2. DERIVACIJA

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \ln a}$
e^x	e^x
a^x	$a^x \ln a$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{tg} x$	$\frac{1}{\cos^2 x}$
$\operatorname{ctg} x$	$-\frac{1}{\sin^2 x}$

$f(x)$	$f'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$
$\operatorname{arcctg} x$	$-\frac{1}{1+x^2}$

$$(c)' = 0$$
$$[c \cdot f(x)]' = c \cdot f'(x)$$
$$[f(x) + g(x)]' = f'(x) + g'(x)$$
$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$
$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$
$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

2.1. Derivirajte sljedeće funkcije:

(a) $f(x) = \sqrt[3]{x^2} - \sqrt{x\sqrt{x}} + \sqrt[7]{e}$

(b) $f(x) = \dfrac{1-\sqrt{x}}{1+\sqrt[3]{x}}$

(c) $f(x) = \dfrac{\operatorname{ctg} x}{x \ln x} + 3xe^x$

(d) $f(x) = \dfrac{\sin x}{x^3} + e^x \cdot \cos x - (x^3+2)\log x$

2.2. Derivirajte sljedeće funkcije:

(a) $f(x) = (x^2+3)^2$

(b) $f(x) = \sqrt[3]{1+2x}$

(c) $f(x) = \sqrt[4]{(x+3)^3}$

(d) $f(x) = \cos^6 x$

(e) $f(x) = \sin x^3$

(f) $f(x) = \log_3(x^2+5)$

(g) $f(x) = \ln(\sin x)$

(h) $f(x) = \sin(\ln(x+2))$

(i) $f(x) = \operatorname{tg}\left(\ln \dfrac{1-x}{1+x}\right)$

2.3. Derivirajte sljedeće funkcije:

(a) $f(x) = \sin(3x+\pi)\cdot\sqrt{1-x^2}$

(b) $f(x) = \dfrac{x^2+\sin(2x)}{\ln x + \cos(2x+3)}$

(c) $f(x) = \sqrt{xe^x}$

(d) $f(x) = \sqrt{4x-1} + \operatorname{arcctg}\sqrt{4x-1}$

(e) $f(x) = \sqrt{x^2-e^x} + \arcsin\dfrac{1}{x}$

(f) $f(t) = \dfrac{1}{4}(\operatorname{tg} t)^4 - \dfrac{1}{2}(\operatorname{tg} t)^2 - \ln(\cos t)$

(g) $f(x) = \arcsin\dfrac{x-1}{x}$

(h) $f(x) = \ln^2(2x+1)$

(i) $f(x) = \ln\sqrt{\dfrac{1+x}{1-x}}$

(j) $f(x) = \ln\ln(x^4+x)$

2.4. Derivirajte sljedeće funkcije:

(a) $f(x) = e^{-x} + 2^{sin\frac{x}{2}} + \sin^2 x$
(b) $f(x) = 2^{\mathrm{arctg}\sqrt{x}}$
(c) $f(x) = \ln\left(x - \sqrt{x^2+1}\right)$
(d) $f(x) = x \cdot \sqrt{\mathrm{tg}\, x}$
(e) $f(t) = \left(\dfrac{t-2}{2t+1}\right)^9$
(f) $f(x) = \mathrm{arctg}(\ln x) + \sqrt{\ln(x^2+1)}$
(g) $f(x) = 2^{x^3+4}$
(h) $f(x) = \dfrac{1}{5^{x^2}}$
(i) $f(x) = e^{\sqrt{xe^x}}$

2.5. Derivirajte sljedeće funkcije:

(a) $f(x) = x^{\sin x}$

(b) $f(x) = \dfrac{(x^2+2x+3)^{15}(2x+5)^{10}}{(5x-9)^{13}}$

(c) $f(x) = (\ln x)^x$

(d) $f(x) = \dfrac{(\cos x)^{\sin x}}{x^2+3}$

(e) $f(x) = \ln \sqrt[x]{\sin x}$

(f) $f(x) = e^{\cos x} + (\cos x)^x$

(g) $f(x) = \dfrac{\sqrt{(x-2)(x-4)}}{(x+1)(x+3)}$

(h) $f(x) = \pi^2 + 2^x + x^2 + \sqrt[x]{x}$

2.6. Odredite n-tu derivaciju sljedećih funkcija i njenu vrijednost u x_0:

(a) $f(x) = x^5$, $x_0 = 0$

(b) $f(x) = \dfrac{1}{x}$, $x_0 = -1$

(c) $f(x) = \cos x$, $x_0 = \pi$

(d) $f(x) = \sin x$, $x_0 = \frac{\pi}{2}$

(e) $f(x) = \sqrt{1-4x}$, $x_0 = 0$

(f) $f(x) = \ln \dfrac{1-4x}{1+4x}$, $x_0 = 0$

2.7. Derivirajte sljedeće implicitno zadane funkcije $y = f(x)$:

(a) $x^3 y + xy^3 = e^x$

(b) $xy + \sin y = e^{x+y}$

(c) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

(d) $\sqrt{x} + \sqrt{y} = \sqrt{2e}$

(e) $\left(x^2 + y^2\right) \cdot y^2 = a \cdot x^2$

$$y'(x) = \frac{\dot{y}}{\dot{x}} \qquad y''(x) = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^3} \qquad (\dot{x} = x'(t), \ddot{x} = x''(t))$$

