Određivanje dinamike fizikalnih sustava pomoću strojnog učenja

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Sadržaj

Uvod

Diferencijalne jednadžbe $\rightarrow$ rješenja

- Newtonova jednadžba

$$\mathbf{F} = m\ddot{\mathbf{r}}$$

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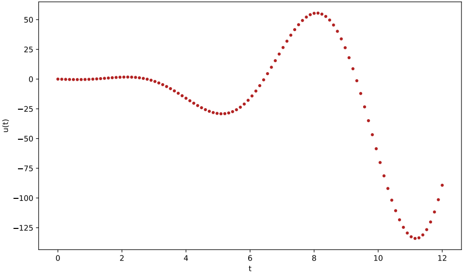
$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0,\ \nabla \cdot \mathbf{B} = \mathbf{0},\ \ldots$$

- Schrödingerova jednadžba (vremenski neovisna)

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) = E\psi(\mathbf{r},t)$$

Uvod

Podatci → diferencijalna jednadžba



Uvod

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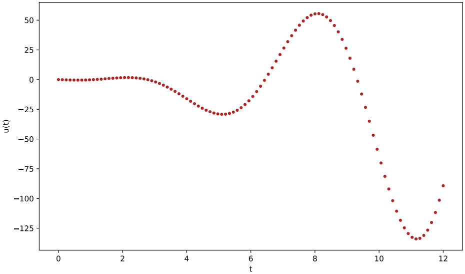
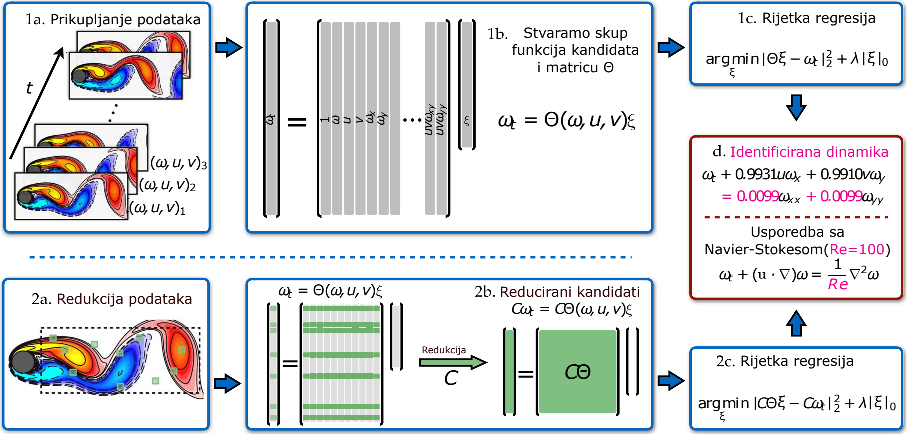


$$\ddot{u} = 1.6\sin(2t) + 2.9u - 3|u|\dot{u} + ...$$

Ilustracija algoritma



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Teorijska pozadina

- Model

$$u_{tt} = F(u, u_x, u_{xy}, ..., \mathbf{r}, t, \mu)$$

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$$x \rightarrow (x_1, x_2, ..., x_n)$$

- Diskretizacija vektora

$$\mathbf{U} \equiv \begin{bmatrix} u(x_0, t_0) \\ u(x_1, t_0) \\ \vdots \\ u(x_{n-1}, t_m) \\ u(x_n, t_m) \end{bmatrix}$$

Teorijska pozadina

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$$\begin{bmatrix} | \\ \mathbf{U_{tt}} \\ | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ \mathbf{1} & \mathbf{U} & \mathbf{U^2} & \mathbf{UU_t} \\ | & | & | & | \end{bmatrix} \begin{bmatrix} | \\ \xi \\ | \end{bmatrix}$$

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Primjer: harmonijski oscilator ($\omega = 1$)

Rješenje: $\xi_0 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \Longrightarrow \ddot{u} = -u$

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- Generalizacija modela

Koeficijenti ξ

- Linearna regresija

$$\mathbf{U}_{tt} = \mathbf{\Theta}(\mathbf{U}, \mathbf{Q})\xi \implies \xi_0 = \underset{\xi}{\operatorname{argmin}} \|\mathbf{\Theta}\xi - \mathbf{U}_{tt}\|_2^2$$

