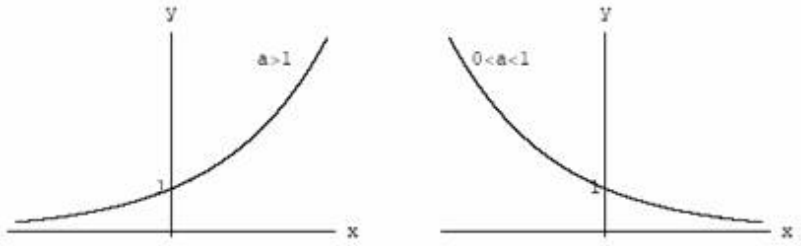
Racionalne funkcije

$f(x) = \frac{p(x)}{q(x)}$, gdje su p i q polinomi.

Domena: $\mathbb{R} \setminus \{x \mid q(x) = 0\}$

Eksponencijalne funkcije

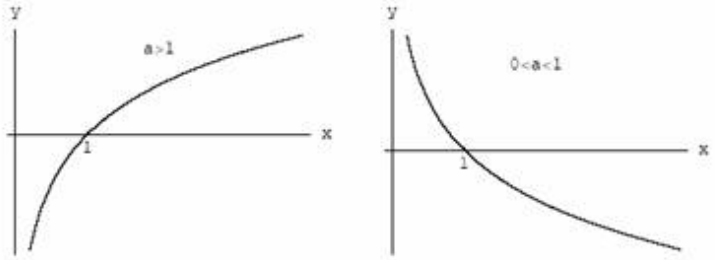
$f(x) = a^x, \quad a > 0, \quad a \neq 1$



Domena: $\mathbb{R}$, slika: $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$.
Funkcija strogo raste za $a > 0$, a strogo pada za $0 < a < 1$.
Svojstva: $a^{x+y} = a^x \cdot a^y, \quad (a^x)^y = a^{x \cdot y}$
Važna baza: $e = 2.718281828459045235360287471352662497757247093699959574966967 6...$

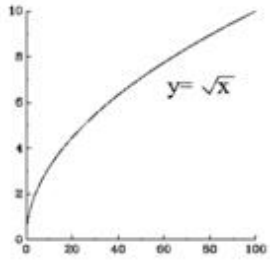
Logaritamske funkcije

$f(x) = \log_a(x), \quad a > 0, \quad a \neq 1$
Za $a = 10$ piše se $f(x) = \log x$, a za $a = e$ oznaka je $f(x) = \ln x$.



Domena: $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$, slika: $\mathbb{R}$.
Funkcija strogo raste za $a > 1$, a strogo pada za $0 < a < 1$.
Logaritamska funkcija je inverzna eksponencijalnoj: $\log_a(a^x) = x, \ \forall x \in \mathbb{R}; \quad a^{\log_a(x)} = x, \ \forall x > 0$
Svojstva: $\log_a(x \cdot y) = \log_a(x) + \log_a(y), \ \log_a(x^y) = y \cdot \log_a(x), \ \log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