2.8. Odredite prve dvije derivacije sljedeće parametarski zadane funkcije:

(a) $x(t) = t - \sin t, y(t) = 1 - \cos t$

(b) $x(t) = t + 2\sin(2t), y(t) = t + 2\cos(5t)$

2.9. Derivirajte sljedeće parametarski zadane funkcije:

(a) $x(t) = \dfrac{t(t+1)}{t+2}, y(t) = \dfrac{t^2 - 4t + 1}{t}$

(b) $x(t) = \sqrt{\sin 3t}, y(t) = \left(\dfrac{t+3}{t-3}\right)^5$

(c) $x(t) = \dfrac{t(t^2+1)}{t^4+1}, y(t) = -\dfrac{t(t^2-1)}{t^4+1}$

Rješenja

2.1. (a) $\dfrac{2}{3\sqrt[3]{x}} - \dfrac{3}{4\sqrt[4]{x}}$

(b) $-\dfrac{\frac{1}{2\sqrt{x}} + \frac{1}{6\sqrt[6]{x}} + \frac{1}{3\sqrt[3]{x^2}}}{\left(1+\sqrt[3]{x}\right)^2}$

(c) $-\dfrac{1}{(\sin^2 x)x\ln x} - \dfrac{\operatorname{ctg} x(\ln x + 1)}{x^2 \ln^2 x} + 3e^x + 3xe^x$

(d) $\dfrac{\cos x}{x^3} - \dfrac{3\sin x}{x^4} + e^x \cos x - e^x \sin x - 3x^2 \log x - x^2 \log e - \dfrac{2\log e}{x}$

2.2. (a) $6x^3 + 12x$

(b) $\dfrac{2}{3\sqrt[3]{(1+2x)^2}}$

(c) $\dfrac{3}{4\sqrt[4]{x+3}}$

(d) $-6\sin x\cos^5 x$

(e) $3x^2\cos x^3$

(f) $\dfrac{2x}{(x^2+5)\ln 3}$

(g) $\operatorname{ctg} x$

(h) $\dfrac{\cos(\ln(x+2))}{x+2}$

(i) $-\dfrac{2}{(1-x^2)\cos^2(\ln\frac{1-x}{1+x})}$

2.3. (a) $3\cos(3x+\pi)\sqrt{1-x^2}-\sin(3x+\pi)(1-x^2)^{-\frac{1}{2}}x$

(b)
$$\frac{x(2x+2\cos(2x)(\ln x+\cos(2x+3))-(x^2+\sin 2x)(1-2x\sin(2x+3))}{x(\ln x+\cos(2x+3))^2}$$

(c) $\dfrac{1}{2\sqrt{xe^x}}(e^x+xe^x)$

(d) $\dfrac{4x-1}{2x\sqrt{4x-1}}$

(e) $\dfrac{2x-e^x}{2\sqrt{x^2-e^x}}-\dfrac{1}{\sqrt{x^4-x^2}}$

(f) $\dfrac{\sin^3 t}{\cos^5 t}-\dfrac{\sin t}{\cos^3 t}+\dfrac{\sin t}{\cos t}$

(g) $\dfrac{1}{\sqrt{x^2(2x-1)}}$

(h) $\dfrac{4\ln(2x+1)}{2x+1}$

(i) $\dfrac{1}{1-x^2}$

(j) $\dfrac{4x^3+1}{x\left(x^3+1\right)\ln\left(x^4+x\right)}$

2.4. (a) $-e^{-x} + \frac{\ln 2}{2} \cdot 2^{\sin \frac{x}{2}} \left(\cos \frac{x}{2}\right) + 2 \sin x \cos x$

