12. INTEGRIRANJE IRACIONALNIH FUNKCIJA

12.1. Izračunajte neodređene integrale:

(a) $\int \frac{dx}{x(1+2\sqrt{x}+\sqrt[3]{x})}$.

(b) $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}\,dx$.

(c) $\int \frac{dx}{(2x+1)^{\frac{2}{3}}-(2x+1)^{\frac{1}{2}}}$.

(d) $\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}\,dx$.

12.2. Izračunajte neodređene integrale:

(a) $\dfrac{dx}{\sqrt{2x^2-6x+5}}$

(b) $\displaystyle\int \frac{\ln x}{x\sqrt{1-4\ln x-\ln^2 x}}\,dx$

(c) $\displaystyle\int \frac{\sin x}{\sqrt{\cos^2 x+4\cos x+1}}\,dx$

12.3. Izračunajte neodređene integrale:

(a) $\displaystyle\int \sqrt{4x^2-4x+3}\,dx$

(b) $\displaystyle\int \sqrt{3-2x-x^2}\,dx$

12.4. Izračunajte sljedeći integral

$$\int_{\frac{\sqrt{2}}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx.$$

Rješenja

12.1. (a) $6\ln\sqrt[6]{x}-\frac{3}{2}\ln\left(\sqrt[6]{x}+1\right)-\frac{9}{4}\ln\left(2\sqrt[3]{x}-\sqrt[6]{x}+1\right)-\frac{3}{2\sqrt{7}}\operatorname{arctg}\frac{4\sqrt[6]{x}-1}{\sqrt{7}}+c$

(b) $\frac{3}{2}\sqrt[3]{x^2}+6\operatorname{arctg}\sqrt[6]{x}+c$

(c) $\frac{3}{2}(2x+1)^{\frac{1}{3}}+3(2x+1)^{\frac{1}{6}}+3\ln\left|\sqrt[6]{2x+1}-1\right|+c$

(d) $-\frac{6}{7}(x+1)^{\frac{7}{6}}+\frac{6}{5}(x+1)^{\frac{5}{6}}+\frac{3}{2}(x+1)^{\frac{2}{3}}-2(x+1)^{\frac{1}{2}}-3(x+1)^{\frac{1}{3}}+6(x+1)^{\frac{1}{6}}+3\ln\left|\sqrt[3]{x+1}+1\right|-6\operatorname{arctg}\sqrt[6]{x+1}+c$

12.2. (a) $\frac{1}{\sqrt{2}}\ln\left|x-\frac{3}{2}+\sqrt{x^2-3x+\frac{5}{2}}\right|+c$

(b) $-\sqrt{1-4\ln x-\ln^2 x}-2\arcsin\frac{\ln x+2}{\sqrt{5}}+c$

(c) $-\ln\left|\cos x+2+\sqrt{\cos^2 x+4\cos x+1}\right|+c$

12.3. (a) $\left(\frac{1}{2}x-\frac{1}{4}\right)\sqrt{4x^2-4x+3}+\frac{1}{2}\ln\left|2x-1+\sqrt{4x^2-4x+3}\right|+c.$

(b) $2\arcsin\frac{x+1}{2}+\frac{x+1}{2}\sqrt{3-2x-x^2}+c.$

12.4. $1-\frac{\pi}{4}$