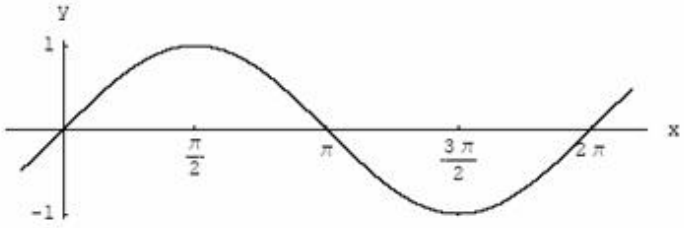
Korijen



Domena: [0, +>, Slika: [0, +∞>

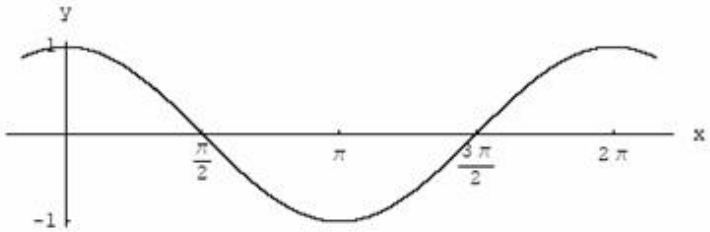
Trigonometrijske funkcije

Sinus: $f(x) = \sin x$



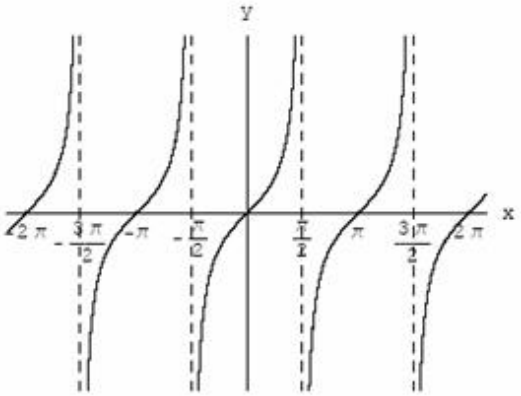
Domena: $\mathbb{R}$; slika: $[-1, 1]$. Funkcija je periodična temeljnog perioda 2π i neparna.

Kosinus: $f(x) = \cos x$

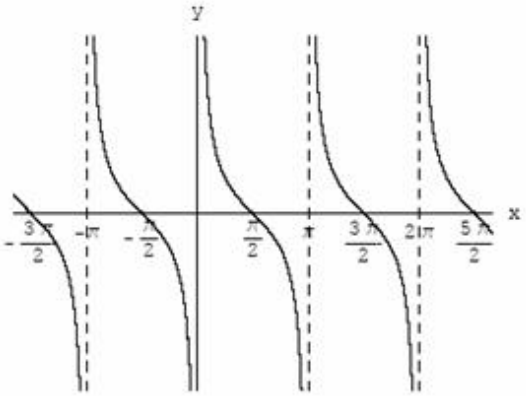


Domena: $\mathbb{R}$; slika: $[-1, 1]$. Funkcija je periodična temeljnog perioda 2π i parna.

Tangens: $f(x) = \operatorname{tg} x = \frac{\sin x}{\cos x}$



Kotangens: $f(x) = \operatorname{ctg} x = \frac{\cos x}{\sin x}$

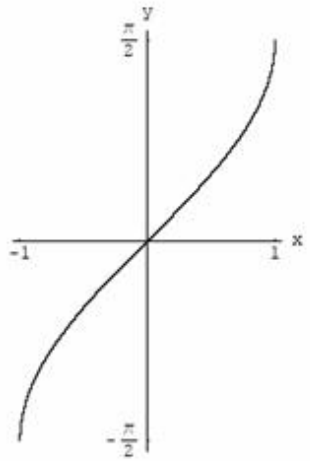


Funkcije su periodične temeljnog perioda π i neparne.

Tangens: domena $\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$, slika $\mathbb{R}$. Kotangens: domena $\mathbb{R} \setminus \{k\pi \mid k \in \mathbb{Z}\}$, slika $\mathbb{R}$.

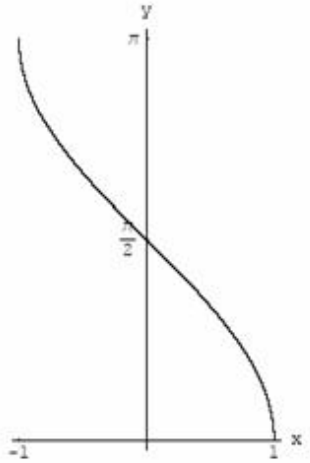
Arkus funkcije

Arkus sinus: $f(x) = \arcsin x$



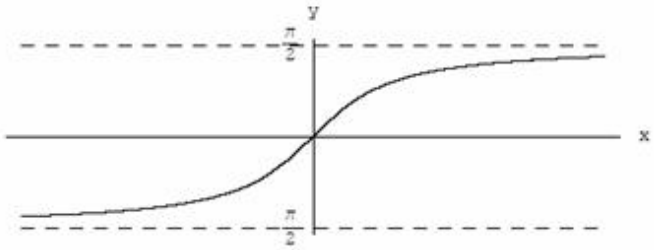
Domena: $[-1, 1]$; slika: $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
Funkcija je rastuća i neparna.