(b) $\frac{\ln 2}{2\sqrt{x}(1+x)} 2^{\operatorname{arctg} \sqrt{x}}$

(c) $-\frac{1}{\sqrt{x^2+1}}$

(d) $\frac{2 \sin x \cos x + x}{2\sqrt{\operatorname{tg} x} \cos^2 x}$

(e) $45 \cdot \frac{(t-2)^8}{(2t+1)^{10}}$

(f) $\frac{1}{x(1+\ln^2 x)} + \frac{x}{(x^2+1)\sqrt{\ln(x^2+1)}}$

(g) $2^{x^3+4} \cdot \ln 2 \cdot 3x^2$

(h) $-\frac{2x \ln 5}{5^{x^2}}$

(i) $\frac{e^x(1+x)}{2\sqrt{xe^x}} e^{\sqrt{xe^x}}$

2.5. (a) $x^{\sin x}\left(\cos x \ln x + \dfrac{\sin x}{x}\right)$

(b) $\dfrac{(x^2+2x+3)^{15}(2x+5)^{10}}{(5x-9)^{13}} \cdot \left(15 \cdot \dfrac{2x+2}{x^2+2x+3} + 10 \cdot \dfrac{2}{2x+5} - 13 \cdot \dfrac{5}{5x-9}\right)$

(c) $(\ln x)^x \left[\ln(\ln x) + \dfrac{1}{\ln x}\right]$

(d) $\dfrac{(\cos x)^{\sin x}}{x^2+3}\left[\cos x \ln(\cos x) - \dfrac{\sin^2 x}{\cos x} - \dfrac{2x}{x^2+3}\right]$

(e) $\dfrac{\cos x}{x \sin x} - \dfrac{\ln(\sin x)}{x^2}$

(f) $e^{\cos x}(-\sin x) + (\cos x)^x[\ln(\cos x) - x \operatorname{tg} x]$

(g) $\dfrac{\sqrt{(x-2)(x-4)}}{(x+1)(x+3)}\left[\dfrac{1}{2(x-2)} + \dfrac{1}{2(x-4)} - \dfrac{1}{x+1} - \dfrac{1}{x+3}\right]$

(h) $2^x \ln 2 + 2x + \sqrt[x]{x} \cdot \dfrac{1-\ln x}{x^2}$

2.6. (a) $f^{(n)}(x) = 0, \quad \text{za } n \geq 6, \quad f^{(n)}(0) = \begin{cases} 5!, & n = 5 \\ 0, & n \neq 5 \end{cases}$

(b) $f^{(n)}(x) = (-1)^n \cdot n! \cdot x^{-(n+1)}, \quad f^{(n)}(-1) = -n!$

(c) $f^{(n)}(x) = \cos\left(x + \dfrac{n\pi}{2}\right), \quad f^{(n)}(\pi) = \begin{cases} 0, & \text{n neparan} \\ -1, & n = 4k \\ 1, & n = 4k+2 \end{cases}$

(d) $f^{(n)}(x) = \sin\left(x + \dfrac{n\pi}{2}\right), \quad f^{(n)}\left(\frac{\pi}{2}\right) = \begin{cases} 0, & \text{n neparan} \\ 1, & n = 4k \\ -1, & n = 4k+2 \end{cases}$

(e) $f^{(n)}(x) = \begin{cases} -2, & \text{n=1} \\ -(2n-3)!! \cdot 2^n \cdot (1-4x)^{\frac{1}{2}-n}, & n \geq 1 \end{cases}$

$f^{(n)}(0) = \begin{cases} -2, & \text{n=1} \\ -(2n-1)!! \cdot 2^n, & n \geq 1 \end{cases}$

(f) $f^{(n)}(x) = -(n-1)!(1-4x)^{-n} \cdot 4^n + (-1)^n(n-1)!(1+4x)^{-n} \cdot 4^n$

$f^{(n)}(0) = 4^n(n-1)![(-1)^n - 1]$

2.7. (a) $\dfrac{e^x - 3x^2y - y^3}{x^3 + 3xy^2}$

(b) $\dfrac{e^{x+y} - y}{x + \cos y - e^{x+y}}$

(c) $-\dfrac{\sqrt[3]{y}}{\sqrt[3]{x}}$

(d) $-\dfrac{\sqrt{y}}{\sqrt{x}}$

(e) $\dfrac{2ax - 2xy^2}{4y^3 + 2x^2y}$

2.8. (a) $y' = \dfrac{\sin t}{1-\cos t}, y'' = -\dfrac{1}{(\cos t - 1)^2}$

(b) $y' = \dfrac{1 - 10\sin(5t)}{1 + 4\cos(2t)},$

$$y'' = \frac{-50\cos(5t) - 200\cos(5t)\cos(2t) + 8\sin(2t) - 80\sin(5t)\sin(2t)}{[1 + 4\cos(2t)]^3}$$

2.9. (a) $y' = \dfrac{\sin t}{1-\cos t}$

(b) $y' = \dfrac{(t^2-1)(t+2)^2}{t^2(t^2+4t+2)}$

(c) $y' = -\dfrac{20(t+3)^4}{(t-3)^6} \cdot \dfrac{\sqrt{\sin(3t)}}{\cos(3t)}$

(d) $y' = -\dfrac{t^6 - 3t^4 - 3t^2 + 1}{t^6 + 3t^4 - 3t^2 - 1}$