

(4.6) Ujetne razdiobe

(X, Y) slu. vektor na (Ω, \mathcal{F})

$\forall y \in \mathbb{R}$ t.d. $\boxed{IP(Y=y) > 0}$ (ISSOP $y \in D_Y$),

ujetna distr. s. var. X uz danu $Y=y$

je ujetna distr. od X uz danu $\boxed{A := \{Y=y\}}$,

te $E[X | Y=y] := E[X | A]$.

(vidi poglavlje (3.5))

Definiramo $g: D_Y \rightarrow \mathbb{R}$

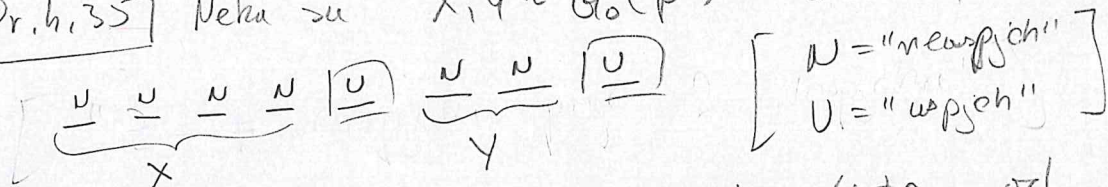
$\boxed{g(y) := E[X | Y=y]}$, $y \in D_Y$,

a slučajna varijabla $E[X | Y]$ defin. sa

$E[X | Y] := g(Y)$, (4.23)

zove se ujetno očekivanje od X uz danu Y .

Pr. 4.35] Neka su $X, Y \sim G_0(p)$ nezavisne, te $Z := X+Y$.



(a) Za $m \in D_Z$, odredite ujetna distr. od X uz danu $Z=m$, te $E[X | Z=m]$.

(b) Odredite $E[X | Z]$.

"negativna binomna"
↓ (3.12)

(Ri) $X, Y \in \mathbb{N}_0 \Rightarrow Z \in \mathbb{N}_0$, te zapravo $Z \sim N(2, p)$
 \uparrow
 \mathbb{N}_0

(a) $\forall m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$,

$$(3.12) \Rightarrow P(Z=m) = \binom{m+2-1}{m+1} q^{m+1} \cdot p^2$$

- za $k \in \mathbb{N}_0, k > m, P(X=k | Z=m) = 0$,
- za $k \in \mathbb{N}_0, k \leq m$

$$P(X=k | Z=m) = \frac{P(X=k, Z=m)}{P(Z=m)} = \frac{P(X=k, Y=m-k)}{P(Z=m)}$$

$$= \frac{P(X=k) P(Y=m-k)}{P(Z=m)} = \frac{q^k p \cdot q^{m-k} p}{\binom{m+2-1}{m+1} q^{m+1} p^2} = \frac{1}{m+1}$$

$X, Y \sim \text{Geo}(p)$

tj.

$$"X | Z=m" \sim \begin{pmatrix} 0 & 1 & \dots & m \\ \frac{1}{m+1} & \frac{1}{m+1} & \dots & \frac{1}{m+1} \end{pmatrix} \quad [\text{Intercept?}]$$

distr. od X uz dane $Z=m$

uniformna rozdzielca na $\{0, \dots, m\}$

Specjalnie,

$$g(m) := E[X | Z=m] = \frac{m}{2}, \quad \forall m \in \mathbb{N}_0$$

$$(b) E[X | Z] = g(Z) = \frac{Z}{2}$$

Prp. 4.36

Ako oba očekivanja postoje, vrijedi

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$$E[E[X|Y]] = E[X]. \quad (4.24)$$

Dokaz] Slijedi odmah iz Tm. 3.42 jer

$$\begin{aligned} E[E[X|Y]] &= \sum_{y \in D_Y} g(y) P(Y=y) \\ &= \sum_{y \in D_Y} E[X|Y=y] P(Y=y), \\ &\quad \uparrow \\ &\quad \text{iz (4.23)} \end{aligned}$$

a $\{ \{Y=y\}, y \in D_Y \}$ je \mathcal{P} -D. \square

[dakle, (4.24) je zapravo formula usjetnog očekivanja iz Tm. -a 3.42!]