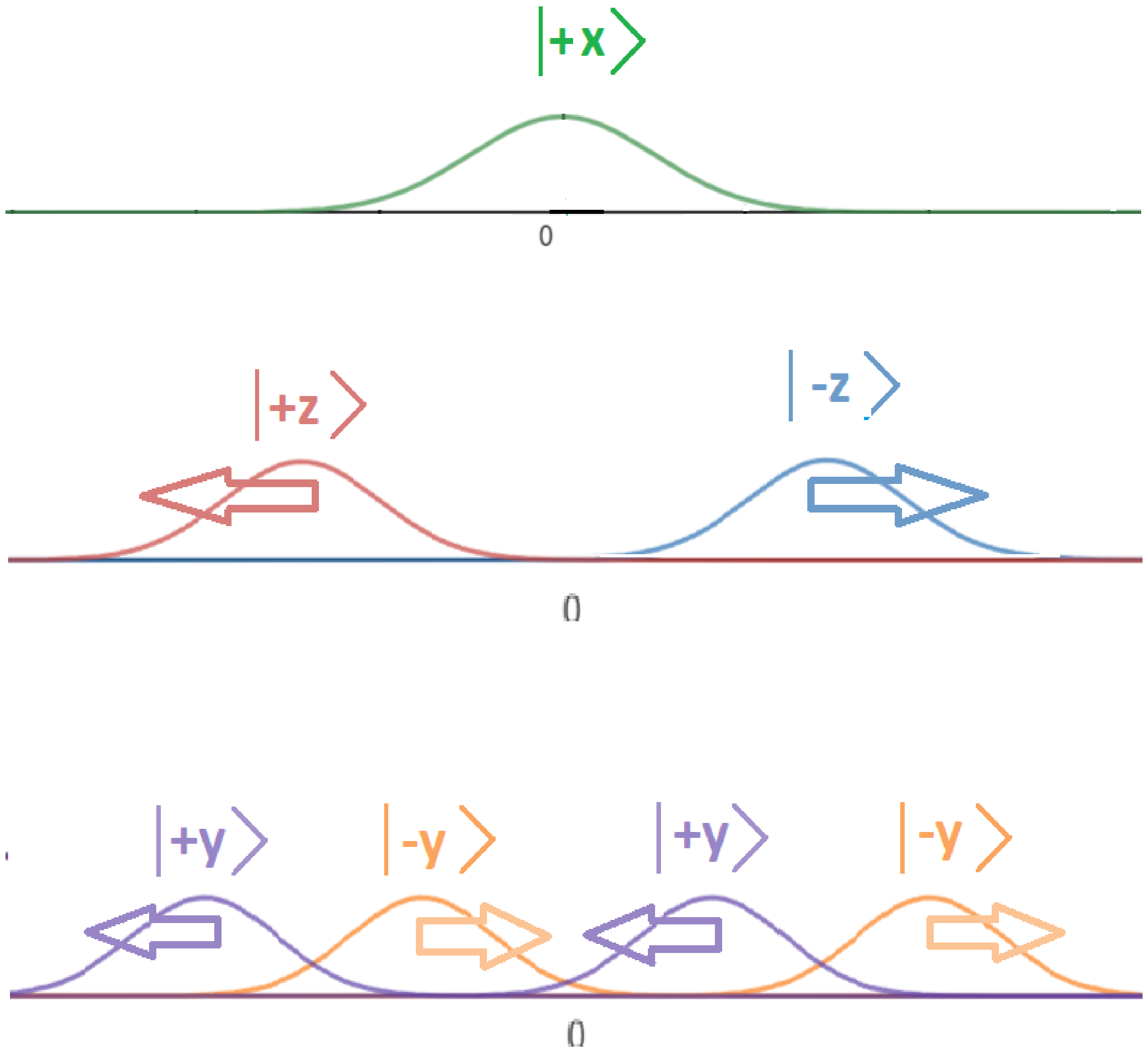
$$b_{01} = b_{02} = 0$$

$$\frac{2E_J}{\hbar\nu} sin(\phi_0) = (2K+1)\frac{\pi}{2}, \quad K \in \mathbb{Z}$$