Arkus kosinus: $f(x) = \arccos x$



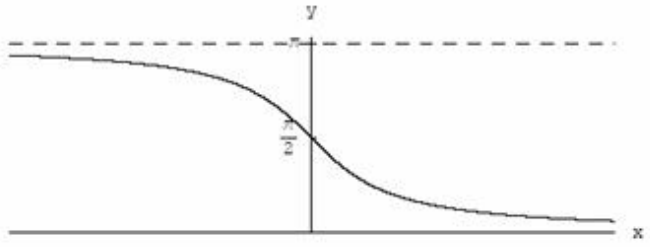
Domena: $[-1, 1]$; slika: $[0, \pi]$.
Funkcija je padajuća.

Arkus tangens: $f(x) = \operatorname{arctg} x$



Domena: $\mathbb{R}$; slika: $\langle -\frac{\pi}{2}, \frac{\pi}{2}\rangle$. Funkcija je rastuća i neparna.

Arkus kotangens: $f(x) = \operatorname{arcctg} x$



Domena: $\mathbb{R}$; slika: $\langle 0, \pi\rangle$. Funkcija je padajuća.

Arkus funkcije su inverzne trigonometrijskim funkcijama:
$\arcsin(\sin x) = x, \ \forall x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$; $\arccos(\cos x) = x, \ \forall x \in [0, \pi]$; $\operatorname{arctg}(\operatorname{tg} x) = x, \ \forall x \in \langle -\frac{\pi}{2}, \frac{\pi}{2}\rangle$
$\operatorname{arcctg}(\operatorname{ctg} x) = x, \ \forall x \in \langle 0, \pi\rangle$; $\sin(\arcsin x) = x, \ \forall x \in [-1, 1]$; $\cos(\arccos x) = x, \ \forall x \in [-1, 1]$
$\operatorname{tg}(\operatorname{arctg} x) = x, \ \forall x \in \mathbb{R}$; $\operatorname{ctg}(\operatorname{arcctg} x) = x, \ \forall x \in \mathbb{R}$

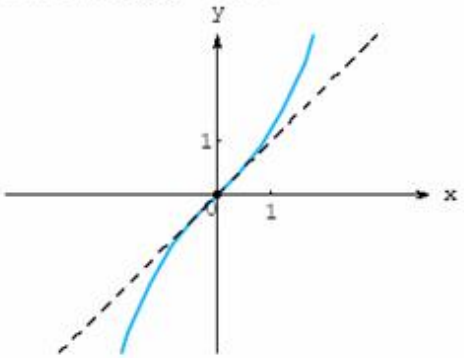
Hiperbolne funkcije

Sinus hiperbolni

Oznaka: sh

$$shx = \frac{e^x - e^{-x}}{2}, \quad sh : R \to R$$

Graf funkcije $y = shx$:

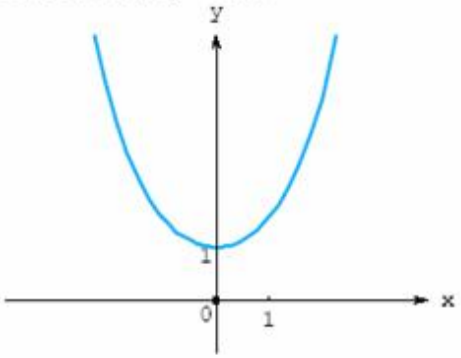


Kosinus hiperbolni

Oznaka: ch

$$chx = \frac{e^x + e^{-x}}{2}, \quad ch : R \to [1, +\infty)$$

Graf funkcije $y = chx$:



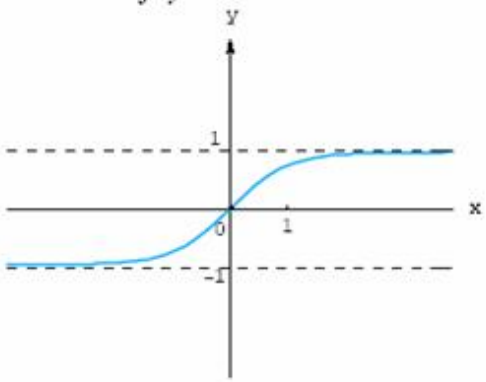
Tangens hiperbolni

Oznaka: th

$$thx = \frac{shx}{chx} = \frac{e^x - e^{-x}}{e^x + e^{-x}},$$

$$th : R \to (-1,1)$$

Graf funkcije $y = thx$:



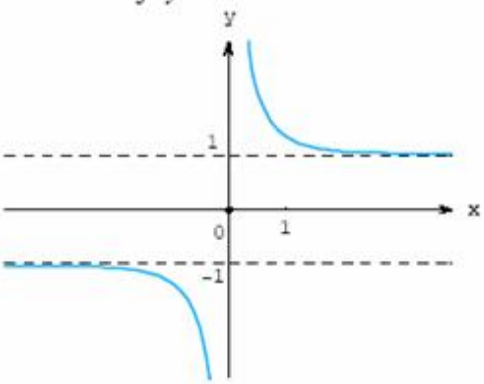
Kotangens hiperbolni

Oznaka: cth

$$cthx = \frac{chx}{shx} = \frac{e^x + e^{-x}}{e^x - e^{-x}},$$

$$cth : R \setminus \{0\} \to R \setminus [-1,1]$$

Graf funkcije $y = cthx$:



Area funkcije

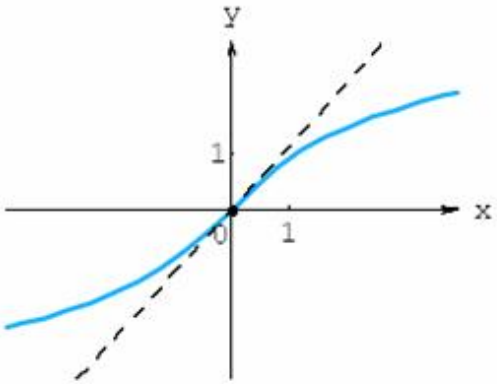
Area funkcije su inverzne funkcije hiperbolnih funkcija.

Area sinus hiperbolni

Oznaka: *Arsh*

$Arsh = sh^{-1} : R \longrightarrow R$

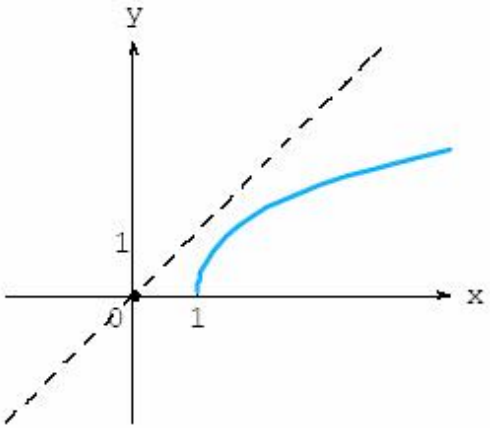
Graf funkcije $y = Arshx$:



Area kosinus hiperbolni

$Arch = Ch^{-1} : [1, +\infty) \to [0, +\infty)$.

Graf funkcije $y = Archx$:

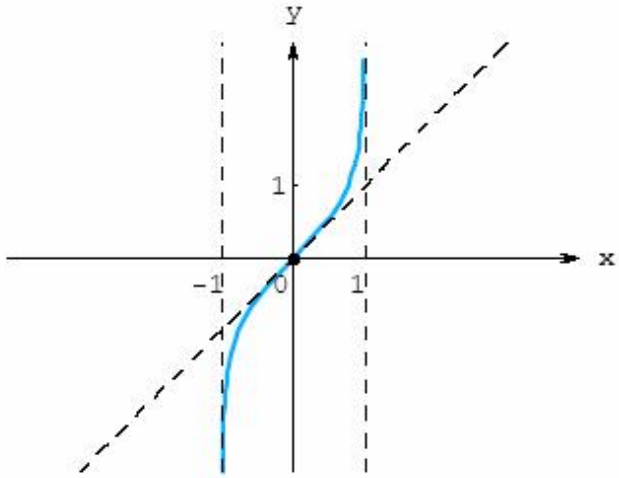


Area tangens hiperbolni

Oznaka: *Arth*

Arth : <-1, 1>$\longrightarrow R$

Graf funkcije $y = Arthx$:



Area kotangens hiperbolni

Oznaka: *Arcth*

$cth : R \setminus \{0\} \longrightarrow R \setminus [-1, 1]$

$Arcth = cth^{-1} : R \setminus [-1, 1] \longrightarrow R \setminus \{0\}$

Graf funkcije $y = Arcthx$: