(2.3) Vjer. prostor za nezavisne pokuse

↳ 2 nezavisna pokusa $(\Omega_i, \mathcal{F}_i, \mathbb{P}_i)$, $i=1,2$ uz

- $|\Omega_i| < \infty$, $\forall i$
- $\mathcal{F}_i = \mathcal{P}(\Omega_i)$, $\forall i$.

Na $(\Omega, \mathcal{F}) := (\Omega_1 \times \Omega_2, \mathcal{P}(\Omega_1 \times \Omega_2))$ def. $\mathbb{P} : \Omega \to \mathbb{R}$ sa

$$\underbrace{\mathbb{P}((\omega_1,\omega_2))}_{=\mathbb{P}(\{(\omega_1,\omega_2)\})} = \mathbb{P}_1(\omega_1)\,\mathbb{P}_2(\omega_2), \quad \forall (\omega_1,\omega_2) \in \Omega. \qquad (2.14)$$

Iz $\mathbb{P}(\omega) \geq 0$, $\forall \omega \in \Omega$ te $\sum_{\omega \in \Omega} \mathbb{P}(\omega) = 1$ (DZ), Prop. 1.14 daje da se $\mathbb{P}$ može ! proširiti do vjerojatnosti na $(\Omega, \mathcal{F})$, a vrijedi

$$\mathbb{P}(A_1 \times A_2) = \sum_{\substack{\omega_i \in A_i,\\ i=1,2}} \mathbb{P}((\omega_1,\omega_2)) \overset{(2.14)}{\underset{DZ}{=}} \mathbb{P}_1(A_1)\,\mathbb{P}_2(A_2), \qquad (2.15)$$

[tj. $(\omega_1,\omega_2) \in A_1 \times A_2$]

$\forall A_i \subseteq \Omega_i$, $i=1,2$.

Za $A_1 \subseteq \Omega_1$ u Ω imamo

$$\{\text{dogodio se } A_1\} = A_1 \times \Omega_2 =: \tilde{A}_1 \subseteq \Omega.$$

Analogno, za $A_2 \subseteq \Omega_2$ defin. $\tilde{A}_2 := \Omega_1 \times A_2$.

Vrijedi

$$\mathbb{P}(\tilde{A}_1 \cap \hat{A}_2) = \mathbb{P}(A_1 \times A_2) \overset{(2.15)}{=} \mathbb{P}_1(A_1)\,\mathbb{P}_2(A_2), \qquad (2.16)$$

$\forall A_i \subseteq \Omega_i$, $i=1,2$

↳ $A_2 = \Omega_2 \Rightarrow \mathbb{P}(\tilde{A}_1) = \mathbb{P}_1(A_1)$

$A_1 = \Omega_1 \Rightarrow \mathbb{P}(\hat{A}_2) = \mathbb{P}_2(A_2)$

$$\overset{}{\underset{(2.16)}{\Rightarrow}} \boxed{\mathbb{P}(\tilde{A}_1 \cap \hat{A}_2) = \mathbb{P}(\tilde{A}_1)\,\mathbb{P}(\hat{A}_2)}.$$

[tj. $\tilde{A}_1$ i $\tilde{A}_2$ su zaista nezavisni]

Analogno se konstruira vjerojatnost na $(\Omega_1 \times \dots \times \Omega_n, \mathcal{P}(\dots))$, te se isto može napraviti i na $(\Omega_1 \times \Omega_2 \times \dots, \mathcal{F})$, $\mathcal{F} \neq \mathcal{P}(\dots)$