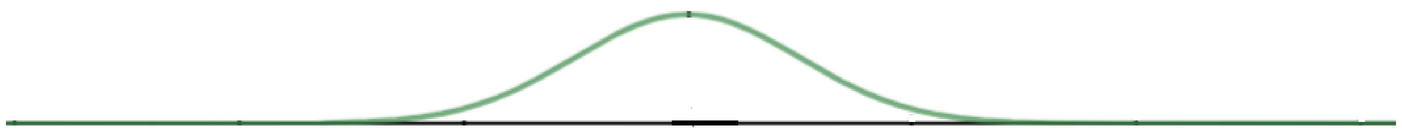
- Nakon toga se koherentna stanja razdvajaju na stanja vezana na svojstvena stanja $\sigma_y$:

$$c_{03} = c_{04} = 0$$

$$\nu t_1 = \arcsin\left(M - \frac{2K+1}{2}\right)\frac{\sin(\phi_0)}{2K+1} + \phi_0$$

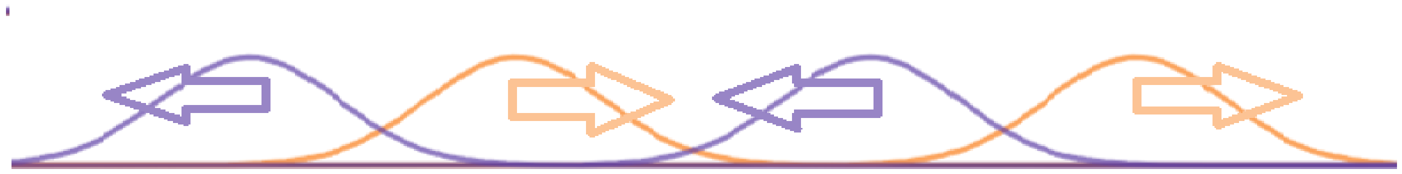


$$|\psi(t=0)\rangle = |+x\rangle\,|0\rangle$$



$$|\tilde{\psi}(t_1)\rangle = \frac{1}{\sqrt{2}}|+z\rangle\,|-\alpha_1(t_1)\rangle + \frac{1}{\sqrt{2}}|-z\rangle\,|\alpha_1(t_1)\rangle$$

$$|\tilde{\psi}(t)\rangle = \frac{1}{2}|+y\rangle\,\{\beta(t,t_1)\,|-\alpha_1(t_1)-\alpha_2(t-t_1)\rangle - i\beta^*(t,t_1)\,|+\alpha_1(t_1)-\alpha_2(t-t_1)\rangle\} + \frac{1}{2}|-y\rangle\,\{\beta^*(t,t_1)\,|-\alpha_1(t_1)+\alpha_2(t-t_1)\rangle + i\beta(t,t_1)\,|+\alpha_1(t_1)+\alpha_2(t-t_1)\rangle\}$$