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- STRidge algoritam

Deriviranje podataka

- Čisti podatci – metoda konačnih razlika

$$f'(x) = \frac{f(x+\Delta) - f(x-\Delta)}{2\Delta} + O(\Delta^2)$$

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- Podatci sa šumom – interpolacija polinoma
- Konvolucija sa Gaussijanom, metoda konačnih razlika

Sadržaj

Primjeri i rezultati

- Stvaranje podataka

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```
PDJ dobivena pomoću STRidge:
u_tt = (-0.051289 +0.000000i)
    + (1.406844 +0.000000i)u_t
    + (-0.952997 +0.000000i)u
    + (0.032605 +0.000000i)u^2
    + (-1.301716 +0.000000i)u^2u_t

PDJ dobivena pomoću STRidge:
u_tt = (1.500808 +0.000000i)u_t
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PDJ dobivena pomoću STRidge:
u_tt = (1.454861 +0.000000i)u_t
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PDJ dobivena pomoću STRidge:
u_tt = (1.440753 +0.000000i)u_t
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    + (-1.370771 +0.000000i)u^2u_t
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Primjeri i rezultati

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- Srednja jednadžba

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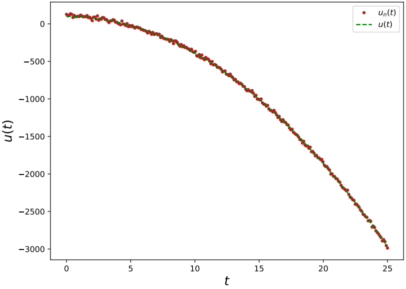
Slobodni pad

Diferencijalna jednadžba

$$\ddot{u} = -g;\ g = 9.81$$



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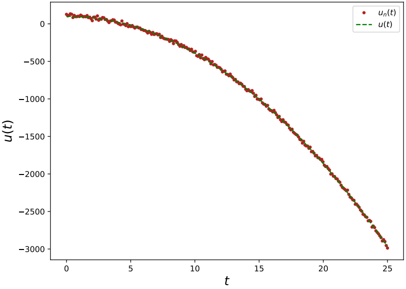

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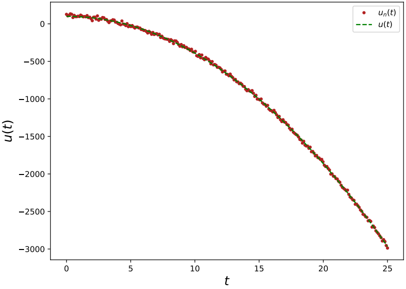


- PDE-FIND

$$\ddot{u} = -9.81$$

$$\Delta\xi = 0\%$$

- PDE-FIND sa šumom $(\mu_N = 0, \sigma_N = 15)$

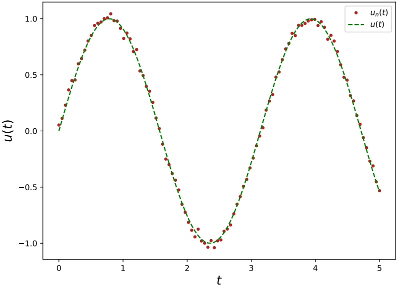
$$\ddot{u} = -9.795279$$

$$\Delta\xi = (1.0 \pm 0.5)\%$$

Harmonijski oscilator

Diferencijalna jednadžba

$$\ddot{u} = -\omega^2 u;\ \omega = 2$$



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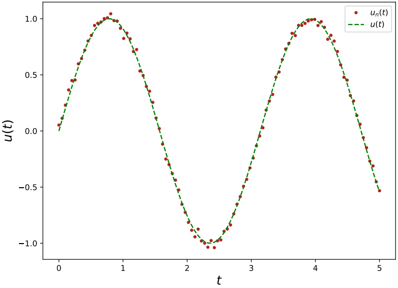

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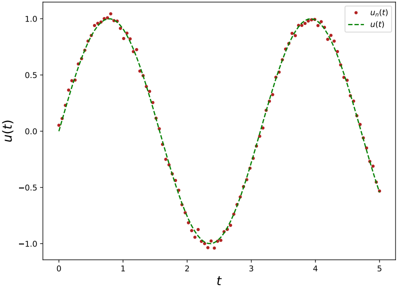


- PDE-FIND

$$\ddot{u} = -3.99687u$$

$$\Delta\xi = 0.08\%$$

- PDE-FIND sa šumom $(\mu_N = 0, \sigma_N = 0.03)$

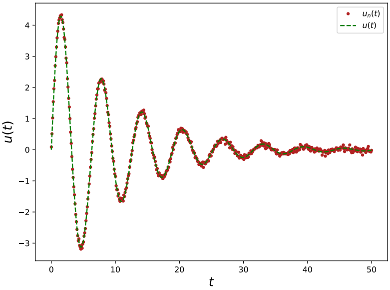
$$\ddot{u} = -3.879961u$$

$$\Delta\xi = (3.0 \pm 1.6)\%$$

Gušeni oscilator

Diferencijalna jednadžba

$$\ddot{u} = -2\zeta\dot{u} - \omega^2 u;\ \omega = 1,\ \zeta = 0.1$$



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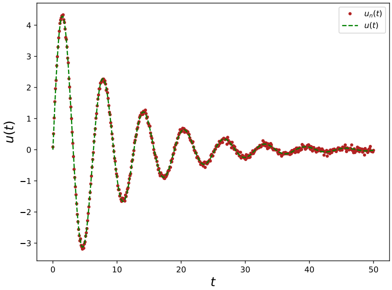

- PDE-FIND

$$\ddot{u} = -0.200179\dot{u} - 0.999128u$$

$$\Delta\xi = 0.09\%$$

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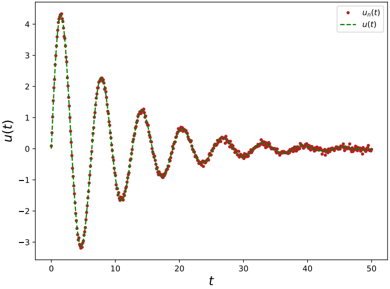

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$$\ddot{u} = -0.200179\dot{u} - 0.999128u$$

$$\Delta\xi = 0.09\%$$

- PDE-FIND sa šumom $(\mu_N = 0, \sigma_N = 0.05)$

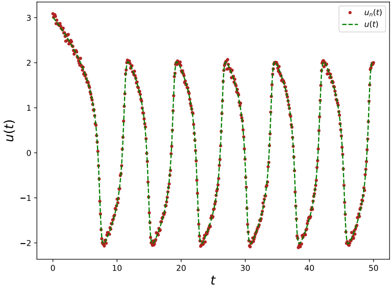
$$\ddot{u} = -0.188519\dot{u} - 0.982673u$$

$$\Delta\xi_{max} = (6 \pm 2)\%$$

Van der Polov oscilator

Diferencijalna jednadžba

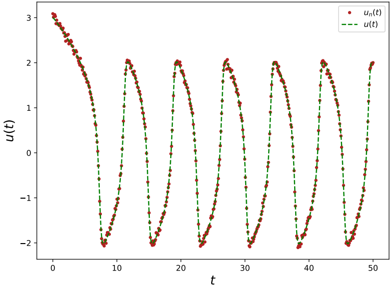
$$\ddot{u} = \mu(1 - u^2)\dot{u} - \omega^2 u;\ \omega = 1$$



Van der Polov oscilator

Diferencijalna jednadžba

$$\ddot{u} = \mu(1 - u^2)\dot{u} - \omega^2 u;\ \omega = 1$$



- 3 parametra μ

$$\mu = 0.1,\ 2 \text{ i } 5$$

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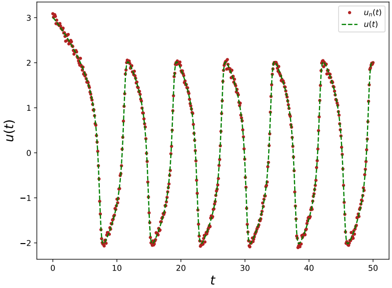


- 3 parametra μ

$$\mu = 0.1,\ 2 \text{ i } 5$$

- 3 početna uvjeta $(x_0, \dot{x}_0)$

$$(5,1),\ (3,0) \text{ i } (0,-5)$$

Van der Polov oscilator

Diferencijalna jednadžba

$$\ddot{u} = \mu(1 - u^2)\dot{u} - \omega^2 u;\ \omega = 1$$

Tablica: Van der Polov oscilatora bez šuma

μ	PDE-FIND	Max. greška
0.1	$\ddot{u} = 0.099726\dot{u} - 0.999290u - 0.099848u^2\dot{u}$	0.27%
2	$\ddot{u} = 1.941602\dot{u} - 0.972596u - 1.942509u^2\dot{u}$	2.92%
5	$\ddot{u} = 4.926123\dot{u} - 0.976853u - 4.905555u^2\dot{u}$	2.31%

