Alpe-Adria Seminar

(3rd meeting)

May 21, 2022

Department of Mathematics
Faculty od Science
University of Zagreb

Schedule

10:00–10:45 Goran Muić: *Hilbert's Irreducibility, Modular Forms, and Computation of Certain Galois Group*

10:50–11:35 Klemen Šivic: *Irreducible components of varieties of commuting matrices*

Coffee break

12:00–12:45 Luka Grubišić: *High Order Approximations of the Operator Lyapunov Equation Have Low Rank*

12:50–13:35 Aljoša Peperko: *On some spectral theory for suprema preserving operators on max-cones*

14:00– Lunch at *Tvornica pljeskavica Kosta*, Savska 107/1

Abstracts

High Order Approximations of the Operator Lyapunov Equation Have Low Rank

Luka Grubišić (joint work with Harri Hakula)
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We present a low-rank greedily adapted hp-finite element algorithm for computing an approximation to the solution of the Lyapunov operator equation $AX + XA = bb^*$. In the case in which the coefficient A is self-adjoint and positive definite, the Lyapunov equation has the unique positive and self-adjoint solution X. Furthermore, in the case in which A has a compact resolvent the operator X is a Hilbert Schmidt operator. We interpret the problem of finding the low rank approximation of X as the problem of approximating the dominant eigenvalue cluster of a bounded self-adjoint operator. We show that there is a hidden regularity in eigenfunctions of the solution of the Lyapunov equation which can be utilized to justify the use of high order finite element spaces. Namely, the eigenvectors of the operator X are A-analytic functions. We utilize this regularity to construct the fully hp adapted finite element space. Our numerical experiments indicate that we achieve eight figures of accuracy for computing the trace of the solution of the Lyapunov equation posed in a dumbbell-domain using a finite element space of dimension of only 104 degrees of freedom. Even more surprising is the observation that hp-refinement has an effect of reducing the rank of the approximation of the solution.

Hilbert's Irreducibility, Modular Forms, and Computation of Certain Galois Group

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We consider congruence subgroups $\Gamma_0(N)$, $N \geq 1$, and the corresponding compact Riemann surface $X_0(N)$ which we initially consider as a complex irreducible smooth projective curve. The $\mathbb{Q}$–structure on $X_0(N)$ is defined in a standard way using j–function i.e., the field of rational functions over $\mathbb{Q}$ on $X_0(N)$ is given by $\mathbb{Q}\left(X_0(N)\right) = \mathbb{Q}(j, j(N\cdot))$. If we take that f, g, h are linearly independent modular forms of same even weight $m \geq 2$ with rational q–expansions for $\Gamma_0(N)$, we can construct an irreducible over $\mathbb{Z}$ homogeneous polynomial with integral coefficients $P_{f,g,h}$ such that $P_{f,g,h}(f, g, h) = 0$ in $\mathbb{Q}\left(X_0(N)\right)$. Let $Q_{f,g,h}$ be its dehomogenization with respect to the last variable. Again, we obtain an irreducible over $\mathbb{Z}$ polynomial with integral coefficients. By means of Hilbert's irreducibility theorem we obtain a family of irreducible over $\mathbb{Z}$ polynomials with integral coefficients $Q_{f,g,h}(\lambda, \cdot)$, where λ ranges over a certain thin set. In this way we obtain a family of number fields determined as splitting fields of these polynomials -all of them have the same Galois group $G_{f,g,h}$ which is the Galois group of the splitting field of $Q_{f,g,h}(g/f, \cdot)$ over $\mathbb{Q}(g/f)$. The goal of present talk is to explain these objects as an application of the theory developed in our earlier papers. We elaborate on example $\Gamma_0(72)$.

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On some spectral theory for suprema preserving operators on max-cones

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Suprema preserving nonlinear operators are interesting from the applicative and theorethical point of view. Although such operators may usually be sensibly defined on the whole (normed or Banach) space, they are often well behaved (suprema preserving, positively homogeneous, Lipschitz, ...) on some smaller cone. In the talk some elements of the spectral theory for such operators will be presented. The talk will be mainly based on three papers which are joint work with Vladimir Müller, Prague:

1. MÜLLER, Vladimir, PEPERKO, Aljoša, On the Bonsall cone spectral radius and the approximate point spectrum, Discrete and Continuous Dynamical Systems – Series A, vol. 37, no 10. (October 2017), 5337–5364, https://doi.org/10.48550/arXiv.1612.01755

2. MÜLLER, Vladimir, PEPERKO, Aljoša, Lower spectral radius and spectral mapping theorem for suprema preserving mappings, Discrete Dinamical Systems – Series A, (August 2018) vol 38, no 8, 4117 -4132, https://doi.org/10.48550/arXiv.1712.00340

3. MÜLLER, Vladimir, PEPERKO, Aljoša, On some spectral theory for infinite bounded non-negative matrices in max algebra, submitted, https://doi.org/10.48550/arXiv.2201.02123

Irreducible components of varieties of commuting matrices

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The set of all n-tuples of $d \times d$ matrices is an affine variety defined by quadratic commutativity relations. This variety is important in various areas of algebra, for example in commutative algebra and in representation theory. However, some basic properties, such as the number and dimension of its irreducible components, are not understood well. In the talk we classify the irreducible components for small d and all n, and compute the dimension of the variety for large n and all d. The talk is based on the following recent preprints:

1. J. Jelisiejew, K. Šivic: Components and singularities of Quot schemes and varieties of commuting matrices,
   https://arxiv.org/abs/2106.13137

2. P. D. Levy, N. V. Ngo, K. Šivic: Commuting varieties and cohomological complexity theory,
   https://arxiv.org/abs/2105.07918