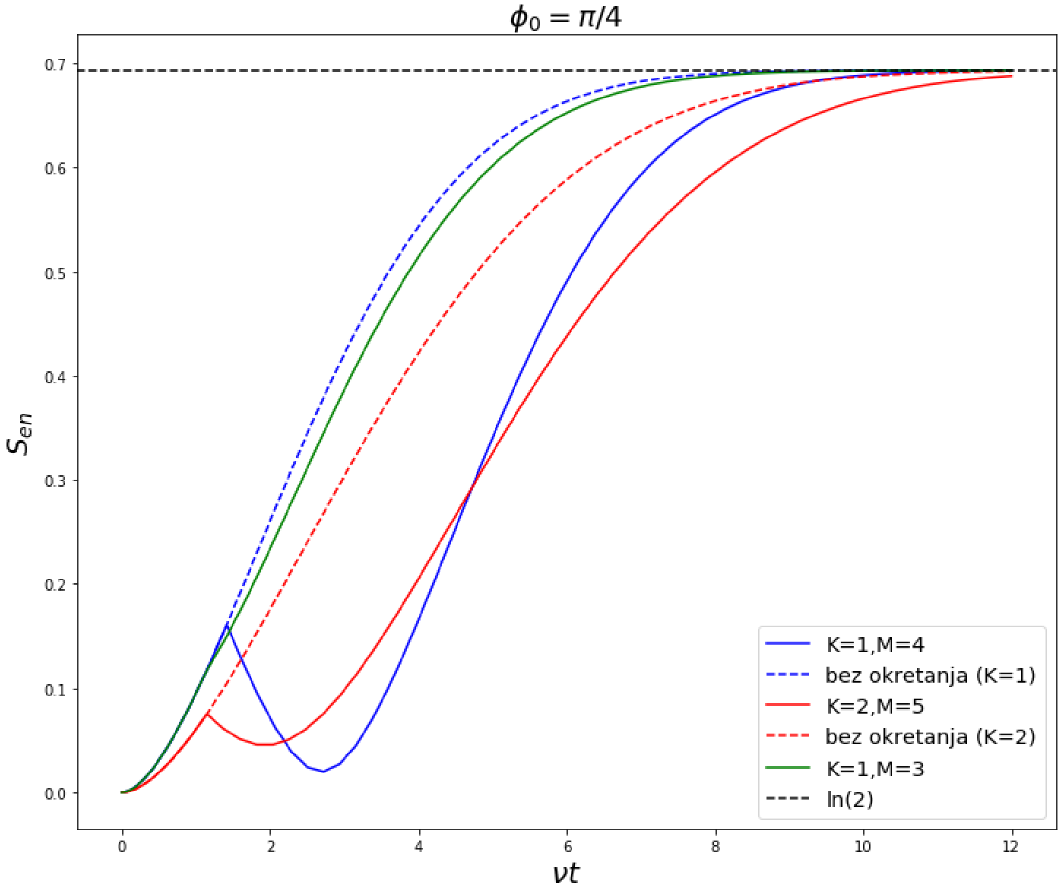
# Entropija kvantne isprepletenosti

- $\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$
  $= |\psi_+(t)\rangle|+y\rangle\langle+y|\langle\psi_+(t)| + |\psi_+(t)\rangle|+y\rangle\langle-y|\langle\psi_-(t)|$
  $+ |\psi_-(t)\rangle|-y\rangle\langle+y|\langle\psi_+(t)| + |\psi_-(t)\rangle|-y\rangle\langle-y|\langle\psi_-(t)|$

- $S = \mathrm{Tr}\left(\hat{\rho}_q ln\hat{\rho}_q\right) = \mathrm{Tr}(\hat{\rho}_m ln\hat{\rho}_m)$

- $\hat{\rho}_q(t) = \frac{1}{2}\begin{pmatrix} 1 & \eta(t) \\ \eta^*(t) & 1 \end{pmatrix}, \quad \eta(t) = \langle\psi_-(t)|\psi_+(t)\rangle$

- $S(t) = \ln(2) - \frac{1}{2}\ln(1-|\eta(t)|^2) - \frac{1}{2}\ln\left(\frac{1+|\eta(t)|}{1-|\eta(t)|}\right)$

