3.8 Nejednakosti

Tm. 3.48 • X diskretna sluč. varijabla

• $h : D_X \to \boxed{[0,+\infty\rangle}$ f-ja

Tada $\forall a > 0$ vrijedi

$$\mathbb{P}(\underbrace{h(X)}_{\geq 0} \geq a) \leq \frac{\mathbb{E}[h(X)]}{a}. \quad (3.32)$$

Dokaz $X \sim \begin{pmatrix} a_1 & a_2 & \cdots \\ p_1 & p_2 & \cdots \end{pmatrix}$

$$\mathbb{E}[h(X)] = \sum_{i \in I} h(a_i)\, p_i = \sum_{i : h(a_i) \geq a} -\text{''}- + \sum_{i : h(a_i) < a} -\text{''}-$$

$h(x) \geq 0$

$$\geq \sum_{i : h(a_i) \geq a} \overbrace{h(a_i)}^{\geq a} p_i \geq a \cdot \sum_{i : h(a_i) \geq a} p_i$$

$h(a_i) \geq 0, \forall i \in I$

$$= a \cdot \mathbb{P}(h(X) \geq a) \quad \blacksquare$$

(!)

Prop. 3.49 Neka je X diskretna sluč. varijabla. Za sve $a > 0$, vrijedi

"Markovljeva nejednakost"

(i) $$\mathbb{P}(|X| \geq a) \leq \frac{\mathbb{E}[|X|]}{a}, \quad (3.33)$$

(ii) $$\mathbb{P}(|X - \mathbb{E}[X]| \geq a) \leq \frac{\mathrm{Var}(X)}{a^2}, \quad (3.34)$$

ako $\mathbb{E}[|X|] < \infty$

"Čebiševljeva nejednakost"

Dokaz (i) Slijedi iz (3.32) uz $h(x) := |x|$.

(ii) $\mathbb{P}(|X - \mathbb{E}[X]| \geq a) = \mathbb{P}(|X - \mathbb{E}[X]|^2 \geq a^2) \leq$

$\leq \left[(3.32) \text{ uz } h(x) := |x - \mathbb{E}[X]|^2, x \in \mathbb{R} \right]$

$$\leq \frac{\mathbb{E}[(X - \mathbb{E}[X])^2]}{a^2} = \frac{Var(X)}{a^2} \quad \blacksquare$$

Pr. 3.50 za X t.d. $\mathbb{E}[|X|]$ i $\forall c > 0$ vrijedi:

$$\mathbb{P}\left(|X - \mathbb{E}[X]| \geq c \cdot \underbrace{\sigma(X)}_{=\sqrt{Var(X)}} \right) \overset{(3.34)}{\leq} \frac{\cancel{Var(X)}}{c^2 \cdot \cancel{\sigma(X)^2}}$$

$$= \boxed{\frac{1}{c^2}}$$

ne ovisi o X!

npr. za $\boxed{c = 3}$, $\boxed{\frac{1}{c^2} \approx 0.11}$ $\blacksquare$