Van der Polov oscilator

Diferencijalna jednadžba

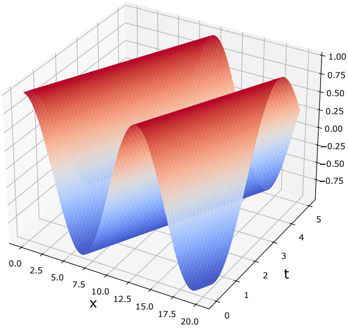
$$\ddot{u} = \mu(1 - u^2)\dot{u} - \omega^2 u;\ \omega = 1$$

μ	PDE-FIND srednja jednadžba	Parametri μ_N i σ_N	Max. greška
0.1	$\ddot{u} = -0.016188 + 0.076330\dot{u} - 0.983806u$ $+ 0.006127u^2 - 0.002214u\dot{u} - 0.088699u^2\dot{u}$	$\mu_N = 0, \sigma_N = 0.05$	$(26 \pm 12)\%$
2	$\ddot{u} = -0.033608 + 1.412393\dot{u} - 0.928338u$ $+ 0.023849u^2 - 0.004304u\dot{u} - 1.357691u^2\dot{u}$	$\mu_N = 0, \sigma_N = 0.05$	$(32 \pm 3)\%$
5	$\ddot{u} = -0.010158 + 3.237894\dot{u} - 1.027299u$ $+ 0.002301u^2 - 0.005109u\dot{u} - 3.034661u^2\dot{u}$	$\mu_N = 0, \sigma_N = 0.05$	$(39 \pm 3)\%$

Valna jednadžba

Diferencijalna jednadžba

$$\dot{u} = cu',\ c = -2$$



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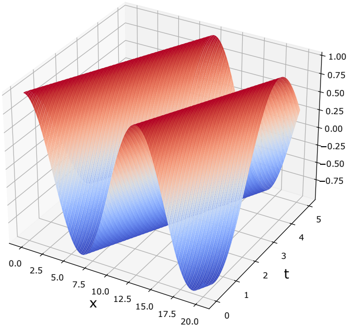

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$$\dot{u} = -2.002414u'$$

$$\Delta\xi = 0.1\%$$

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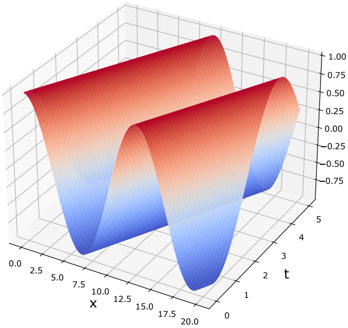

- PDE-FIND

$$\dot{u} = -2.002414u'$$

$$\Delta\xi = 0.1\%$$

- PDE-FIND sa šumom $(\mu_N = 0, \sigma_N = 0.1)$

$$\dot{u} = -1.969783u'$$

$$\Delta\xi = (1.5 \pm 0.3)\%$$

Sadržaj

Zaključak

Identifikacija dinamike

PDE Functional Identification of Nonlinear Dynamics

Zaključak

Identifikacija dinamike

PDE Functional Identification of Nonlinear Dynamics

Pozitivno

- Višedimenzionalni podatci i podatci sa šumom
- Brz, efikasan i robustan
- Fizika, kemija, biologija, ekonomija, ...

Zaključak

Identifikacija dinamike

PDE Functional Identification of Nonlinear Dynamics

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- Višedimenzionalni podatci i podatci sa šumom
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Negativno

- Visoka rezolucija
- Puno parametara
- Osjetljivost

Reference I

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Henning U. Voss, Paul Kolodner, Markus Abel, and Jürgen Kurths.
Amplitude equations from spatiotemporal binary-fluid convection data.
Phys. Rev. Lett., 83:3422–3425, Oct 1999.

Kraj prezentacije

https://github.com/VinkoGitHub/Samostalni-seminar