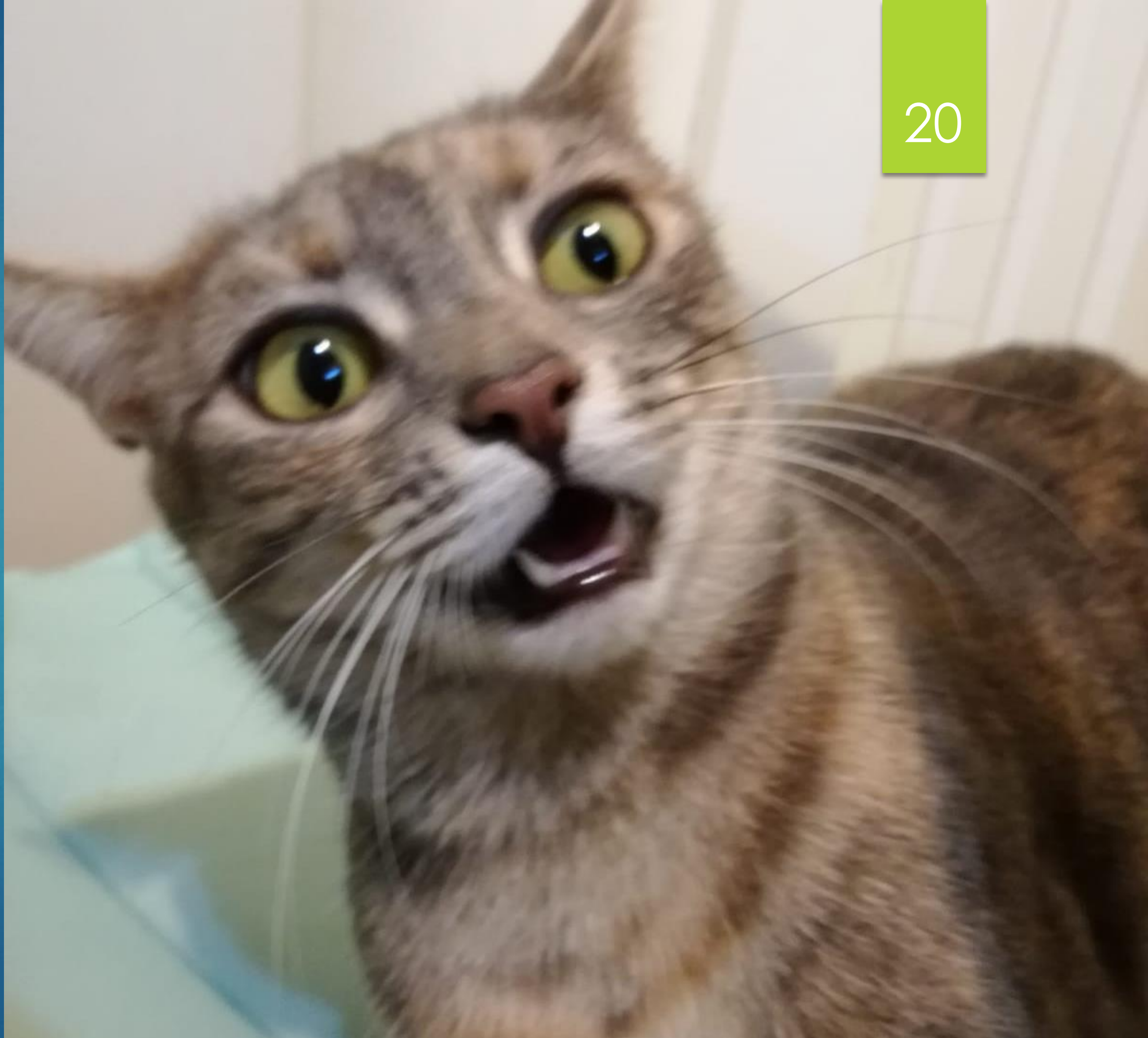


# Zaključak



- Sustav u kojem kvantna supravodljiva točka (uz odabir napona vrata ponaša se kao qubit) harmonički titra između supravodljivih kontakata → koherentno tuneliranje Cooperovih parova sa supravodljivih kontakata na kvantnu točku → ispreplitanje mehaničkih i qubitnih stupnjeva slobode
- Protokol okretanja prednapona → stvaranje isprepletenog stanja qubitnih stupnjeva slobode s mehaničkim ''cat - stanjima''
- Što dalje istražiti?
  - Osjetljivost sustava na odsustvo rezonancije Josephsonove i mehaničke frekvencije
  - Modeliranje tranzijentnog perioda okretanja napona
  - Primjena: kodirati informaciju u ''cat-stanja''

# Pitanja?



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